

Chapter 9. Growth through Technological Progress

UMSL

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Facts: Positive Trend Growth Rate

- Ramsey (1928) World, economic growth assumed zero.
 - in real GDP growth rate.
 - "Classical": Ricardo, J.S. Mill, & Marx uses zero growth world.
 - Also famous: Malthus (1798) Essay on Principle of Population, population will grow so as to keep per-capita output constant (no growth).
 - Ramsey's (1928) in line with trend growth rate experience, at the time
- Industrial revolution started in Great Britain around 1750's,
 - with positive growth now normal in developed economies.
- Brought about rethinking of facts of economy.
 - With growth in real GDP, growth in real consumption
 - and in real investment had to keep pace real GDP.
 - This way have stable shares of cons. & inv. in GDP.
- So add: that growing variables all grow at same rate.

Stylized Growth Rate Facts

- 1: output grows at some steady rate.
- 2: All variables that grow over time grow at same rate as GDP output:
 - output y , consumption c , investment i , & capital stock k & wage w .
 - Gives the *trend growth rate* of economy.
- Trend growth rate depends on time period and country.
 - 2% commonly used for US past century; 3.5% US Post-1959.
- Europe & Japan; developed countries similar growth trend to US.

Europe, Japan, US Post WWII Growth Rates

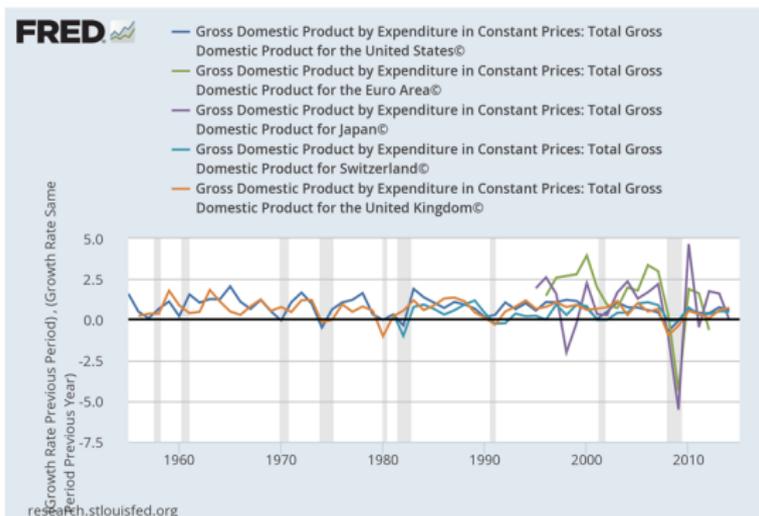


Figure: Real GDP Growth Rate, annual basis, for Four Industrial Nations Plus the Euro-zone Countries.

Solow Growth Facts

- Stylized growth facts explained by Solow.
- Solow starts with constant growth rate, such as 2%.
- Adds real wage growing over time.
- Real interest rate stable over time.
- Rising real wage goes with rising per capita income.
- Stable real interest rate goes with output to capital ratio being constant.
- Uses theory of marginal products of capital & labor.
 - Marginal product of capital is set fraction of y/k :
 - so output to capital ratio is constant since
 - Real interest rate equals marginal product of capital.
 - Real wage is Marg Product of labor=fraction of y/l .
 - So y/l growing implies w growing.

Given Constant Output Growth Rate: 4 facts

- 1 Real wage w rises steadily over time at real output growth rate.
- 2 Real interest rate r remains constant on average over time.
- 3 Per capita income, or output divided by total labor supply, rises at same output growth rate.
- 4 Output to capital stock ratio remains constant on average over time.

Harberger & Falling Relative Cost of Output

- Harberger (1998) argues: real cost of goods falls over time
 - due to Solow-type technological progress.
- Harberger says "real cost reduction" lowers
 - percent of real GDP used up in production.
- Capture this is by comparing Ramsey-Solow cost of goods
 - (and capital) relative to labor.
- *Steady technological progress* makes real price of good relative to labor lower over time (Harberger, 1988).
- CPI index divided by the nominal wage rate gives real relative price of goods to labor,
 - so $1/w$ should fall over time.
- Rise of real wage over time, one of Solow's Facts, automatically gives
 - fall of real price of goods relative to labor over time.

Solow Plus Growth Facts

- 1 The relative price of output to labor falls over time.
 - 1 Real price of output relative to labor, $1/w$, falls over time.
 - 2 Is satisfied if Solow Growth Facts are met, as include rising wage w .
- 2 Average hours worked per week very gradually decrease.
- 3 Time spent in education steadily rises over time.
- 4 Time spent for leisure falling over time.

Theory: Zero Growth to Positive, Constant, Growth

- Allow for exogenous technological progress.
- Stated in terms of productivity parameter of output production function
 - that increases at constant rate over time.
 - Causes variables to grow at constant growth rate.
- Just assume but do not explain productivity increase.
 - May say progress due to research & development,
 - to human capital accumulation, or rising aggregate "knowledge".
- Model known as *exogenous growth model*.
- Simple extension of Ramsey World to Ramsey-Solow World.
- Productivity parameter A now rises at steady rate.
- "Solow Growth theory": explains 4 stylized Solow growth facts.

Facts along a Balanced Growth Path Equilibrium

- Equilibrium called "*balanced growth path*" equilibrium.
 - output, consumption, investment & capital all grow at same rate.
- To explain business cycles: used changes in two parameters,
 - output productivity & time endowment.
 - Interpreted these changing two main resources:
 - goods & time endowments.
- Apply same two "*comparative static*" changes for growth process.
- Productivity increase used to explain 4 Solow growth facts.
- Time endowment change added to productivity increase
 - to explain additional 4 Solow Plus growth facts,
 - while maintaining original four Solow growth facts.

