Tax Evasion, Human Capital, and Productivity-Induced Tax Rate Reduction

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Growth in the human capital sector's productivity explains in part how US postwar growth and welfare could have increased while US tax rates declined. Modeling tax evasion within an endogenous growth model with human capital, an upward trend in goods and human capital sectors gradually decreases tax evasion and allows for tax rate reduction. Using estimated goods and human capital sectoral productivities, the model explains 30 percent of the actual decline in a weighted average of postwar US top marginal personal and corporate tax rates. The productivity increases are asymmetric in a fashion related to that of McGrattan and Prescott.

I.   Introduction

It is well known that postwar US top marginal personal and corporate income tax rates have trended downward while US government revenue as a proportion of GDP has remained stable. Figure 1 (solid line) shows that a weighted average of the top US marginal personal (x's) and corporate (squares) income tax rates fell from 75 percent to 35 percent from 1951 to 2012. Figure 1 also shows that federal tax revenue as a per-
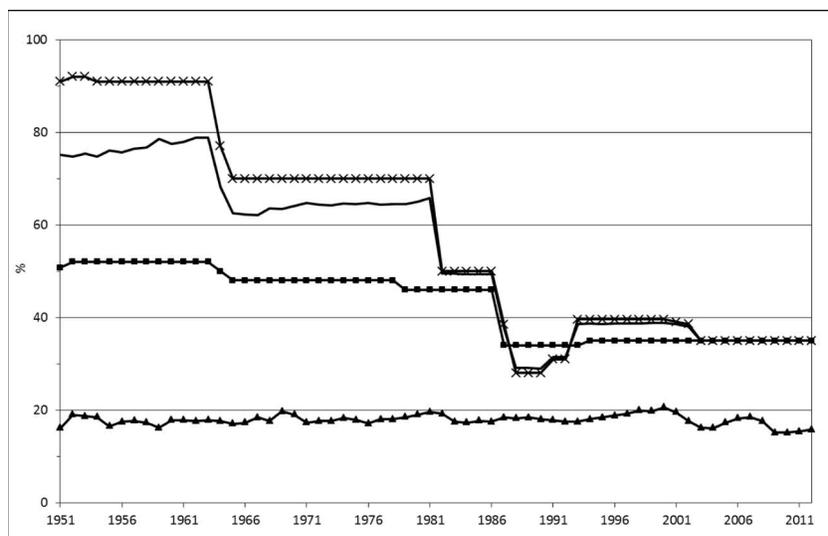
Figure 1.—US postwar marginal personal and corporate income tax rates, 1951–2011. Sources for the top marginal tax rates: Tax Foundation tables, U.S. Federal Individual Income Tax Rates History, 1913–2013 (Nominal and Inflation-Adjusted Brackets) and Federal Corporate Income Tax Rates, Income Years 1909–2012. Weights are calculated using shares in OMB historical table 2, Percentage Composition of Receipts by Source: 1934–2018. Tax receipts as a fraction of GDP are from OMB historical table 1.3; 2012 is their estimate.

centage of GDP (triangles) varied little from its average value of 18 percent over this postwar period. Similarly, the average personal income tax rate for the top 0.5 percent of taxpayers (accounting for an estimated 31.67 percent of federal personal income tax receipts in 2010) fell from 56 percent to 34 percent between 1960 and 2004, the average US corporate income tax rate fell from 52 percent to 27 percent between 1951 and 2011, and the weighted average declined from about 52 percent to 33 percent from 1960 to 2004 (average rates are shown in Sec. A of App. A, fig. A1, and online App. B).[1] Such tax trends are also found for the United Kingdom.

The incentive effect of top marginal tax rates, or the average rate on the highest-income taxpayers, is stressed by many from McGrattan (2012) to Saez, Slemrod, and Giertz (2012).[2] Besides documenting the decline in US postwar tax rates, Saez et al. also document a more than doubling of the share of the top 1 percent of US income earners from a steady 8 percent

[1] The historical contribution of the personal income tax to total US taxes was around 7 percent in 1949 and in 2012 with some variation across time, while the corporate tax share of taxes has fallen some from about 4 percent to 2 percent from 1949 to 2012. See App. tables B1–B5 for details.

[2] McGrattan (2012) focuses on how higher tax rates on dividend income, paid mainly by those in higher income brackets, can help to explain key facets of the Great Depression, while, e.g., Romer and Romer (2010) and Cloyne (2013) find that tax cuts have a large impact on output.

level from 1960 to 1981 to 18 percent in 2006, with acceleration upward after the 1981 tax reductions and after the 1986 tax reductions. Saez et al. find this income share rise puzzling as it apparently is not explained by "real" factors such as labor supply response. They conclude instead that the cause is that tax evasion, or avoidance, decreased as the tax rate fell and the "tax base" of reported income rose.[3] As Thornton (2012, 449) puts it, "Higher marginal tax rates also provide a stronger incentive to go 'underground.'" With evasion a part of the explanation, the elasticity of reported income to the tax rate would be expected to be higher than if there were no evasion. Saez et al. also present evidence (their table 1) showing the existence of high estimated elasticities of the income share of the top 1 percent to the tax rate after the US 1986 tax act, as consistent with high elasticity evidence in Mertens and Ravn (2013, forthcoming). High elasticities appear difficult to explain theoretically in standard models without tax evasion, such as those reviewed by Saez et al.[4]

This paper models tax evasion, explains how reported income elasticities to tax rates are higher by making the tax rates a function of the degree of evasion, and provides an analysis of how postwar US tax rates may have fallen. Exploiting the role of human capital investment is key to our explanation of the downward US tax rate trend while also implying rising growth and welfare. Assuming flat rates of taxes on capital and labor income, assuming a constant share of government revenue as a percentage of income (Lucas 2000, secs. 4, 5), and modeling tax evasion in a general equilibrium with human capital investment, we show that tax rates decrease as sectoral productivity rises. Tax evasion creates a higher elasticity (magnitude) of the share of taxable income compared to no evasion that depends on the ratio of unreported to reported income. Increases in productivity cause a lesser degree of evasion, a lower tax rate elasticity of taxable income, and given the constant share of tax revenue in output, a decrease in the tax rate. Estimated postwar productivity trends for the goods and human capital investment sectors are based on the Baier, Dwyer, and Tamura (2006) database. With this evidence the US calibrated model can explain 30 percent of the figure 1 downward trend in postwar US tax rates. This fraction is more than 30 percent if we define the empirical tax rate decline more narrowly, such as found in Saez et al. But this fraction and our ability to explain the postwar tax trend downward drop significantly without the human capital investment sector and endogenous growth.

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[3] "While such policy options may have little impact on real responses to tax rates (such as labor supply or saving behavior), they can have a major impact on responses to tax rates along the avoidance or evasion channels" (Saez et al. 2012, 42). Slemrod and Weber (2012) review the evasion literature.

[4] In between the US 1981 and 1986 tax reforms was the lesser-known US Tax Reform Act of 1984, which broadened the tax base by ending a decade-long congressional deadlock on Internal Revenue Service determination of nonstatutory fringe benefits; the act specified exactly how a variety of such benefits should be taxed by the IRS.

II.   Methodology and the Role of Human Capital

The representative agent economy is a human capital–based "second-generation" endogenous growth economy as in Lucas (1988) but without externalities, with flat taxes as in King and Rebelo (1990), and with a wasteful activity as related to the political capital for corruption in Ehrlich and Lui (1999), except that here this activity takes the form of a decentralized tax evasion service.[5] The consumer's degree of tax evasion determines the curvature of the tax revenues per unit of output as graphed against the tax rate, and this curvature in turn translates directly into the elasticity of the reported income share relative to the tax rate.[6] Increases in goods and human capital productivities reduce the degree of evasion and induce a lower tax rate in order to keep the share of government revenue in output constant while increasing the stationary time spent in human capital investment and stationary growth and welfare.

The human capital sector is key for four main reasons. First, the estimated human capital productivity increase causes a larger simulated tax rate decline, by itself, than does the estimated goods sector productivity increase. This results in the sense that the human capital productivity increase is found to be five times larger than that coming from the goods sector while having half the effect as that of the goods sector productivity in lowering tax rates per unit of productivity increase. Second, the goods sector productivity increase taken by itself causes a significantly larger decrease in the tax rate in the endogenous growth baseline model with human capital investment as compared to a similarly calibrated exogenous growth model in which the human capital grows exogenously. Therefore, the model with human capital investment provides a fuller explanation of the actual US downward tax trend both because it includes human capital productivity increases along with goods sector productivity increases and because the latter have a stronger effect on tax rates within the human capital–based endogenous growth economy than in the exogenous growth economy. Third, the estimated productivity increases as combined with the implied tax reduction also imply that the time spent in human capital gradually rises over time, consistent with how average time spent in education apparently has trended upward. Fourth, only within our human capital investment model do we find rising stationary growth and welfare from the productivity increases.

The way the evasion service works is that reported and unreported income are perfect substitutes for buying goods once the unreported income has been "laundered" through the competitive evasion intermediary. This intermediary has a rising marginal cost of evasion so that as the tax rate increases, there is a greater waste of resources lost to evasion ac-

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[5] What we call tax evasion in terms of avoiding legal taxes can also be interpreted to include avoidance through various means that lower the effective tax rate.

[6] At a given tax rate, the tax elasticity equals the slope of the output-normalized tax revenue "Laffer curve," as Agell and Persson (2001) call it, divided by the slope of a ray from the origin, or the marginal change divided by the average change.

tivity, comparable to the resource cost of the political capital investment time lost to corruption in Ehrlich and Lui (1999). Given perfect substitutability between reported and laundered unreported income, the rising marginal cost of the evasion service conversely is shown to imply an equilibrium outcome similar to a rising tax elasticity of reported income. This gives a bigger tax elasticity than without evasion at any given tax rate, as episodal evidence may suggest. This tax elasticity is also such that the higher it is, the greater the reduction in evasion that results from a given productivity increase and the greater the subsequent decrease in tax rates given a constant share of tax revenue in output. This mechanism when viewed more broadly implies that there is a link between low tax evasion and high productivity. This link is consistent with lower tax evasion being widely found in developed countries and higher evasion in developing countries (Schneider and Enste 2000) since higher productivity is found in developed countries and lower productivity in developing countries (Klenow and Rodríguez-Clare 1997; Hall and Jones 1999). And the link of less evasion with lower tax rates is consistent with the movement toward lower flat taxes as designed to broaden the tax base, for example, as seen starting in 1993 in Eastern Europe.[7]

Estimates of upward trends in sectoral productivities are widespread in the Solow growth accounting/real business cycle framework for the goods sector and are emphasized, for example, in terms of human capital by Baier et al. (2006) and Guryan (2009). Beaudry and Francois (2010) emphasize how standard growth accounting has understated the role of human capital productivity. McGrattan and Prescott (2010) present an alternative growth accounting framework that adds in the productivity of the intangible capital sector, which has been interpreted in part as including human capital. We similarly use extended growth accounting to estimate the productivities of the goods and human capital sectors, using the data set of Baier et al. for the growth rate of output, physical capital, and human capital (see App. A, Sec. E).

In Section III, the tax evasion model is first presented with only physical capital, as an *Ak* model. The paper analytically derives the *Ak* elasticity features and shows how productivity increases tax revenue per GDP at any given tax rate and so induces tax rate reduction. As it quantitatively cannot account for human capital sectoral productivity trends, Section IV extends

[7] Tax evasion here occurs in a similar manner to the inflation tax avoidance in the literature going back to Bailey (1956) and Cagan (1956); in both of these there is a rising interest elasticity of money demand as the inflation rate rises, supported empirically in international money demand evidence by Mark and Sul (2003). Tax evasion takes place in a competitively decentralized market (see also Becker 1968; Ehrlich 1973), analogous to inflation tax avoidance through a decentralized competitive exchange credit intermediary in Benk, Gillman, and Kejak (2010), in which there is an equilibrium relation of a rising interest elasticity of money demand. This rising inflation tax elasticity feature is the basis of Cagan's (1956) explanation of hyperinflation, Gillman and Kejak's (2005) explanation of the negative long-run inflation-output growth relation found in evidence, and Eckstein and Leiderman's (1992) explanation of Israeli inflation tax revenue.

the model with a human capital investment sector and Cobb-Douglas production for both the goods and human capital sectors; this is similar to how McGrattan and Prescott (2010) extend their basic model to account for increases in its productivity in the intangible investment sector. Section V provides the US calibration, and Section VI presents the simulation results of how productivity changes affect evasion, tax rate reduction, growth, and welfare. Section VII estimates postwar growth rates in the goods and human capital investment productivities and uses these in turn to estimate what proportion of the observed downward trend in US postwar tax rates can be explained by the extended economy. Section VIII provides discussion, and Section IX presents conclusions.

III. *Ak* Model with Evasion

In this representative agent economy, the consumer invests in physical capital k_t and rents it to the goods-producing firm and to the intermediary that provides tax evasion services. With the share of capital going to the goods sector denoted by s_{Gt} and that going to the evasion sector by s_{Et}, we have that $s_{Gt} + s_{Et} = 1$. With r_t the competitively determined rental price of capital goods, the rental income that the consumer receives is $r_t(s_{Gt} + s_{Et})k_t = r_t k_t$. The representative agent places deposits d_t, equal to all income $r_t k_t$, into the intermediary:

$$d_t = r_t k_t. \tag{1}$$

By choosing the fraction of income to report to the tax authority, $a_t \in [0, 1]$ (similarly to Fullerton and Karayannis [1994]), the household pays taxes on $a_t r_t k_t$ and demands tax evading services for the income equal to the remainder, $(1 - a_t) r_t k_t$. The statutory tax rate on capital income is τ_k and the competitive market price for the tax evasion service in per-unit terms is denoted by p_{Et}. The income that evades tax net of the price of evasion is $(1 - p_{Et})(1 - a_t) r_t k_t$. However, as the agent owns the intermediary, the profit produced by the evasion intermediary is paid back to the consumer in the form of a return per unit of deposits, denoted by r_{Et}, and thus total profit returned to the consumer is $r_{Et} d_t$. This makes the actual average cost of evasion less than p_{Et} once the intermediary's dividend payments are accounted for. Using the sum of after-tax reported income, after-evasion unreported income, and dividends from the intermediary, the agent decides how much new investment to make in capital, denoted by i_t, and the level of goods consumption c_t. Assuming a depreciation rate of δ_K on capital, the capital accumulation equation is

$$\dot{k}_t = i_t - \delta_K k_t. \tag{2}$$

The representative consumer also receives a government transfer, denoted by v_t; hence the representative consumer's budget constraint is

$$\dot{k}_t = (1 - \tau_k)a_t r_t k_t + (1 - p_{Et})(1 - a_t)r_t k_t + r_{Et}d_t - c_t$$
$$- \delta_K k_t + v_t. \tag{3}$$

The representative consumer derives utility only from consumption goods, c_t, and maximizes lifetime utility $V(k_0)$ at time 0:

$$V(k_0) = \max_{c_t, a_t, d_t, k_t} \int_0^\infty \ln c_t e^{-\rho t} dt \tag{4}$$

subject to the deposit (1) and budget (3) constraints given the initial capital stock k_0.

The production of the output of goods, denoted by y_{Gt} and with $A_G > 0$, is a linear function in only the physical capital allocated to the goods sector $(s_{Gt}k_t)$:

$$y_{Gt} = A_G s_{Gt} k_t. \tag{5}$$

In this Ak model, the representative agent as goods producer takes the price of capital services, r_t, as given and maximizes profit Π_{Gt} by choosing the capital input:

$$\max_{s_{Gt}k_t} \Pi_{Gt} = A_G s_{Gt} k_t - r_t s_{Gt} k_t, \tag{6}$$

so that in equilibrium $r_t = A_G$.

The government receives tax revenue $a_t \tau_k r_t k_t$ from reported capital income, it transfers the lump sum v_t to the consumer, and it consumes an amount Γ_t:

$$a_t \tau_k r_t k_t = v_t + \Gamma_t. \tag{7}$$

The intermediation sector produces the tax evasion service that enables the consumer to report only a fraction of the capital income; it is owned by the representative agent, just as in the goods producer. A Leontief one-to-one "household" production technology is implicitly assumed such that a unit of the tax evasion service and a unit of "laundered" income are combined to yield a unit of untaxed income that the consumer can use for goods purchases. Therefore, the quantity of evasion services, denoted by κ_t, equals the quantity of unreported income: $\kappa_t = (1 - a_t)r_t k_t$.[8]

The intermediary takes as given prices p_{Et} and r_t and maximizes profit Π_{Et}, which equals total revenue $p_{Et}\kappa_t$ minus the rental costs of capital used in producing the intermediation service, $r_t s_{Et} k_t$, and minus the dividend

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[8] A related Leontief approach is formalized in Gillman and Kejak (2005, eqq. 8–11), as based on Becker's (1965) household production technology.

payouts on the income deposits $r_{Et}d_t$. There is zero profit after paying out the residual dividend income:[9]

$$\max_{s_{Et}k_t,d_t} \Pi_{Et} = p_{Et}\kappa_t - r_t s_{Et}k_t - r_{Et}d_t. \tag{8}$$

Note that the consumer owns the intermediary because, as with a mutual bank, each dollar deposited buys an ownership share in which the price per share is fixed at one.[10]

Given $\omega_E \in [0, 1)$, the technology of the intermediary's tax evasion service is assumed to be constant returns to scale (CRS) in its inputs of physical capital and deposited funds (a form of "financial" capital; see Berger and Humphrey 1997):

$$\kappa_t = A_E\left(s_{Et}k_t\right)^{\omega_E}\left(d_t\right)^{1-\omega_E}. \tag{9}$$

Per unit of deposits, the production function, $\kappa_t/d_t = A_E(s_{Et}k_t/d_t)^{\omega_E}$, exhibits diminishing returns to the normalized capital input factor and so has an upward-sloping marginal cost per unit of deposits and a unique equilibrium.[11]

The first-order conditions imply that the cost of capital equals its marginal product, $r_t = p_{Et}\omega_E A_E(s_{Et}k_t/d_t)^{\omega_E-1}$; that the residual return on deposits equals its marginal product, $r_{Et} = p_{Et}(1 - \omega_E)A_E(s_{Et}k_t/d_t)^{\omega_E}$; and that the unique solution for the normalized input ratio is $s_{Et}k_t/d_t = (\omega_E A_E p_{Et}/r_t)^{1/(1-\omega_E)}$. Substituting this ratio into the production function in equation (9) yields the equilibrium ratio of tax evasion dollars to deposits:

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[9] The evasion intermediation activity, or avoidance more broadly, can be viewed as taking place in a branch of the firm, in a small segment of the banking sector, or in other ways whereby the income is reprocessed into nontaxable income through an intermediary (see also Gillman and Kejak 2011). This income can be from legal enterprises or criminal industries such as drugs, trafficking, and illegal arms trade; presumably most large sums of both legal and illegal income are deposited in banks. Tax evasion through banks is the focus of ongoing US Congress hearings and continuous news reports. See, e.g., Minority Staff of the Permanent Subcommittee on Investigations Report on Correspondent Banking, "A Gateway for Money Laundering," February 5, 2001, Senate Hearing 107-84; the report appears in vol. 1 of the hearing record entitled "Role of U.S. Correspondent Banking in International Money Laundering" held on March 1, 2, and 6, 2001. Also, the *Wall Street Journal* (2013) reports a "detailed account of a system the bank allegedly helped put in place to allow some wealthy French people to evade taxes."

[10] We assume that dividends are not taxed since the total value added of the intermediary equals the factor income $s_{Et}r_tk_t$ that is used up in production, and this is already subject to full taxation (although some is evaded); taxation of dividends $r_{Et}d_t$ would amount to a type of double taxation that we prefer to avoid in this context.