Growth Puzzle that Solow Solved

- All variables growing at same rate raises question:
 - How can output grow if labor cannot.
- Labor constrained by stationary, or constant, time endowment.
 - If labor share of time continually grows, no leisure time left.
 - consumer would not choose to allow this.
- If capital alone grows, with labor constant,
 - then marginal product of capital gets increasingly small.
 - Follows from diminishing product of labor:
 - Adding more capital causes smaller increase in output.
- Diminishing marginal product: no constant rate of growth in output
 - unless capital grows exponentially.
 - But faster capital growth than output growth
 - inconsistent with balanced growth equilibrium.
 - Also inconsistent with Growth fact: constant output to capital ratio.
- Problem solved by Solow with "augmented-labor"
 - & capital inputs growing by same amount,
 - with productivity parameter growth "augmenting" labor.

A Comparative Static Productivity Increase

- *Comparative static* change of Solow's Growth theory
- is increase in productivity.
- Here illustrate this initially with single 5% increase
- in production function productivity parameter A .
- Causes marginal productivity of both labor & capital to increase.
- Marginal cost of goods falls, AS curve shifts out:
- more output supplied for each relative price $1/w$.
- Higher wage from higher marginal product of labor,
- consumer gets more capital wealth
- & increases demand for consumption; AD curve shifts out.
- "Supply Side Economics": AS shifts out by more than AD ; falling $1/w$.

Productivity Increase Shifts Out AS & AD

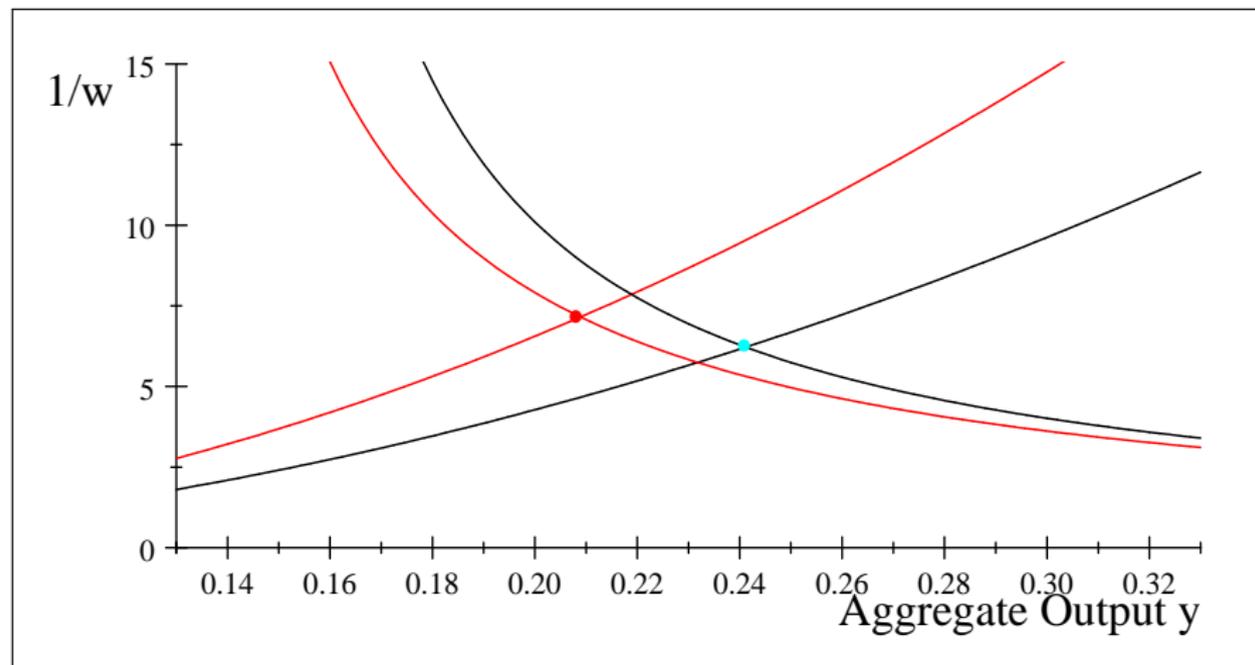


Figure: AS – AD Equilibrium with Goods Productivity Increase (in Black) as Compared to the Original (in Red).

Productivity Change Effect on Labor Market

- Labor market: demand shifts out for labor
- because higher marginal productivity of labor.
- Supply shifts back because consumer has higher capital wealth k ,
- so unwilling to work as much at any given wage rate.
- Net effect: equilibrium quantity of labor unchanged,
- consumer working same amount of time,
- while real wage rises.