[11] With $\omega_E = 1$, it can be shown that no unique equilibrium exists. See also Sealey and Lindley (1977), Clark (1984), Hancock (1985), and Gillman and Kejak (2005, 2011) for the intermediation approach. The "productivity" parameter is shorthand for the myriad of factors affecting the evasion that Becker (1968) and Ehrlich (1973, 1996) detail more generally for participation in an illegal activity. Our service production is for the evasion industry itself, while Ehrlich's (1973) production function is for a certain probability of the good state (apprehension of criminal activity).

$$\kappa_t / d_t = A_E (\omega_E A_E p_{Et} / r_t)^{\omega_E / (1 - \omega_E)}.$$

Given $\kappa_t = (1 - a_t) r_t k_t$ and $d_t = r_t k_t$, this implies an equilibrium fraction of unreported income of

$$1 - a_t = A_E (\omega_E A_E p_{Et} / r_t)^{\omega_E / (1 - \omega_E)}.$$

One can rewrite these equilibrium conditions to show that the marginal cost of the evasion service (MC) is equated to the price p_{Et}, with MC defined as the marginal factor price divided by the marginal factor product:

$$p_{Et} = r_t / \left[\omega_E A_E (s_{Et} k_t / d_t)^{\omega_E - 1} \right] \equiv \text{MC}. \tag{10}$$

A.   Equilibrium

A competitive equilibrium for this economy consists of a set of allocations $\{c_t, a_t, k_t, s_{Gt}, s_{Et}, d_t\}$, a set of prices $\{r_t, p_{Et}, r_{Et}\}$, the government's policy $\{\tau_k, v_t, \Gamma_t\}$, and the initial condition k_0 such that (i) given $r_t, p_{Et},$ and $r_{Et},$ the consumer maximizes utility $V(k_0)$ in equation (4) with respect to $\mathbf{u}_t \equiv (c_t, a_t, d_t, k_t)$ subject to the deposit constraint (1) and the budget constraint (3); (ii) given $r_t,$ the goods-producing firm maximizes profit Π_{Gt} in (6), with respect to $s_{Gt} k_t$; (iii) given $r_t, r_{Et},$ and $p_{Et},$ the intermediary maximizes its profit Π_{Et} in (8) subject to (9) with respect to $s_{Et} k_t$ and d_t; (iv) the government budget (7) is always satisfied; and (v) all markets clear at given prices.

B.   Balanced Growth Path Growth and Welfare

Along the balanced growth path (BGP), the variables $k_t, c_t, y_{Gt}, \kappa_t,$ and d_t grow at the constant rate g and remain time indexed while the other shares and factor prices, $s_G, s_E, a, r,$ and $r_E,$ are constant.

PROPOSITION 1.   A necessary condition for an interior solution for the fraction of reported income $a \in (0, 1)$ is that the competitive equilibrium price of tax evasion services for capital income tax evasion equals the tax rate:

$$p_E = \tau_k. \tag{11}$$

Proof.   This follows directly from the consumer's first-order condition with respect to the fraction of reported income, $a.$[12] QED

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[12] The first-order condition with respect to a_t implies that if $p_E > \tau_k,$ then $a = 1$ and the consumer will report the whole income and not use any tax evasion services; excluding the case $a = 0$ rules out having no taxes paid.

A competitive equilibrium market price for illegal evasion services that is equal to the tax rate relates to the literature of Becker (1968) and Ehrlich (1973, 1996), with an analogous result found in inflation tax theory (Gillman and Kejak 2005).[13]

Solving for $s_{Et}k_t/d_t$ from equation (9) and using that $\kappa_t/d_t = 1 - a$, $s_E k_t/d_t = [(1 - a)/A_E]^{1/\omega_E}$ results. Substituting this input ratio back into (10), using that $p_E = \tau_k$ and that $r = A_G$, gives an upward-sloping MC in price-quantity space $(\tau_k, 1 - a)$:[14]

$$\tau_k = A_G(\omega_E A_E^{1/\omega_E})(1 - a)^{(1-\omega_E)/\omega_E} \equiv \text{MC}. \tag{12}$$

When plotted, the area under the MC curve represents the total resource cost of tax evasion, $rS_E k_t$, while the producer surplus that is returned to the consumer is the dividend, $r_E d_t$. At the margin the cost of using reported income to purchase goods is equal to the cost of using unreported income for the same purpose, with the key incentive being to optimally evade the income tax. Solving for a from equation (12), we get

$$a = 1 - A_E(\omega_E A_E \tau_k/r)^{(1-\omega_E)/\omega_E}. \tag{13}$$

The higher the tax rate, the lower the equilibrium fraction of income that is reported. While the consumer is a competitive price taker with an infinitely elastic demand for evasion at the price $p_E = \tau_k$ as in proposition 1, the equilibrium outcome for $a(\tau_k)$ is a steady-state relation that is a "downward-sloping" function of the price.

The parameters of the evasion intermediary technology tie down what we will call the BGP "equilibrium tax rate elasticity for the reported income," or just tax elasticity for short, in a precise fashion. To see this, define the economy's total income as the value added from both goods and evasion intermediary sectors, so that $y_t \equiv s_G r k_t + s_E r k_t = r k_t$, and derive the elasticity with it denoted by $\eta_{\tau_k}^a$.

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[13] Ehrlich (1996) notes that he does "not necessarily mean a physical setting where such illegitimate transactions are contracted" (45), but in general where "a person's decision to engage in an illegal activity i can be viewed as motivated by the costs and gains from such activity" (46). He focuses not on the implicit tax caused by the law itself but rather on expenditure to reduce the benefit of evading the law. This approach also gives a tax equals market price result, but it is with respect to the margin of the protection/enforcement activity: "Private self-protection and public law enforcement set a 'price,' or 'tax,' on criminal activity by reducing the marginal net return to the offender" (51). We take a more primitive, related, approach by explicitly modeling the market for an illegal activity but abstract from more detailed modeling of the protection/enforcement activity by reflecting the outcome of all such activity in the productivity parameter of the evasion intermediary sector: the statutory tax rate reduces the marginal return to reporting income and induces "offending" in the form of evading the tax up until the marginal cost of the share of unreported income equals the tax rate itself.

[14] See, e.g., Ehrlich (1973, eq. 2.2), the "aggregate supply curve of offenses," with eq. (3.1) being normalized to the rate of offense; we use a similar normalization in that $1 - a$ is the percentage of income not reported.

PROPOSITION 2.   The elasticity of the taxable income as a share of total income relative to the tax rate equals

$$\eta_{\tau_k}^a \equiv (\partial a/\partial \tau_k)(\tau_k/a) = -[(1-a)/a][\omega_E/(1-\omega_E)] \leq 0;$$

it approaches zero as τ_k approaches zero and a approaches one.

Proof.   Given that taxable income equals the reported income of ay_t and that as a share of total income the ratio of taxable to total income is $ay_t/y_t = a$, take the derivative with respect to the tax rate using equation (13), and the proof follows. QED

COROLLARY 3. $\partial \eta_{\tau_k}^a / \partial \tau_k = -[\omega_E/(1-\omega_E)]^2 (1-a)/(\tau_k a^2) < 0.$

The absolute value of the tax elasticity rises (becomes more negative) as τ_k increases and a falls, with marginally more substitution toward unreported income. The consumer in equilibrium becomes increasingly sensitive to the tax and substitutes away from it through greater use of the evasion service.[15] This means that the elasticity rises at an increasing rate as the tax rate rises, as related to Cagan (1956) and Gillman and Kejak (2005) inflation tax elasticity features.[16]

The tax elasticity also affects how the BGP growth rate g responds to the tax. To see this, consider that the "after-evasion effective tax rate" is less than the actual tax rate because the intermediary returns $r_E d_t$ to the consumer as dividends. Defined here as the statutory rate τ_k minus r_E, this effective rate in the BGP equilibrium is given by $\tau_k - r_E = \tau_k - [\tau_k(1-\omega_E)(1-a)] < \tau_k$, where $1-a$ is given by equation (13). It can be shown that the effective rate rises as τ_k rises, falls as evasion productivity A_E rises, and falls as goods sector productivity A_G rises. Also, it can be rewritten as a weighted average of the unit cost of reported and unreported income, with weights a and $1-a$, and with the average cost when reporting income equal to τ_k and when not reporting income equal to $\tau_k \omega_E$; that is, $\tau_k a + \tau_k \omega_E (1-a)$.[17]

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[15] An increase in the statutory tax rate also increases the elasticity of substitution between reported income and unreported income, defined as

$$\varepsilon \equiv \{\partial [a/(1-a)]/\partial [\omega_E \tau_k (A_E)^{1/\omega_E}/A_G]\}\{[a/(1-a)]/[\omega_E \tau_k (A_E)^{1/\omega_E}/A_G]\},$$

with the relative price being $\omega_E \tau_k (A_E)^{1/\omega_E}/A_G$, where $\varepsilon = -[\omega_E/(1-\omega_E)]/a = \eta_{\tau_k}^a/(1-a)$. A price-theoretic form of Alfred Marshall's factor input laws results: $\eta_{\tau_k}^a = \varepsilon(1-a)$ (see Layard and Walters 1978; Gillman and Kejak 2005).

[16] This is analogous to the consumer in monetary theory taking the nominal interest rate (the inflation tax) as given while facing a downward-sloping money demand per unit of consumption (e.g., Lucas 2000; Gillman and Kejak 2005). With m denoting the money income ratio and r the interest rate, Lucas writes, "Let $m(r)$ denote the m value that satisfies (3.7), expressed as a function of the interest rate. Throughout the paper, it is this kind of steady state equilibrium relation $m(r)$ that I call a 'money demand function,' and that I identify with the curves shown in Figures 2 and 3" (256). In Gillman and Kejak (2005, 2011), eq. (12) is analytically synonymous with a "Baumol" (1952) type condition that equalizes the marginal cost of the different exchange means of money and interest-bearing bank deposits while optimally avoiding the inflation tax, and from which the money demand equilibrium relation is derived.

[17] To compute the average cost of unreported income, divide the capital rental cost for evasion production by the quantity of evasion services produced, or $r s_E k_t / \kappa_t$. Since the share of

Tax evasion lowers this "effective tax rate" because of the lower average cost of unreported income, which in turn allows for a higher BGP rate of growth g.

The BGP growth rate g depends on the effective tax rate, given by

$$g = r(1 - \tau_k + r_E) - \delta_K - \rho. \tag{14}$$

It can be shown that $\partial g / \partial \tau_k = -ra$, with

$$\partial^2 g / \partial \tau_k^2 = r[(1 - a)/\tau_k][\omega_E/(1 - \omega_E)] > 0.$$

The growth rate falls at a decreasing rate as τ_k rises except when $a = 1$.

The welfare effect of including evasion is shown by deriving the BGP welfare W:

$$W \equiv \int_0^\infty \ln c_t e^{-\rho t} dt = \frac{1}{\rho} \{\ln k_0 + [\ln(c_t/k_t) + (g/\rho)]\}. \tag{15}$$

From equations (3) and (7), $c_t/k_t = \rho + \tau_k ra$. Since $\partial(c_t/k_t)/\partial \tau_k = ra(1 + \eta_{\tau_k}^a)$ and $\partial g / \partial \tau_k = -ra$, the effect of τ_k on welfare is simply $\partial W / \partial \tau_k = -ra[1 - \rho(1 + \eta_{\tau_k}^a)]/\rho$. An increase in A_E lowers the effective tax rate and so increases g and W.

Imposing the assumption that government tax revenue is a constant share of output effectively endogenizes the tax rate and changes the welfare effects of increasing evasion productivity. Given the consistency of this assumption with the US empirical trend, for the rest of the paper we assume a constant share of revenue and denote this share by $\gamma \in (0, 1)$. With $y_t \equiv s_G rk_t + s_E rk_t = rk_t$ and with government revenue v_t transferred back to the consumer in lump-sum fashion, the government budget constraint becomes

$$v_t + \Gamma_t = \tau_k ark_t = \gamma y_t. \tag{16}$$

Given $y_t = rk_t$, this implies that $\tau_k = \gamma/a$, where $a \leq 1$ and $\tau_k \geq \gamma$, with $A_E = 0$, $a = 1$, and $\tau_k = \gamma$.

PROPOSITION 4. Given equation (16) and $|\eta_{\tau_k}^a| < 1$, a marginal increase in A_E decreases welfare W.

See Appendix A, Section B, for a proof. A more productive evasion sector requires a higher tax rate in order to keep revenue the same fraction of output, causing growth and welfare to fall as resources are increasingly used up in tax evasion. This result is analogous to the BGP loss of re-

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capital in evasion sector output is the factor cost divided by the value of evasion output, or $rs_E k_t/(\tau_k \kappa_t) = \omega_E$, then $rs_E k_t/\kappa_t = \tau_k \omega_E < \tau_k$.

sources, growth, and welfare in Ehrlich and Lui (1999) when their productivity of producing political capital increases.[18]

C.   Revenue Curve, Tax Rate, and Productivity Change

Following Agell and Persson (2001), we now derive the relation of the tax rate to the total tax revenue per unit of output. In the BGP equilibrium, revenue per output is simply $a\tau_k(A_G k_t)/y_t = a\tau_k$. When $a\tau_k$ is graphed against τ_k, the peak occurs at $\eta^a_{\tau_k} = -1$.

Assuming $\omega_E = 0.72$, $\delta_k = 0.07$, $A_E = 0.46$, $A_G = r = 0.176$, $\rho = 0.02$, and $\gamma = 0.31$ as is similar to our calibration for the extended model detailed in Section V below, figure 2 graphs $a\tau_k$ as the solid line; with $A_E = 0$, the straight 45-degree dashed ray results. A 10 percent increase in A_G causes an increase in the ratio of tax revenue to output at any given tax rate as seen in the dashed curve. As long as $\gamma = 0.31$ intercepts the baseline curve to the left of its peak, then when the curve pivots upward because A_G increases, the rate τ_k needs to be reduced to keep $\gamma = 0.31$. This result would not follow without tax evasion as on the 45-degree line. The possible tax reduction becomes smaller for a given A_G increase as the tax elasticity falls (in magnitude).

PROPOSITION 5.   Under the condition of the fixed share of tax revenues in output, γ, a marginal increase in A_G causes a decrease in the statutory tax rate τ_k as given by $d\tau_k = -\{A_G[(1/|\eta^a_{\tau_k}|) - 1]\}^{-1} dA_G$. For $|\eta^a_{\tau_k}| < 1$, that is, being on the upward-sloping part of the normalized tax revenue curve, the size of the decrease in the statutory tax rate is smaller, the smaller the elasticity of reported income $|\eta^a_{\tau_k}|$.

Proof.   See Appendix A, Section C.

IV.   Extension with Human Capital Investment

Productivity increases are empirically documented to be significant in both goods and human capital investment sectors. Therefore, the qualitative result that a goods productivity increase allows for a lower tax rate can be better quantified by extending the economy to include human capital investment. Then productivities of both sectors can be included in simulation results of an economy calibrated for postwar US data.

The extended economy consists of three sectors. The goods sector produces output, the human capital sector produces gross investment in human capital, and both sectors use CRS production with inputs of physical capital and human capital, with the human capital sector more human capital intensive than the goods sector. The third sector is evasion intermediation that uses CRS production with inputs of physical capital k_t, human capital h_t, and deposited income d_t.

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[18] Ehrlich and Lui (1999, S274) have a parameter related to our γ in their θ, which is the "portion of all transactions subject to government intervention."
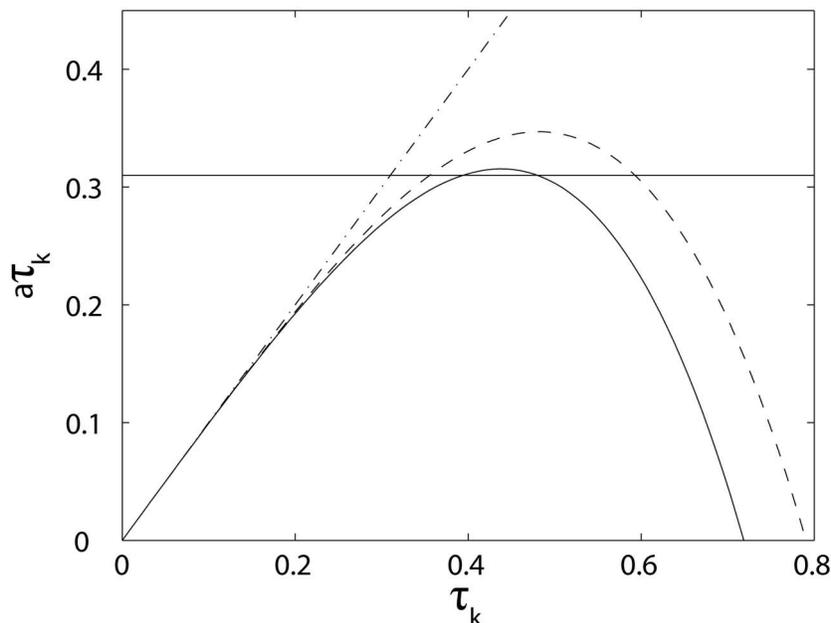
Figure 2.—Tax revenue normalized by output and τ_k

The representative consumer allocates one unit of time across work in goods production, l_{Gt}, in human capital investment, l_{Ht}, and in evasion, l_{Et}, with leisure time, x_t, the residual; $l_{Gt} + l_{Ht} + l_{Et} = 1 - x_t$. The quantity of human capital input in three sectors is $l_{Gt}h_t$, $l_{Ht}h_t$, and $l_{Et}h_t$. Similarly, the share of physical capital allocated to goods production is denoted by s_{Gt}, the share to human capital production by s_{Ht}, and the share to the evasion intermediary sector by s_{Et}; $s_{Gt} + s_{Et} + s_{Ht} = 1$. The quantity of physical capital input in each sector is $s_{Gt}k_t$, $s_{Ht}k_t$, and $s_{Et}k_t$.

With productivity parameters $A_G > 0$ and $A_H > 0$, labor shares $\beta \in (0, 1)$ and $\varepsilon \in (0, 1)$, where $\beta > \varepsilon$, and the depreciation rate of human capital given by $\delta_H \in [0, 1]$, let goods output be denoted by y_{Gt} and its production function be given by

$$y_{Gt} = A_G(l_{Gt}h_t)^\beta (s_{Gt}k_t)^{1-\beta}. \tag{17}$$

Let the gross production of human capital investment be given by

$$\dot{h}_t + \delta_H h_t = A_H[(1 - x_t - l_{Gt} - l_{Et})h_t]^\varepsilon [(1 - s_{Gt} - s_{Et})k_t]^{1-\varepsilon}. \tag{18}$$

While the human capital investment sector is a "home production" sector in this representative agent framework, the goods and evasion sectors are decentralized such that the consumer rents human capital to them at the wage rate w_t and physical capital at the rate r_t. Human capital

income is thereby $w_t(l_{Gt} + l_{Et})h_t$, and capital income is $r_t(s_{Gt} + s_{Et})k_t$. In order to avoid taxes, the consumer reports again only a fraction a_t of the human and physical capital income, where we denote this income by

$$y_t \equiv w_t(l_{Gt} + l_{Et})h_t + r_t(s_{Gt} + s_{Et})k_t.$$

The consumer also pays the fee p_{Et} per unit of evasion service, κ_t, which in turn by the implicit Leontief technology is equal to the quantity of income that evades taxes, $\kappa_t = (1 - a_t)y_t$. The taxes in the economy are the proportional tax rates on capital income, τ_k, and on labor income, τ_l. The consumer also receives dividends from the evasion intermediary at the rate of r_{Et} per unit of deposits d_t and a government transfer of v_t. Income is used for gross physical capital investment, $\dot{k}_t + \delta_K k_t$, where the depreciation rate for physical capital is given as $\delta_K \in [0, 1]$, and for consumption goods purchases c_t. The budget constraint is

$$
\begin{aligned}
\dot{k}_t = \; & a_t[(1 - \tau_l)w_t(l_{Gt} + l_{Et})h_t + (1 - \tau_k)r_t(s_{Gt} + s_{Et})k_t] \\
& + (1 - a_t)(1 - p_{Et})[w_t(l_{Gt} + l_{Et})h_t + r_t(s_{Gt} + s_{Et})k_t] \\
& + r_{Et}d_t + v_t - \delta_K k_t - c_t.
\end{aligned}
\tag{19}
$$

The first term on the right-hand side shows the reported income on which taxes are paid and the next term the usable unreported income after paying the fee p_{Et} to the evasion intermediary. The household deposits in the evasion intermediary are equal to its total income, as given by

$$d_t = w_t(l_{Gt} + l_{Et})h_t + r_t(s_{Gt} + s_{Et})k_t. \tag{20}$$

Given (k_0, h_0), $\alpha > 0$, and $\rho \in (0, 1)$, the representative consumer maximizes lifetime welfare $V(k_0, h_0)$:

$$V(k_0, h_0) = \max_{\{c_t, x_t, d_t, k_t, h_t, l_{Gt}, l_{Et}, s_{Gt}, s_{Et}, a_t\}_{t=0}^{\infty}} \int_0^{\infty} (\ln c_t + \alpha \ln x_t)e^{-\rho t}\,dt,$$

subject to the human capital accumulation constraint (18), budget constraint (19), and deposit constraint (20); see Appendix A, Section D, for the first-order conditions.