Real Wage Up, Employment Unchanged

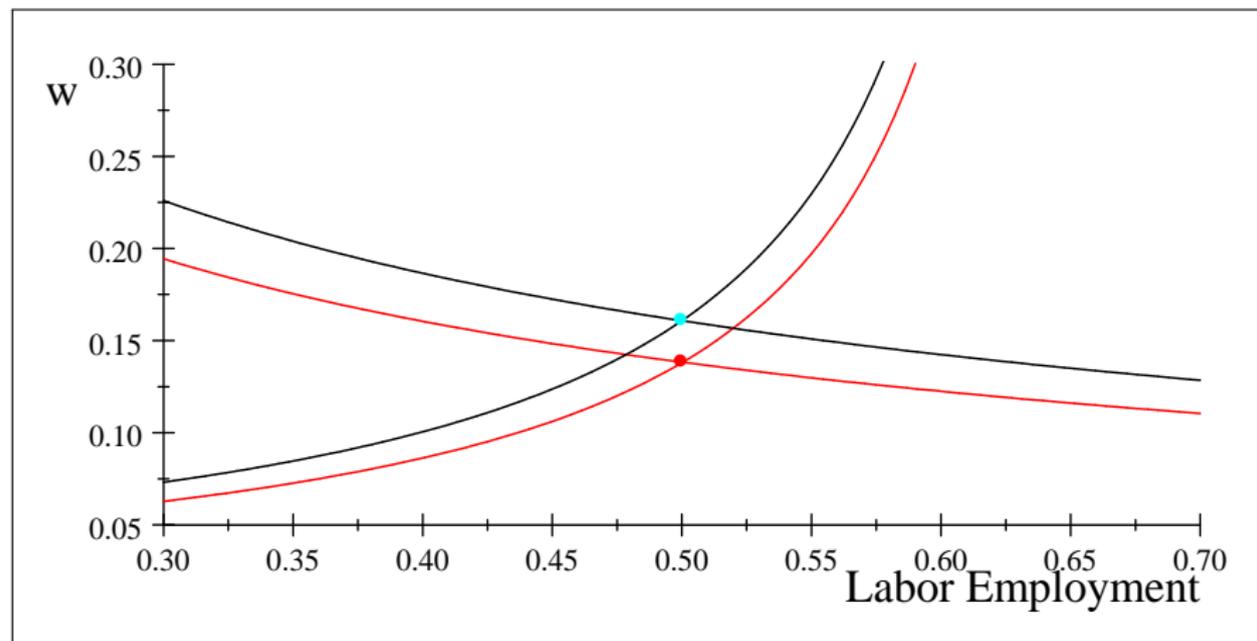


Figure: Increase in Productivity (Black Curves) Raises w and Leaves Employment Unchanged.

Real Interest Unchanged, Investment Up

- Capital market: marginal product of capital rises
- & demand for capital shifts out
- while interest rate remains fixed (at 6% in the example).
- Equilibrium capital k & investment (δk) rise.
- Fixed r because of exogenous growth assumption.
 - Interest rate fixed in equilibrium by assumed growth rate:
 - this is fact of Ramsey-Solow Exogenous growth theory.
- Note: With extension to human capital, productivity rise is
 - endogenous, as is growth rate, & real interest rate is not fixed.

Productivity Change Effect on Capital Market

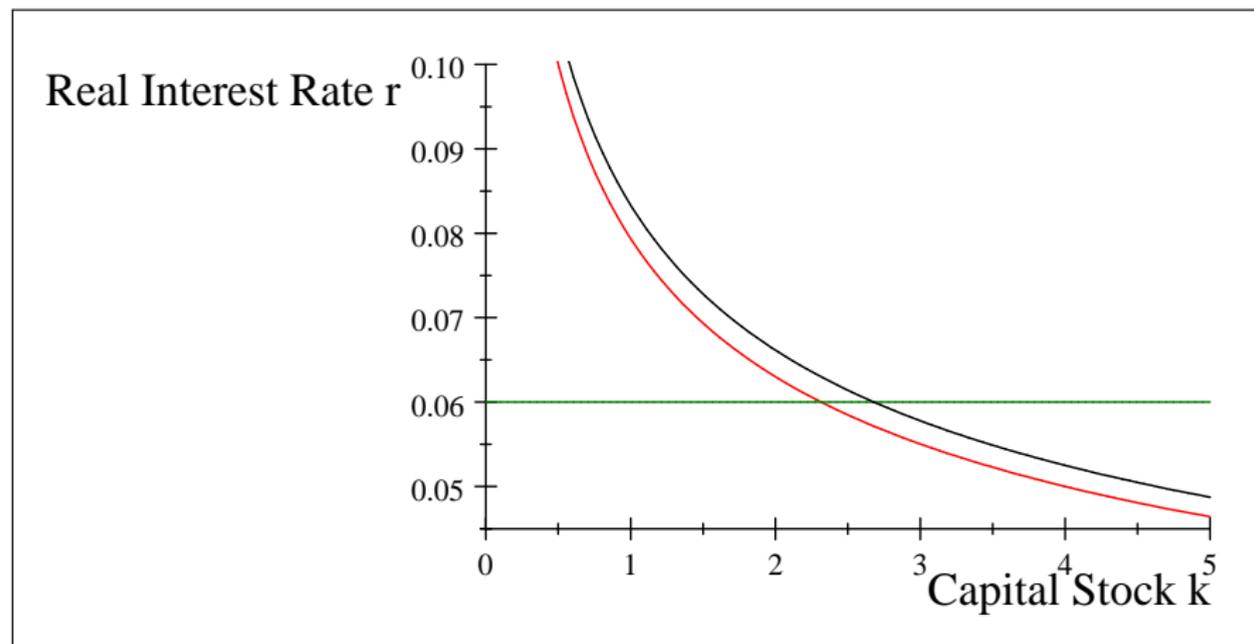


Figure: Capital Market Shows Shift Out of Demand for Capital (Black Curve) When Factor Productivity A Increases 5% from Original value (Red Curve).

Productivity Effect on Utility & Production

- Consumption & output rise by same amount
 - so "propensity to consume", or conversely savings rate, remain same. Here consumption rate is two-thirds: $c/y = 0.67$
- Production function pivots up;
- utility level rises, indifference curve shifts up.
- Budget line actually pivots upwards,
- since its slope rises: w/r is slope & w rises, r stays same.
- Labor employment unchanged at 0.5.