The government receives taxes, spends (unproductively) on government consumption Γ_t, and returns the rest as a transfer, v_t, such that

$$a_t[\tau_l w_t(l_{Gt} + l_{Et})h_t + \tau_k r_t(s_{Gt} + s_{Et})k_t] = \Gamma_t + v_t.$$

Additionally, assume that the size of government consumption is a fixed fraction $\gamma \in [0, 1]$ of the value of market output such that

$$\Gamma_t + v_t = \gamma y_t. \tag{21}$$

The goods-producing firm takes r_t and w_t as given, maximizes revenue minus cost, and has the first-order conditions

$$w_t = \beta A_G (s_{Gt} k_t)^{1-\beta} (l_{Gt} h_t)^{\beta-1},$$
$$r_t = (1 - \beta) A_G (s_{Gt} k_t)^{-\beta} (l_{Gt} h_t)^{\beta}.$$

The competitive intermediary is owned by the consumer and maximizes profit Π_{Et} subject to its production function. With $A_E > 0$, $\omega_l \in (0, 1)$, $\omega_k \in (0, 1)$, and κ_t denoting the amount of evasion services provided to the consumer by the evasion intermediary, the production function for these services is given by

$$\kappa_t = A_E (l_{Et} h_t)^{\omega_l} (s_{Et} k_t)^{\omega_k} d_t^{1-\omega_l-\omega_k}. \tag{22}$$

The quantity of evasion services corresponds to the quantity of unreported income "laundered" into income that can subsequently be used to purchase goods. Implicitly assuming a Leontief production function, combining one unit of the evasion service with one unit of laundered income yields that

$$\kappa_t = (1 - a_t)[(s_{Gt} + s_{Et}) r_t k_t + (l_{Gt} + l_{Et}) w_t h_t].$$

The intermediary problem is

$$\max_{\{l_{Et} h_t, s_{Et} k_t, d_t\}} \Pi_{Et} = p_{Et} \kappa_t - w_t l_{Et} h_t - r_t s_{Et} k_t - r_{Et} d_t \tag{23}$$

subject to (22). The first-order conditions are

$$w_t = \omega_l p_{Et} A_E (l_{Et} h_t / d_t)^{\omega_l - 1} (s_{Et} k_t / d_t)^{\omega_k}, \tag{24}$$

$$r_t = \omega_k p_{Et} A_E^{\omega_l} (l_{Et} h_t / d_t) (s_{Et} k_t / d_t)^{\omega_k - 1}, \tag{25}$$

$$r_{Et} = (1 - \omega_k - \omega_l) p_{Et} A_E^{\omega_l} (l_{Et} h_t / d_t) (s_{Et} k_t / d_t)^{\omega_k}. \tag{26}$$

The solution for the degree of tax evasion follows as

$$1 - a_t = A_E^{1/(1-\omega_l-\omega_k)} (p_{Et} \omega_l / w_t)^{\omega_l/(1-\omega_l-\omega_k)} (p_{Et} \omega_k / r_t)^{\omega_k/(1-\omega_l-\omega_k)}; \tag{27}$$

in addition, the CRS property implies $r_{Et} = (1 - \omega_l - \omega_k) p_{Et} (1 - a_t)$.

From the consumer problem, the price of tax evasion services p_{Et} is a weighted average of the capital and labor tax rates:

$$p_{Et} = [\tau_l w_t (l_{Gt} + l_{Et}) h_t / d_t] + [\tau_k r_t (s_{Gt} + s_{Et}) k_t / d_t].$$

In the case of a uniform tax rate for both capital and labor income, $\tau_l = \tau_k \equiv \tau$, this reduces to $p_{Et} = \tau$, as in proposition 1; then $r_{Et} = (1 - \omega_l - \omega_k) \tau (1 - a_t) < \tau$. In this case, the BGP equilibrium solution for the growth

rate is $g = r(1 - \tau + r_E) - \delta_K - \rho$. As the tax rate τ rises, the consumer is increasingly less willing to substitute from goods consumption to leisure and more willing to evade income tax. This causes the BGP growth rate to decline at an increasingly lower rate as the tax rate rises compared to the economy without evasion, as in the *Ak* economy.

In the BGP equilibrium, all growing variables evolve at the same rate g, with k_t/h_t constant, and the BGP welfare W is equal to

$$W = \int_0^\infty (\ln c_t + \alpha \ln x_t) e^{-\rho t} dt = \frac{1}{\rho} \left\{ \ln k_0 + \ln \left[\left(\frac{c_t}{k_t} \right) x^\alpha \right] + \frac{1}{\rho} g \right\}. \quad (28)$$

With both human and physical capital in the extended economy and an assumed common tax rate τ, the output normalized tax revenue curve is $a_t\tau$ as in the *Ak* economy. But now increased human capital productivity also increases the ratio of tax revenue to output for any given tax rate and forces a reduction in τ given equation (21); it also increases growth and welfare.

The revenue per output for any given tax rate depends on the degree of evasion a and tax elasticity. Denoting the tax elasticity of reported income by η_τ^a and of unreported income by η_τ^{1-a} and using equation (27), one can show that $\eta_\tau^a = -[(1 - a)/a]\eta_\tau^{1-a} \simeq \eta_{\tau_k}^a$ of the *Ak* model in the previous section.[19] The higher the tax elasticity magnitude, the stronger the revenue increase from improved productivity, and the larger the tax rate decrease required to keep a constant γ.

V.   Calibration

The BGP equilibrium of the model is calibrated annually on the basis of Trabandt and Uhlig's (2011) US averaged data from 1995 to 2007; we get targets from these data and then make adjustments to these targets so that we can better capture the entire postwar period rather than just 1995–2007, which is the end of the period when tax rates have already fallen. We also follow Gomme and Rupert (2007), who refine the calibration methodology in general and in particular for a two-sector market and nonmarket household economy; our human capital investment sector is the nonmarket "household" sector.[20] As such, in table 1 we present 12 in-

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[19] That is,

$$-[(1 - a)/a]\eta_\tau^{1-a} = -[(1 - a)/a][(\omega_l + \omega_k)/(1 - \omega_l - \omega_k)] \\ \times [1 - 0.5(\eta_\tau^w + \eta_\tau^r)],$$

where $\eta_\tau^w + \eta_\tau^r$ denotes the sum of the wage and interest rate elasticities to τ. Quantitatively, in our calibration below, $\eta_\tau^w + \eta_\tau^r$ is negligible, so the approximation results.

[20] We do not independently estimate time in human capital investment, although data are becoming more available for this as a task for future research; instead we use Gomme and Rupert's concept of a much lower leisure time share around 0.5 relative to one-sector exogenous growth economies that typically set leisure above 0.7; and we set a time share for

TABLE 1
Target Values of the Baseline Calibration for the US Benchmark Model

Calibration Targets	Target Variable	Target Value	Achieved Value	Target Source
Output growth rate	g	.02	.02	Trabandt and Uhlig (2011)
Inverse of intertemporal elasticity of substitution	θ	1–2	1	Trabandt and Uhlig (2011)
Government revenue to GDP	γ	.26	.31	Trabandt and Uhlig (2011)
Average income tax rate	τ	.31	.4	Trabandt and Uhlig (2011)
Government consumption to GDP	Γ/y	.18	.20	Trabandt and Uhlig (2011)
Government transfers to GDP	v/y	.08	.11	Trabandt and Uhlig (2011)
After-tax net real interest rate	r'	.04	.04	Trabandt and Uhlig (2011)
Capital to output ratio	k/y	2.38	2.38	Trabandt and Uhlig (2011)
Leisure time	x	.5	.5	Jones et al. (2005)
Labor time	l_G	.17	.16	Jones et al. (2005)
Fraction of income reported	a	.78	.78	Waud (1988)
Tax elasticity ETI	$\eta_{1-\tau}^a$	.4–3.0	1.08	Saez et al. (2012), Feldstein (1995)

dependent pieces of information on our variables from different sources, eight from Trabandt and Uhlig, two on leisure and labor time from Jones et al. (2005), one on the fraction of reported income from Waud (1988), and one on tax elasticity from Feldstein (1995) and Saez et al. (2012). These form our calibration targets, which in turn enable us to uniquely pin down the model's parameter values. Table 2 presents 12 parameters for which values are assigned in order to get the "achieved" calibration targets of table 1. Note that parameter calibration depends often on the solution to BGP variables from implicit equations; for example, x, a, and $s_G k/l_G h$ are solved only implicitly within a system of three nonlinear equations.

As in Trabandt and Uhlig (2011), the target value of the real growth of output, g, is set to 2 percent. Denoting by r' what Trabandt and Uhlig call the annual real interest rate, we define this as consistent with their usage as $r' \equiv (1 - \tau_k + r_E)r - \delta_K$ and set it at $r' = 0.04$ as in their calibration. Given our assumption of log utility, a special case of one given in Trabandt and Uhlig's paper, together g and r' imply a time preference rate of $\rho = 0.02$. As in Trabandt and Uhlig's table 2, we set the share of labor income in the goods sector such that $\beta = 0.62$. The labor share of the human capital sector, ε, is equal to 0.80, similarly to, for example, Bowen (1987) and Pecorino (1995). Also following Trabandt and Uhlig's table 1, we target $\gamma = 0.26$ as the sum of their government consumption plus investment, $\Gamma/y = 0.18$, plus their government transfer of $v/y = 0.8$.

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that spent in household production similar to theirs, although their household output is not specified to be human capital investment.

TABLE 2
Parameter Values of the Baseline Calibration for the US Benchmark Model

Parameter	Variable	Baseline Value
Time preference rate	ρ	.02
Labor share in goods sector	β	.62
Inverse of intertemporal elasticity of substitution	θ	1
Depreciation rate of physical capital	δ_K	.07
Depreciation rate of human capital	δ_H	.07
Productivity in goods sector	A_G	1
Productivity in human capital sector	A_H	.29
Productivity in evasion sector	A_E	1.44
Weight of labor in preferences	α	2.5
Labor share in education sector	ε	.8
Capital share in evasion sector	ω_k	.36
Labor shares in evasion sector	ω_l	.36

Then using Trabandt and Uhlig's β and their τ_l and τ_k, we assume an average tax rate τ from the labor and capital tax equal to

$$\tau = \tau_l \beta + \tau_k (1 - \beta) = (0.28)(0.62) + (0.36)(0.38) = 0.31.$$

Accounting for a depreciation tax element found in Trabandt and Uhlig's paper, we would revise this τ upward somewhat; we impute the depreciation rate according to our equilibrium conditions such that $\delta_K = 0.07$.[21]

In our model, it is the share of reported tax revenue per output that equals the spending share γ. Therefore, we have $a\tau = \gamma$, while in Trabandt and Uhlig (2011) it is implicit that $\tau = \gamma$ in our notation. For targeting a we use 0.78 since Waud (1988) reports that 22 percent of federal income tax was lost in 1981 as a result of unreported income (total federal corporate and personal income), implying $1 - a = 0.22$; Fullerton and Karayannis (1994) report that 20 percent of noncorporate income evades taxation in the United States; other estimates abound, for example, Schneider and Enste (2000). Given $a = 0.78$, we now have $a\tau = \gamma = 0.26$, or $\tau = 0.26/0.78 = 0.33$.

We then make adjustments in order to try to better capture the entire postwar period. We chose a higher tax rate than the rate that existed in Trabandt and Uhlig's data set from 1995–2007, or as implied at $\tau = 0.33$. We set the baseline τ at $\tau = 0.40$ as an approximation of the midpoint of

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[21] Trabandt and Uhlig assume that capital income taxes are levied on dividends net of depreciation, i.e., $\tau_k[1 - \beta - \delta_K(k/y)]$, so that the weighted average is $\tau = \tau_l \beta + \tau_k[1 - \beta - \delta_K(k/y)] = 0.33$ given $\delta_K = 0.07$ instead of 0.31. We have also ignored their consumption tax of 0.05. Our BGP equilibrium implies that we impute δ_K as

$$\delta_K = (y/k)[1 - \tau + (1 - \omega_l - \omega_k)\tau(1 - a)][\omega_k \tau(1 - a)]$$
$$\times \{1 + [\omega_k/(1 - \beta)]\tau(1 - a)/[1 - \tau(1 - a)(\omega_l + \omega_k)]\}.$$

the postwar economy tax rates, in that this increase puts us approximately in the midrange of the 20 points between lowest and highest weighted average tax rates, of 30 percent and 50 percent, on the top 1 percent of income that we find for postwar United States in Appendix figure A1. This implies $a\tau = (0.4)(0.78) = 0.31$, and so $\gamma = 0.31$ instead of the $\gamma = 0.26$ target. This increase is distributed between government spending of Γ/y going to 0.20 and v/y to 0.11, our achieved values, instead of the Trabandt and Uhlig values of 0.18 and 0.08.

In calibrating the evasion intermediary technology factors, Slemrod and Weber (2012) make clear that there is a great deal of uncertainty over reliable micro-based or macro-based evidence that can be used for such a purpose. In order to calibrate the labor and capital shares in the evasion sector, we first assume that they are equal, so that $\omega_l = \omega_k = \omega$, as seen in Benk et al. (2010). Then in order to pin down ω precisely, while being given $a = 0.78$, we make use of the large literature reviewed by Saez et al. (2012) on the tax elasticity of reported income. Here Saez et al. focus on estimation of the elasticity of reported income with respect to the net-of-tax rate, or $1 - \tau$; this uses the acronym ETI, and in our notation this is $\eta^a_{1-\tau}$.

Saez et al. (2012) report that substantial variance in the ETI is found in the literature, depending on the year, the percentile income share, and econometric methodology. For the period of the 1986 tax act, Feldstein (1995) finds an ETI between 1 and 3, while Moffitt and Wilhelm (2000) obtain a range of 0.35–0.97. Saez et al. report how the ETI is found to be significantly lower over the long-run period, even though for the long run they find "no truly convincing estimates" of the ETI. Still, they put the upper end of this long-run range at 0.4 for the top 1 percent percentile. Our baseline calibration is designed to get an average effect for the postwar period, including the high responses during tax reform periods. And so we chose an intermediate value equal to 1.08 that is between certain reform period point estimates and the long-run estimates of the ETI. Then using the fact that $\eta^a_\tau = -[\tau/(1 - \tau)]\eta^a_{1-\tau}$ and given our baseline τ, the implied tax elasticity of reported income is $\eta^a_\tau = 0.76$. In turn this implies an approximation for the input share in the intermediary sector that gives $\omega = 0.36$.[22] The other part of the evasion technology is A_E; this is set at 1.44 to achieve the elasticity target in conjunction with $\omega = 0.36$, while giving a share of labor time in evasion equal to less than 1 percent of total time and so achieving the other targeted time allocations.

Leisure time is targeted at $x = 0.5$, on the basis of Jones et al. (2005), Gomme and Rupert (2007), and Ramey and Francis (2009); Ragan (2013) also argues that leisure is 51 percent of 14 hours a day. Labor time l_G is targeted at the Jones et al. value of 0.17. We assume $\delta_K = \delta_H$, although some estimates place human capital depreciation at a lower rate than phys-

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[22] That is, $\omega \simeq 0.5\{1 + [(1 - a)/a][\tau(1 - \tau)]/\eta^a_{1-\tau}\}$.

ical capital. Given that $l_H = \varepsilon(1 - x)(g + \delta_k)/(r' + \delta_k)$, using the target values for g, r', and x, $\theta = 1$, the imputed value of $\delta_K = \delta_H = 0.07$ and the standard value for ε of 0.2, we then impute a standard value for leisure preference of $\alpha = 2.5$ so that $x = 0.5$, $l_G = 0.16$, and $l_H = 0.33$.

VI.   Simulation Results

Tables 3 and 4 present the baseline calibration results, with $g = 0.02$, $\tau = 0.40$, $1 - a = 0.22$, $\gamma = 0.31$, and $\omega = 0.72$, of simulations of the effects of a 10 percent productivity increase in each of the goods and human capital sectors, in terms of A_H and A_G rising, and of a 10 percent decrease in the productivity of the evasion sector, in terms of A_E falling. This is done for the case in which there is a single common tax rate τ on both labor and capital income in table 3 and for the cases in which there are separate taxes on labor income τ_l and on capital income τ_k in table 4. The difference is that in table 3 the common tax rate responds to changes in productivity while in table 4, first the capital tax τ_k is held constant at the baseline and only the labor tax τ_l is allowed to fall, and then the labor tax τ_l is held constant at the baseline and only the capital tax is allowed to fall. The two tables present the new levels of τ, g, and $1 - a$, plus the percentage change in l_H, l_G, s_H, s_G, k/h, $s_G k/(l_G h)$, $s_H k/(l_H h)$, and x, as induced by the productivity changes. And each table includes an exogenous growth special case for comparison; here human capital is specified to grow exogenously at the baseline rate $g = 0.02$, while assuming that $A_H = 0$ and $\delta_h = 0.$[23] Table 3 also shows the results both for the baseline $\omega = 0.36$ and when it is increased to 0.39.

   Increases in goods and human capital investment sector productivities induce a lesser degree of tax evasion $1 - a$, and this in turn allows a lower tax rate in all cases for all models. In table 3, the increase in A_H allows τ to fall by 2 points, with g rising from 2 percent to 3.57 percent, while the A_G increase causes a 4-point fall in the tax rate but a smaller growth rate increase. The growth and welfare (not shown) gain is highest from the A_H increase in all cases.

   Table 3 shows that the factor reallocation from both goods and human capital sector productivity increases is away from leisure and more toward the human capital sector. The input ratios are given in column 9. The capital to effective labor ratio decreases with an A_H increase and increases with an A_G increase; the input factor ratio w/r falls with an A_H increase and rises with an A_G increase; and in column 8, k/h decreases with an A_H increase and increases with an A_G increase.[24] With ω higher, the tax elasticity

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[23] This is similar to an exogenously increasing productivity parameter defined as $A_{Gt} \equiv A_G(h_t)^\beta$; the exogenous growth case with the same baseline calibration gives a larger leisure time allocation, such that $x = 0.8$ and $l_G = 0.19$, close to values used in standard exogenous growth models; the same baseline but with human capital investment gives $x = 0.5$, close to the two-sector household economy of Gomme and Rupert (2007).

[24] It can be shown that w/r depends linearly and positively on the capital to labor ratio.

TABLE 3
PRODUCTIVITY EFFECT ON TAX RATE, GROWTH, AND EVASION

ω	After-Reform Rates			After-Reform Percentage Changes						
	τ (1)	g (2)	$1-a$ (3)	l_H (4)	l_G (5)	s_H (6)	s_G (7)	k/h (8)	$s_G k/l_G h,$ $s_H k/l_H h$ (9)	x (10)
	10 Percent Increase in A_H									
Endogenous growth:										
.36	38.2	3.57	18.9	8.4	2.9	3.8	−1.5	−11.8	−15.6	−6.2
.39	36.6	3.97	15.2	12.7	6.7	5.0	−.6	−10.7	−16.8	−9.7
	10 Percent Increase in A_G									
Endogenous growth:										
.36	35.8	3.25	13.5	10.8	7.9	3.7	1.0	14.1	6.7	−8.9
.39	34.3	3.63	9.5	14.9	11.8	4.9	2.0	15.6	5.5	−12.3
Exogenous growth:										
.36	36.3	2.00	14.6	NA	8.0	NA	3.5	29.6	24.1	−1.4
	10 Percent Decrease in A_E									
Endogenous growth:										
.36	34.6	3.11	10.4	12.1	10.5	3.6	2.2	2.8	−5.0	−10.3
.39	33.4	3.42	7.2	15.4	13.7	4.6	3.1	4.0	−5.7	−13.0
Exogenous growth:										
.36	34.9	2.00	11.1	NA	11.5	NA	4.9	16.0	9.1	−2.0

is higher, and so the degree of lesser evasion and the tax rate reduction are greater from the goods and human capital sector productivity increases.

The time spent in human capital investment l_H increases by more than l_G does for all productivity changes in tables 3 and 4. This is consistent with the human capital sector being relatively more labor intensive than the goods sector, and so it is the sector to expand more as leisure is reduced. The increase in s_H is larger than in s_G for all productivity changes in table 3, but not in table 4 when s_H falls with a capital tax decrease alone.

For exogenous growth in table 3, there is no human capital sector and the good sector productivity increase shows patterns similar to those with endogenous growth. The exception is that for exogenous growth the decline in leisure in column 10 is much smaller and the increase in the capital to labor ratio in column 9 more than three times bigger; similarly, the k/h increase is double what it is in endogenous growth when A_G increases. This leads to what we interpret as greater diminishing returns being experienced in the goods sector during the sectoral reallocation so that it becomes more productive relative to the evasion sector but by a lesser degree than in the case with endogenous growth. This can explain why the tax evasion decrease is less in the exogenous growth case and the tax rate decrease also significantly less.