Productivity Pivots Up Production Function

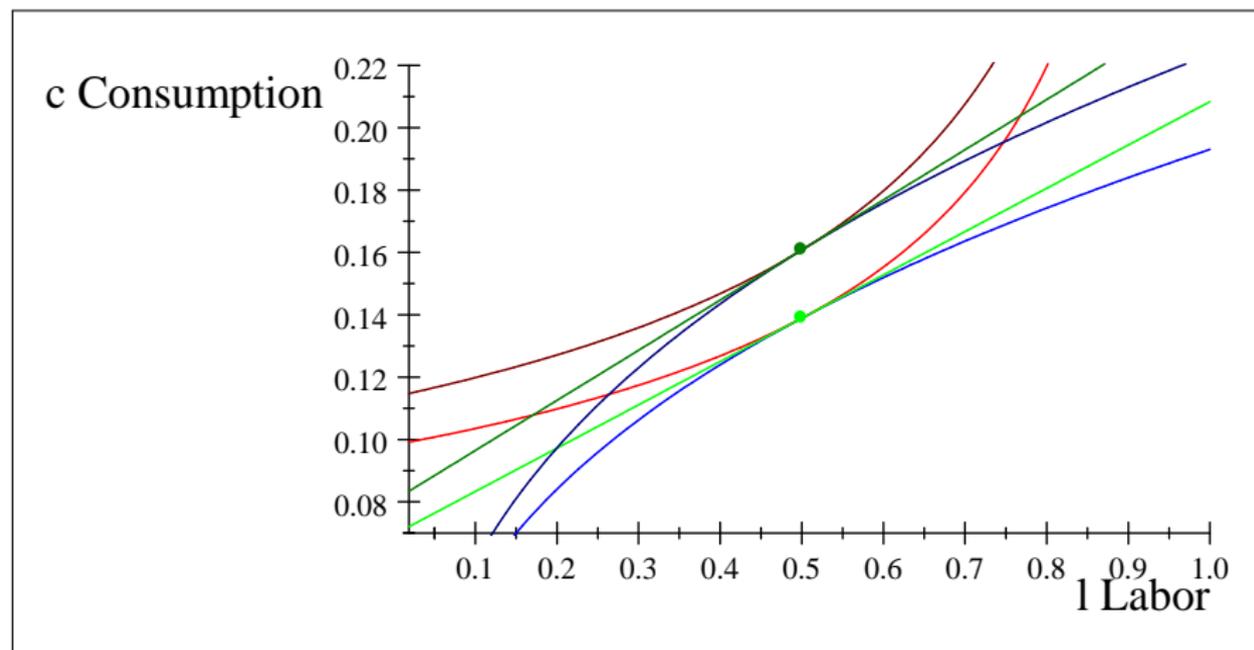


Figure: General Equilibrium Production, Utility Levels and Budget Lines with Goods Productivity Increase (Darker Curves) Compared to Baseline (Lighter Curves).

Factor Input Diagram

- Factor input market equilibrium isoquant & isocost lines,
- along with factor input ratios, at original equilibrium
- & after 5% productivity increase.
- Isoquant level curve (Blue) shifts up as output level increases,
- isocost line (Red) pivoting up as slope of w/r increases,
- factor input ratio of k/l (Green) rises as capital stock increases
 - and labor is unchanged.

Isoquant, Isocost, Factor Ratio: Productivity Effect

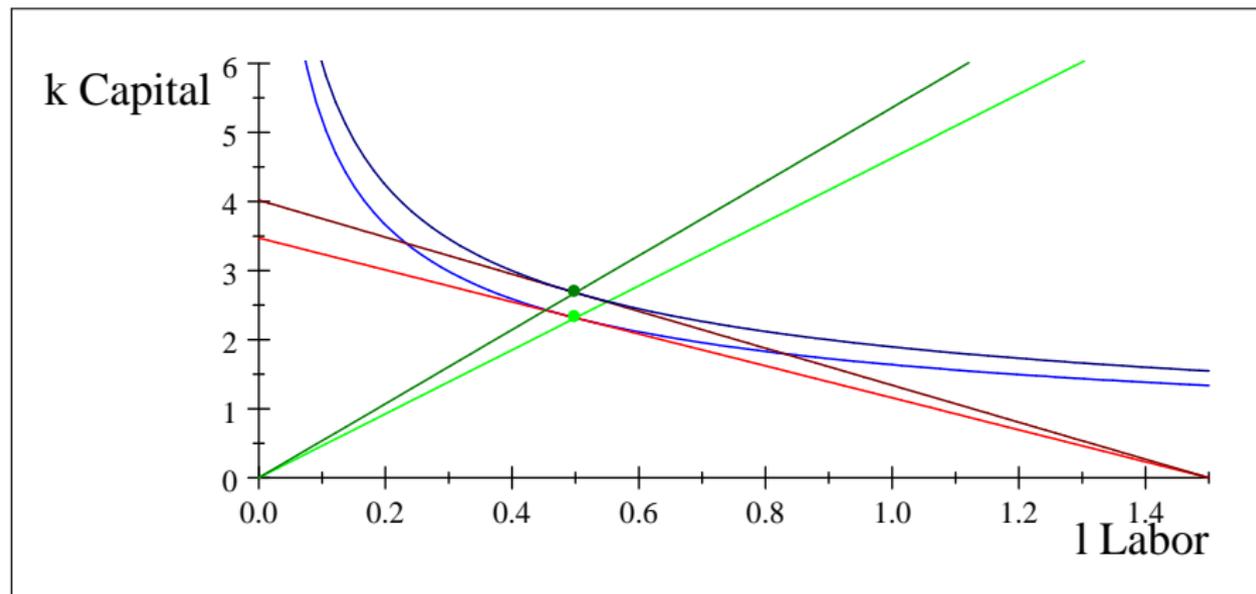


Figure: Factor Market Equilibrium with Goods Productivity Increase (Darker Curves) Compared to Original (Lighter Curves).

AS-AD with Solow Growth

- Continual increase in productivity parameter A
 - with targeted constant growth rate of output of 2%.
- Growth illustrated over 4 time periods in goods & labor markets,
 - using $AS - AD$ & supply, demand in labor market (First time).
 - Equilibrium output & wage rate rises,
 - labor employment remains unchanged.
 - $AS - AD$ & labor market depend on equilibrium capital stock k
 - with k rising each period.
- Ramsey-Solow growth theory means optimization methods
 - combined with technological progress of Solow.
 - Full Ramsey-Solow optimization World:
 - in 1965 by Cass & Koopmans independently.
 - "Ramsey-Cass-Koopmans (RCK) model of neoclassical growth."
- Add trend down in time endowment to productivity increase.
 - Captures time used up by trend upwards in education time.
 - Explains how labor time trends downwards slightly.
 - Still explains other standard growth facts: eg. rising wage.

Solow Growth from Ramsey World

- Real wage w rises;
- output to capital ratio y/k constant
- since both output y & capital k grow at same rate;
- output to labor ratio, per-capital income, y/l ,
- grows since output y is growing
- & time allocation to labor l constant.

AS-AD with Continuous Technological Progress

- Productivity parameter increase at constant rate.
- Aggregate supply continually shifts out
- by more than aggregate demand each period.
- Consistent with one-time increase in productivity
- used to explain business cycle expansion.
- AS shifts out by more than AD
- & real price of goods relative to labor $1/w$ falls.
- & with rising output y , investment (δk) & capital stock k

AS-AD with Solow Growth Productivity Increases

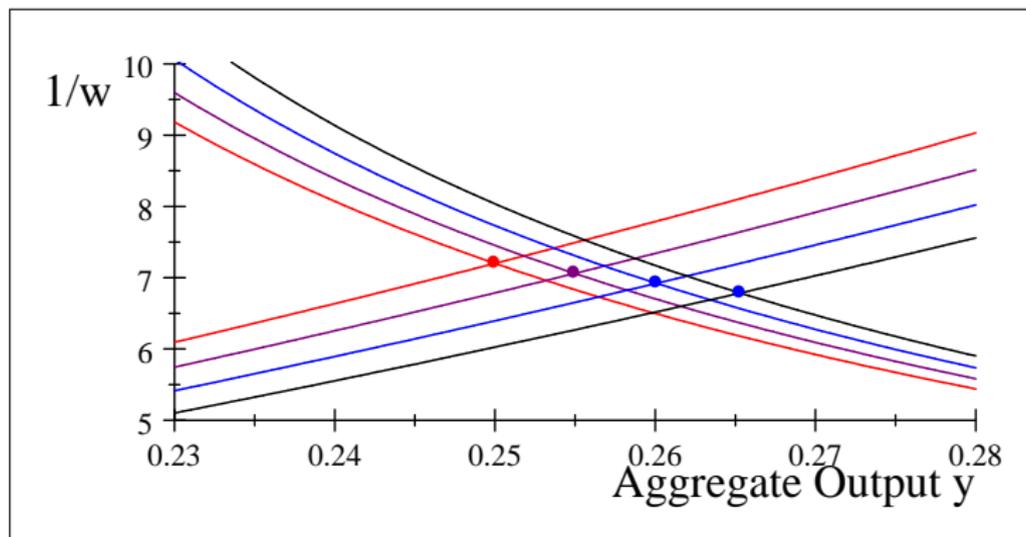


Figure: AS – AD Equilibria Over Time With 2% Exogenous Growth Example; Moving From Red to Black Curves over 4 years.

Labor Market: Solow Growth Productivity Increases

- Labor supply & labor demand shift over time,
- Labor demand shifts out \longrightarrow
- & labor supply shifts back \longleftarrow .
- Leaves labor employment unchanged,
- & wage rate rising.

Real Wage Rises, Employment Unchanged

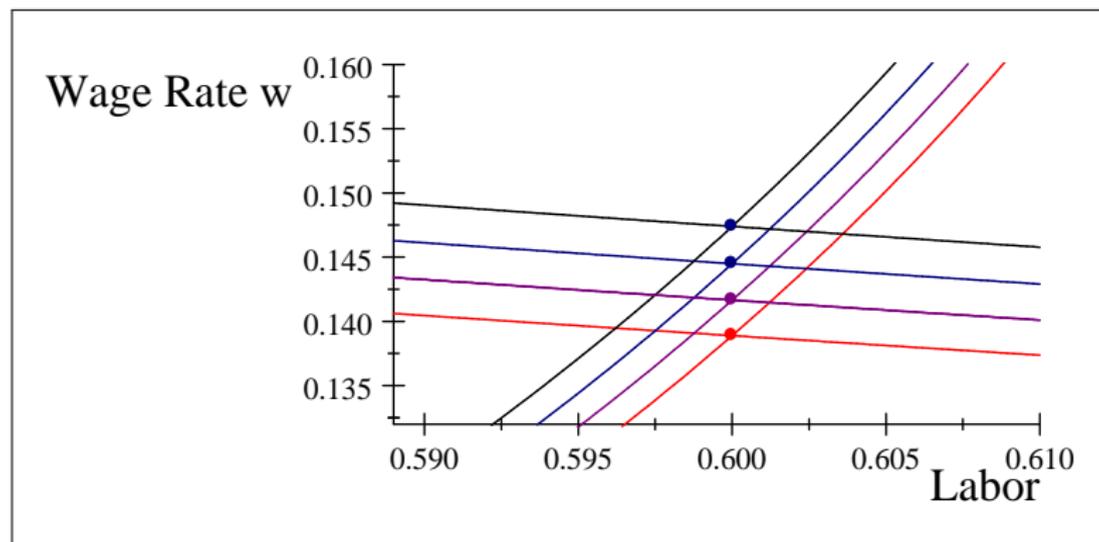


Figure: Labor Market with 2% Exogenous Growth and Rising Real Wage, Constant Employment, Over Time.

Isoquant, Isocost, Factor Input Ratio

- Capital & labor input ratios in isoquant-isocost diagram.
- Labor unchanged as capital k rises by 2% per year,
- equilibria with curved isoquant shifting up
- as indicating that output level is rising.
- Isoquant shows combo of labor & capital for given output level.
- Slope of isocost line is $-w/r$.
- Slope gets steeper as real wage w rises & r .
- Factor input ratio of capital to labor k/l rises,
- as do slopes of lines colored Green.