A 10 percent reduction in the evasion sector productivity A_E in table 3 shows that the evasion degree falls by more, and the growth rate increases

TABLE 4

SEPARATE CAPITAL AND LABOR TAX EFFECTS FROM PRODUCTIVITY CHANGES

	After-Reform Rates				After-Reform Percentage Changes							
ε	τ_K (1)	τ_L (2)	g (3)	$1-a$ (4)	l_H (5)	l_G (6)	s_H (7)	s_G (8)	k/h (9)	$s_G k/l_G h$ (10)	$s_H k/l_H h$ (11)	x (12)
10 Percent Increase in A_H												
Endogenous growth:												
.36	40	37.9	3.68	18.3	9.8	3.9	6.2	−3.2	−13.6	−19.5	−16.4	−7.4
	36.8	40	3.41	19.7	6.9	1.7	1.3	.1	−10.1	−11.6	−14.8	−4.9
10 Percent Increase in A_G												
Endogenous growth:												
.36	40	32.9	3.43	12.8	13.0	9.5	9.3	−3.4	8.4	−4.4	4.8	−10.8
	30.7	40	2.97	14.5	7.5	5.6	−3.6	6.4	21.8	22.8	9.2	−6.1
Exogenous growth:												
.36	40	33.7	2.00	14.0	NA	10.0	NA	3.8	21.9	15.0	NA	−1.8
	31.6	40	2.00	15.4	NA	5.8	NA	3.2	40.4	36.7	NA	−.9
10 Percent Decrease in A_E												
Endogenous growth:												
.36	40	31.1	3.31	9.8	14.5	12.2	10.8	−3.6	−4.0	−17.5	−7.0	−12.4
	27.3	40	2.78	11.4	8.1	7.7	−6.3	10.0	12.9	15.2	−2.1	−6.9
Exogenous growth:												
.36	40	31.5	2.00	11.0	NA	13.8	NA	5.2	5.7	−2.3	NA	−2.5
	27.8	40	2.00	12.0	NA	8.5	NA	4.7	31.4	26.7	NA	−1.3

by less as compared to the goods and human capital sector productivity increases; welfare also rises, but by the least amount as compared to A_H and A_G increases (not shown).

Table 4 shows how allowing for a reduction in either the labor tax or the capital tax alone compares to instead decreasing a common tax rate as in table 3. Increasing either A_H or A_G while lowering the labor tax τ_l causes a larger decline in the degree of evasion and a bigger growth rate increase than does lowering the capital tax τ_k. This happens even as there is a smaller decrease in the labor tax as compared to the capital tax. For exogenous growth, as compared to the endogenous growth baseline, the A_G increase again causes a smaller fall in the degree of evasion and in the tax rate, for both taxes.

Table 4 also shows that the large leisure decrease again induces the l_H time to increase by more than l_G for all the productivity changes reported and for either the labor or capital tax rate reduction, in part because of the large leisure decrease in endogenous growth. For an A_H increase, the share s_H rises absolutely and relatively by more than s_G for either tax rate reduction, and $s_H k/(l_H h)$ falls. However, for an A_G increase or an A_E decrease, s_H expands or contracts depending on whether it is a labor tax reduction or a capital tax reduction, and $s_H k/(l_H h)$ rises for either tax reduction.

Table 4 may be interpreted as indicating an advantage of a labor tax versus a capital tax. As a result of an increase in either A_H or A_G, the associated per-unit decrease in the labor tax allows for less evasion and higher growth, compared to the capital tax per unit decrease, even though the total tax reduction per unit of productivity increase is greater with a capital tax reduction.

The 10 percent A_E decrease in table 4, with a labor tax decrease, comes with a decrease in the capital to labor ratio in the goods sector, in column 10; a capital tax decrease in contrast comes with an increase in the capital to labor ratio in the goods sector. For exogenous growth with an A_E decrease and either a labor or capital tax decrease, the results are similar to those for endogenous growth for a common tax τ. The exception is a rise in k/h for the labor tax in exogenous growth and a fall in k/h for the labor tax in endogenous growth, related to the lower leisure reduction and larger capital to labor ratios that are induced in exogenous growth.

Figure 3 shows the simulated tax revenue curve as normalized by output in the baseline case as the solid curve, along with the 45-degree line, which would apply with no tax evasion. The dashed line shows how the ratio of tax revenue to output increases at any given tax rate following a 10 percent increase in A_H (labeled $A_H \times 1.1$); the dash-dot line shows this for an A_G increase (labeled $A_G \times 1.1$). The dotted line shows the largest increase of the ratio per τ following a 10 percent decrease in A_E (labeled $A_E \times 0.9$).

Figure 4 shows the annual effect over time of permanent 10 percent productivity increases in A_H and A_G, plus a 10 percent decrease in A_E and in γ: on evasion, $1 - a$, the growth rate of y_G (denoted by g_y here), and the
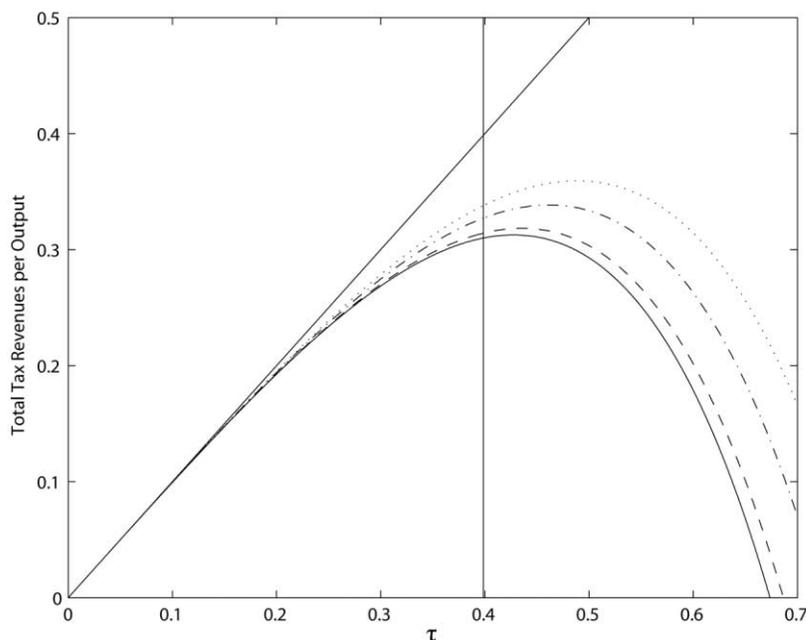
Figure 3.—Tax revenue curve, 10 percent changes in A_H (dashed line), A_G (dash-dot line), and A_E (dotted line); 45-degree line ($A_E = 0$).

$\ln y_G$ (denoted $\ln y$). The dashed lines are the original BGP equilibria, and the solid lines show the new equilibria over time after the permanent parameter change. The decrease in A_E causes the largest decrease in $1 - a$, while the increases in A_H and A_G lead to larger increases in the growth rate. A decrease in the size of government, as summarized by the parameter γ, causes large evasion and growth effects that would involve a movement down a given normalized tax revenue curve. In practice, such a movement could follow from privatization, deregulation, or greater government program efficacy, for example. The A_H increase causes the largest long-run increase in both growth and welfare (not shown). The A_H increase, A_G increase, and A_E decrease cause a 22 percent, 11 percent, and 6 percent increase in welfare, respectively; in exogenous growth (not shown) the welfare increases for A_G and A_E fall to 3 percent and 5 percent.

VII.   Estimate of Tax Rate Reduction

We now calculate what proportion of the observed decline in US postwar top marginal tax rates can be explained by increases in goods sector and human capital sector productivity. We assume a single tax rate on labor and capital income and compare our estimate of the postwar decline in this single rate to an observed composite top marginal rate over the same
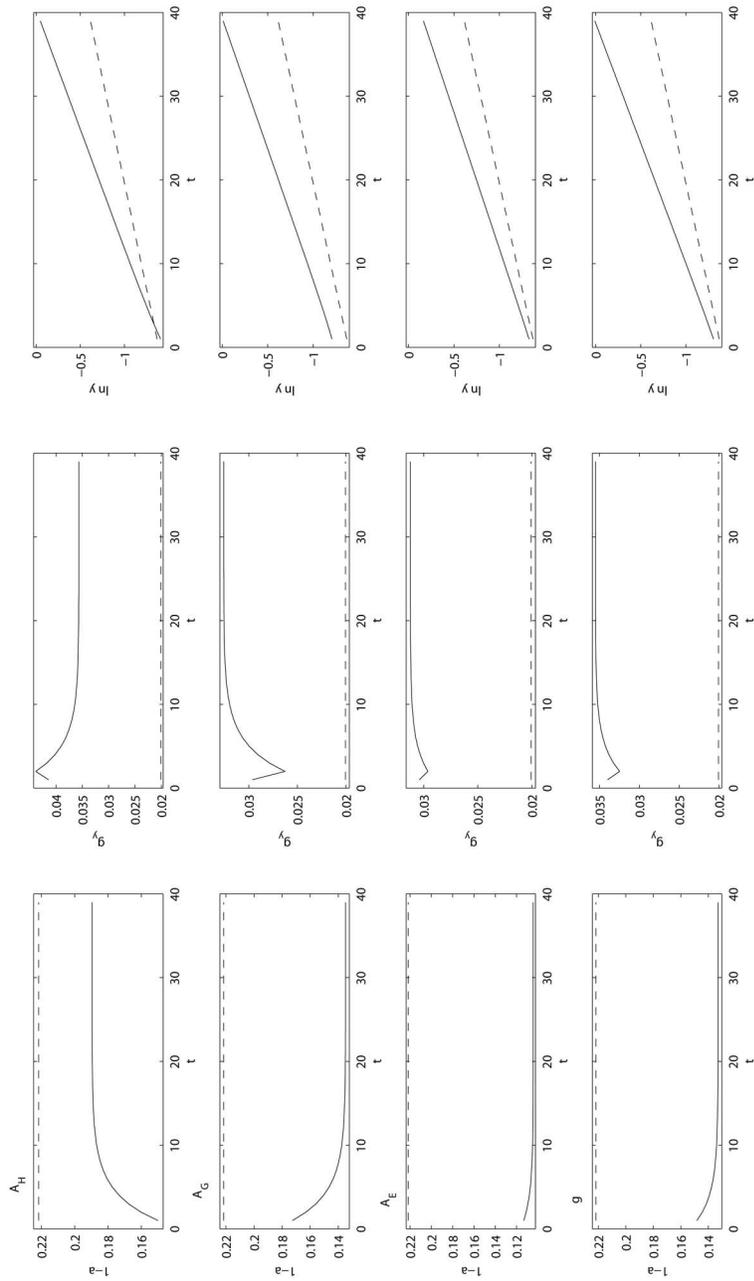
Figure 4.—10 percent increase in A_H and A_G; 10 percent decrease in A_E and γ; and $1 - a$, g, and $\ln y$

period. Using data graphed in figure 1 (see also online App. tables B1–
B5), a weighted average of the top marginal rates on personal and corpo-
rate income falls from 75 percent in 1951 to 35 percent in 2012, a 40-point
drop. For the average weighted rate on the top 0.5 percent of the income
distribution, Appendix figure A1 shows that the postwar rate drops from
around 50 percent to 30 percent.