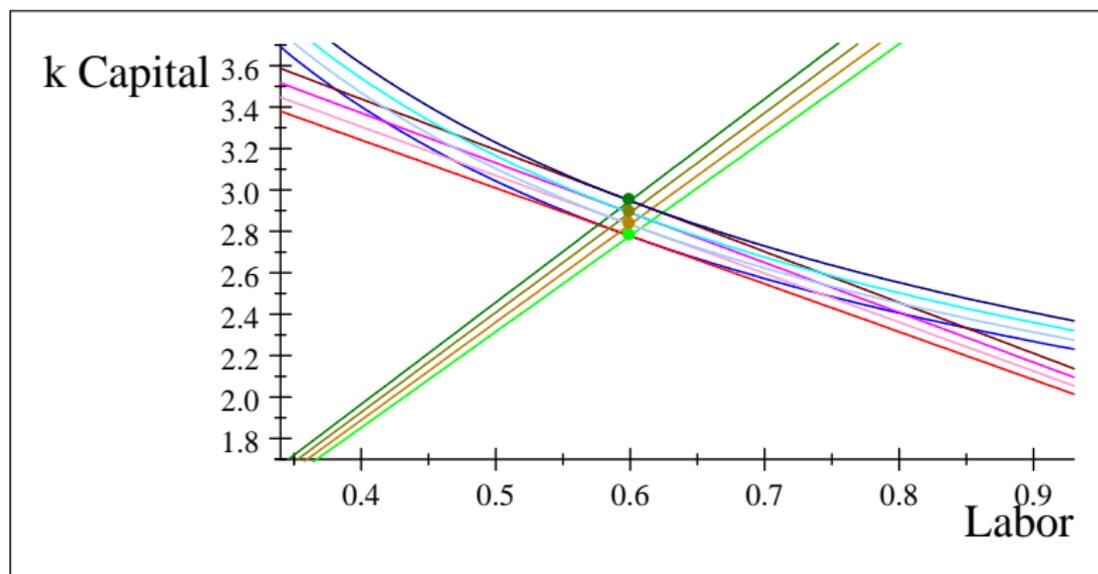
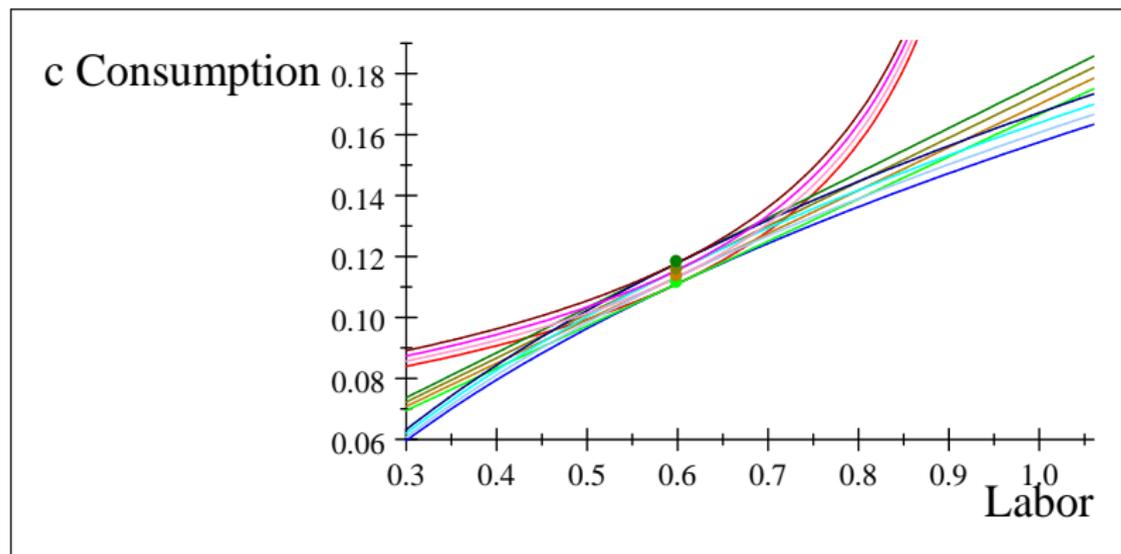


Figure: Factor Market Equilibrium with 2% Exogenous Growth, rising Capital to Labor Ratios, Real Wages and Output Levels, and a Constant Employment Rate and Real Interest Rate.

Production and Utility Levels

- Consumption rises, & labor unchanged.
- Utility level shifts up from Red to Brown;
- production function pivots up, in Blue.
- Budget line shifts up, in Green.



Exogenous Growth with Rising Production, Consumption and Utility Levels and Constant Employment.

Solow Facts in Figures

- Rising wage rate w seen in Figures;
 - constant real interest rate r seen in Figures,
 - rising output y seen in Figures,
 - with a constant labor l seen in Figures.
- Comprise 3 of Solow's growth facts
 - of wage rate rising,
 - interest rate being constant,
 - & output per labor rising.
- 4th is output to capital ratio constant.
 - Established by y & k growing at same rate.
- Also see Solow Plus Harberger Fact:
 - falling relative price of output $1/w$.
 - As wage rises.

Application: Adding a Trend Down in Time Endowment

- Assume time endowment T falls very slightly by 0.18% per year.
 - Matches in Ramsey-Solow World decline in work time
 - by 12% over 40 years, an estimate found for US.
- Continual decrease in T allows us to explain
 - 1. very slowly falling hours of work per week in developed countries.
 - Leaves real wage rising & real price of goods to labor falling.
 - 2. Leisure slightly falls as well.
- Interpretation: increase in our time spent in education over lifetime.
- Balanced path output growth rate now 1.8%, lower than 2%
 - because time endowment is falling.

Real Price of Goods Still Falling, Output Growing

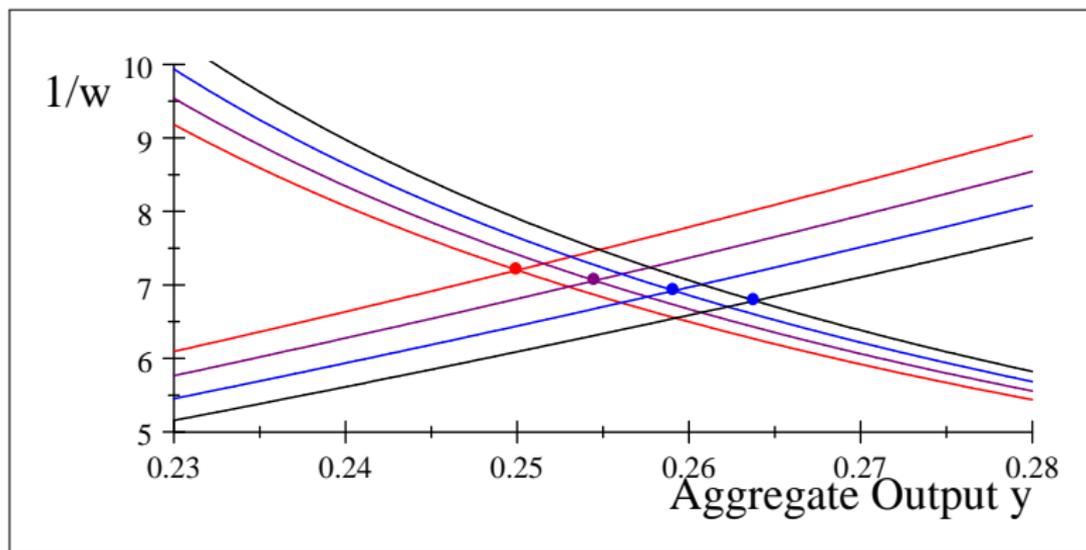


Figure: AS – AD Equilibria With Productivity A Trending Up and Time T Trending Down.

Labor Market with Productivity Rise, T fall

- Labor employment l falls slightly over time,
- while wage rate w rising.
- Falling employment time represents Solow Plus growth fact
- of slightly falling working hour week over time.
- Leisure time also falls (not shown).

Wage Rate Rising, Slight Decline in Hours

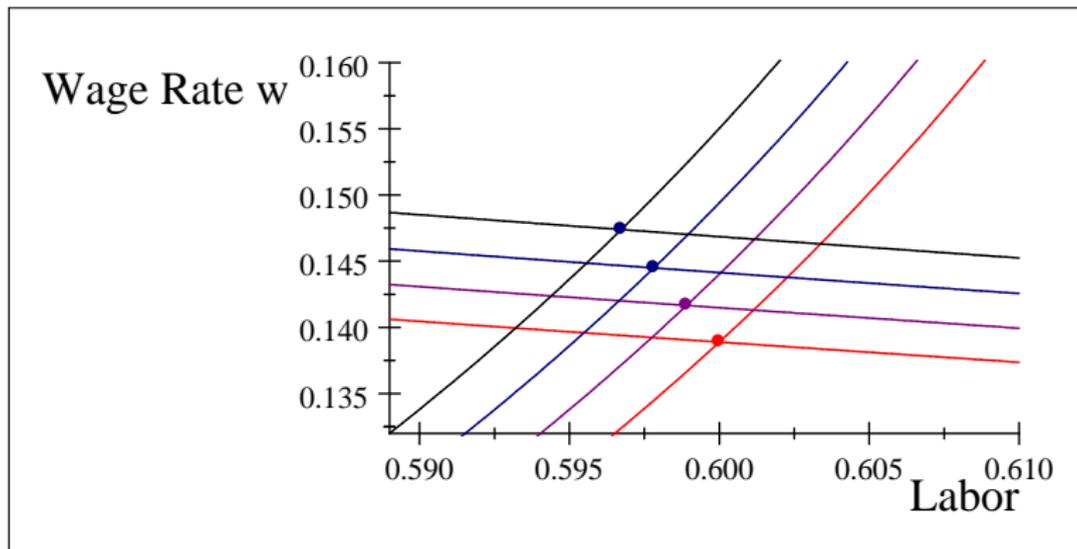
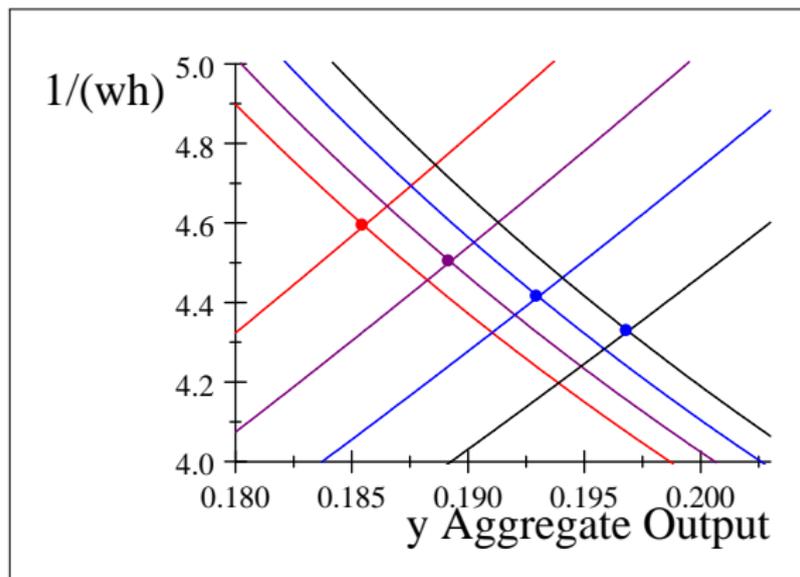


Figure: Labor Market with A Trending Up and T Trending Down.

Appendix: Endogenous Growth with Human Capital

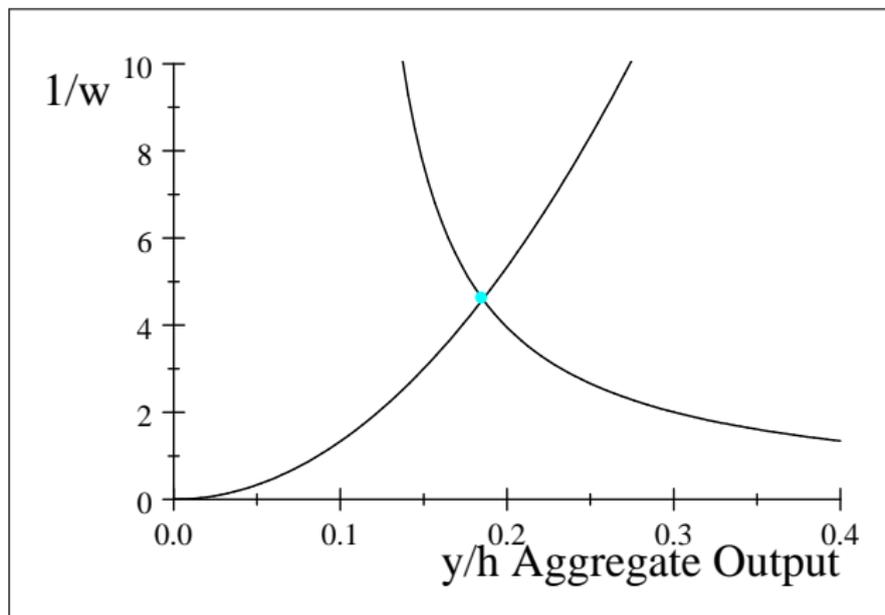
- Ramsey World with human capital accumulation.
 - No exogenous technological change now.
- Instead human capital h rises,
 - "augmenting" our raw Labor, so augmented labor grows
 - at same rate as physical capital.
- Solves balanced path growth rate endogenously,
 - based on resources spent accumulating Human capital.
 - No parameters depend on time but still sustained growth rate.
- Call this "*Ramsey-Lucas World*" with "effective wages"
 - augmented by human capital level: or wh .
- Can graph the $AS - AD$ with y & $1/wh$, or y/h & $1/w$.
- Same return on human & physical Capital drives equilibrium.

Solow Growth Facts with Endogenous Growth



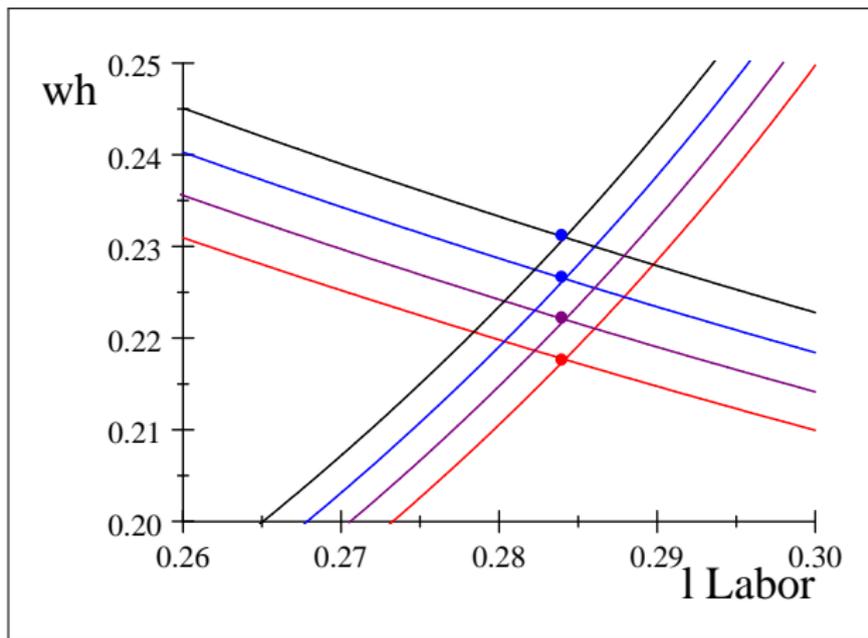
Endogenous Growth shifts in $AS - AD$ over Four Time Periods.

Stationary AS-AD with $1/w$ and y/h



AS – AD with Human Capital and Endogenous Growth with the Raw Real Wage w on the Vertical Axis and Output/Human-Capital on the Horizontal Axis

Labor Market with Human Capital



Labor Market with Endogenous Growth as Graphed with the Effective Wage on the Vertical Axis.

Human Capital and Solow Growth Facts

- If assume exogenous rise in productivity of human capital sector,
- then replicate rest of Solow Plus growth facts.
- Time spent working falls very gradually.
- Time spent in education gradually rises.
- Time spent in leisure falls.
- And: growth rate of economy will gradually rises.
- Captures gradual rise in growth of Industrial Revolution.

Summary

- Stylized growth facts set out.
 - Solow's theory explains, put in Ramsey's World:
 - Solow's steady rise in productivity parameter for output.
 - Explain facts using $AS - AD$.
- Comparative static increase in productivity conducted.
 - AS shifting out by more than AD , $1/w$ falling.
- Then constant Productivity increase each period.
 - produces Solow growth facts using $AS - AD$ & labor market.
- Add trend down in time endowment for Solow Plus growth facts.
- Extension to include human capital,
 - get Ramsey-Lucas endogenous growth,
 - with Solow Growth Facts.
- Steady rise in productivity of human capital sector (education)
 - gives Solow-Plus growth facts with this single parameter change,
 - continuously over time.

Questions

- 1 Explain the stylized facts of economic growth known as the Solow Growth Facts.
- 2 What additional facts are the "Solow Plus" growth facts that we also are able to explain?
- 3 Explain what happens to labor employment over time with the only change being that goods productivity steadily rises.
- 4 Explain what happens to output and consumption over time with the only change being that goods productivity steadily rises.
- 5 What variables move together on a balanced growth path equilibrium and what variables do not change?

Rest of Questions

- 1 What happens to output and labor employment if both the goods productivity factor rises over time and the time endowment falls slightly over time?
- 2 Offer an explanation of why hours per week trend down over the long historical period in developed economies while at the same time the time spent in education trends upwards.
- 3 Graph the labor market when both goods productivity rises and the time endowment falls over time, both at a constant rate.
- 4 How does endogenous human capital accumulation change the explanation of any of the Solow growth facts?
- 5 How does endogenous human capital accumulation change the explanation of any of the Solow Plus growth facts?