Using the database of Baier et al. (2006), we estimate that human capi-
tal productivity has risen at an average rate of 3.69 percent per decade,
from 1950 to 2009, while goods sector productivity has risen at an average
rate of 0.756 percent per decade over the same period (see table A1).
These estimates suggest a 4.9-fold asymmetry between productivity gains
in the human capital sector and productivity gains in the goods sector
over this period. The size of this asymmetry is comparable to empirical
findings in the related intangible capital investment model of McGrattan
and Prescott (2010). For 1993–2000, they report US goods sector produc-
tivity growth of 0.7 percent per year compared to intangible goods sec-
tor productivity growth of 2.7 percent per year, amounting to a 3.9-fold
asymmetry.

Next, we use postwar US data to estimate by how much the hypo-
thetical common tax rate (τ) falls while keeping the share of government
revenue in output unchanged. We simultaneously use the estimated
changes in A_H and A_G for the 1950–2009 period. We choose the year
2000 as the benchmark period; that is, $A_H(2000) = A_H = 0.2935$, $A_G(2000)$
$= A_G = 1$, and $\tau(2000) = \tau = 0.3974$. Given the estimated average growth
rate of A_H equal to 3.69 percent per decade, over the six decades, we let
$[A_H(2009)/A_H(1950)] - 1 = (1.0369)^6 - 1 = 0.2429$, or 24.3 percent. It
follows that $A_H(1950) = A_H(2000)/(1.0369)^5 = 0.24486$ and $A_H(2009) =$
$A_H(2000)(1.0369) = 0.030433$. Similarly, given the average growth rate
of A_G at 0.756 percent per decade, we let $[A_G(2009)/A_G(1950)] - 1 =$
$(1.00756)^6 - 1 = 0.0462$, or 4.62 percent. And it follows that $A_G(1950)$
$= A_G(2000)/(1.00756)^5 = 0.96307$ and $A_G(2009) = A_G(2000)(1.00756)$
$= 1.00756$. Ideally, we would use the boundary values for A_H and A_G and
compute the implied changes in the tax rate under the condition that tax
revenue as a fraction of output remains constant. However, given compu-
tational boundaries for the simulation of the baseline BGP equilibrium of
$A_H \geq 0.287$ and $A_G \geq 0.99457$, we simulate the increase of A_H in the range
0.2870 to $A_H(2009) = 0.30433$ (i.e., only 6.04 percent of the total 24.3 per-
cent) and the increase of A_G in the range 0.99457 to $A_G(2009) = 1.00756$
(i.e., only 1.31 percent of the total 4.62 percent).

The implied decrease in τ from increasing both A_H and A_G simulta-
neously within the simulation range along the BGP is 0.0317, starting at
$\tau = 0.4176$ and going down to $\tau = 0.3859$. Since the simulated change in
A_H forms only $6.04/24.3 = 0.2486$ of the total change and in A_G it forms
$1.31/4.62 = 0.2835$ of the total change, magnitudes that are close to
each other, we take their simple average, that is, $(0.24877 + 0.2827)/2 =$
0.2657, and extrapolate the 0.0317 tax rate decrease for the six decades

to $3.17/0.2657 = 11.93$ points. The estimated reduction in the tax rate of 12 points accounts for 30 percent of the 40-point decline in the weighted top marginal tax rate observed in postwar US data. The estimated decline in the tax rate would double for the weighted average tax rate decline in figure A1. However, the estimates would be smaller if we built a lower tax elasticity magnitude into the baseline calibration. Note that in this simulation we extrapolate the total six-decade change in h/k to be a 17 percent increase.

VIII.   Discussion

Our approach is driven by the well-articulated goal of satisfying the "input justification criterion" that McGrattan and Prescott (2010, 90) put forth: "requiring our exogenous inputs to be consistent with micro and macro empirical evidence." The use of the rise in goods and human capital sector productivities in our extended model is closely related to McGrattan and Prescott's extended two-sector model, which uses unbalanced productivity growth between the goods and intangible capital investment sectors to explain the 1990s expansion in the United States. Our human capital investment sector might be viewed as partially encompassing the intangible capital investment sector that McGrattan and Prescott highlight. Similarly to their intangible capital, all our human capital stock is used as an input for goods production. As in their work, the inclusion of a separate investment sector enables a better empirical explanation of growth episodes, in our case the postwar decline in US tax rates, due to a broader consideration of US postwar productivity increases. And as in McGrattan and Prescott, our evidence implies unbalanced productivity growth favoring the investment sector.

The main qualification not yet addressed is that in explaining 30 percent of the decline in postwar US top marginal tax rates, our model also implies an increase in the BGP growth rate of 5.42 percentage points. This is high compared to the relatively stable 2 percent measured per capita postwar US growth rate, which some have argued has even indicated a slight decline in US living standards. But the higher growth rate within our economy can be viewed as being consistent with the McGrattan and Prescott (2010) 0.7 percent per year increase in goods sector total factor productivity (TFP) and their 0.8 percent higher labor productivity per year. While McGrattan and Prescott base their productivity estimates on transition dynamics in an exogenous growth setting of a high-growth episode, we abstract from transition dynamics, use comparative static changes in the BGP equilibrium within an endogenous growth setting, and consider a broader postwar expansion period. Consequently, in our model, output growth rates and labor productivity coincide along the BGP. If the McGrattan-Prescott 1990s average growth rate was normalized and extended across the entire US postwar period, then presumably there could result a growth rate more comparable to the 5 percent that we find.

The results in McGrattan and Prescott (2010) are driven by a shift of resources to the intangible capital sector with a subsequent rise in per capita hours worked and a decrease in leisure time, which they emphasize as being plausible during an expansion. Similarly, from our productivity increases we find a shift in resources toward the human capital sector, more labor time, and less leisure (see also Beaudry, Doms, and Lewis 2010; Beaudry and Francois 2010). And also comparable to the McGrattan-Prescott results, our productivity changes cause significant leisure time decreases in the endogenous growth baseline model, but leisure decreases only slightly in the exogenous growth version without human capital investment. Such significant declines in leisure time are key to their result of more labor time and in our model are key to increasing the size of the human capital sector and the BGP growth rate.

Buera and Shin (2013) explain long historical periods of accelerated growth resulting from productivity TFP increases, which is related to how we view the postwar US experience when including the human capital sector. Although Buera and Shin emphasize the role of financing costs, which we abstract from, they attribute such increases in TFP to periods of tax reduction and regulation reform, as is related to our focus. While within an exogenous growth framework, tax rate reform may not influence long-run economic growth (e.g., Trabandt and Uhlig 2011), we allow for tax reform's positive effect on growth.

The estimated postwar tax rate decline depends on our simplified assumption of unchanged productivity in the evasion sector. If this productivity were to have fallen/risen, say because of greater ease of enforcement/evasion through information technology advances, then our estimate of the possible tax rate decrease would be larger/smaller. As evidence on evasion productivity change is lacking, instead we use macro evidence to target estimates of the US degree of tax evasion and the tax elasticity of reported income.

The tax evasion literature does not generally use micro evidence. For example, Chen (2003) extends a Becker (1968) and Ehrlich (1973) style of illegal tax evasion within an Ak endogenous growth economy in which a transactions cost for evasion enables a lower effective tax rate and higher growth; we have such effects with the intermediation sector instead.[25] A related theoretical treatment of tax evasion by Dhami and al-Nowaihi (2010) improves on the seminal Yitzhaki (1974) approach by modifying expected utility to provide a theory consistent with broad micro evidence that suggests that tax evasion rises with tax rates and that evasion is sizable relative to the size of the economy; we capture these features with log utility and the evasion intermediation (see also Allingham and Sandmo 1972; Roubini and Sala-i-Martin 1995; Slemrod 2001). Micro evidence on evasion sector technology parameters is difficult to obtain by its nature, but

[25] According to Chen (2003, 384), a transaction cost is "hiring CPAs and lawyers to dodge taxes, and particularly bribing tax officials and law administrators, along with other concealing activities."

this sets out a well-identified task for future research, as emphasized by Slemrod and Weber (2012). A promising direction is experimental inference of evidence on the evasion sector (Saez 2012).

IX.   Conclusion

Using a model calibrated to US postwar data, the paper shows how growth, welfare, and time in human capital investment can trend upward as a result of productivity increases while tax rates trend downward. The paper employs an endogenous growth economy as extended with taxes and a decentralized competitive intermediation sector that provides tax evasion services. It shows how an increase in productivity induces less tax evasion, which causes the ratio of tax revenue to GDP to increase at any given tax rate. In turn, this implies that the tax rate must fall if government revenue as a proportion of GDP is to remain constant in a manner consistent with the data. Upward productivity trends imply a downward tax rate trend.

Using the estimated upward US postwar productivity trends in the goods and human capital sectors and assuming a uniform tax rate on labor and capital income, the model explains 30 percent of the reduction in a weighted average of US postwar tax rates. In simulations, the paper relaxes the assumption of a uniform tax to examine a labor tax reduction versus a capital tax reduction, showing that in either case the time in human capital investment increases. The human capital to physical capital ratio rises with human capital investment productivity increases and falls with goods sector productivity increases; as simulated with our estimated productivities, the postwar human to physical capital ratio rises by about 17 percent.

Without the human capital sector, our explanation of the tax trend would be significantly less strong. With an increase in only the goods sector productivity, a lower tax reduction is found in the exogenous growth special case, as compared to the endogenous growth human capital economy. And our estimate of the effect of US postwar productivity growth would be several times smaller if we took only goods sector data from Baier et al. (2006) and ignored human capital productivity data. Without human capital we would also be inconsistent with the direction of McGrattan and Prescott in using unbalanced goods and intangible capital sector productivity to explain a US growth period, and with the many views and estimates of the impact of rising education and human capital levels in the postwar US experience (e.g., Guryan 2009).

We calibrate tax evasion to fit the United States, a developed country, and find that evasion falls as goods and human capital productivity increases. This is consistent with the idea that developing countries experience less evasion and more growth and welfare as they become more developed through rising human capital accumulation. Tax rate reduction then becomes a natural consequence of a relatively constant share of government revenue in output. Even though there may be many other reasons for the observed decline in top marginal and average tax rates, few

studies model how this might occur. We provide a potential explanation of this stylized trend based on productivity gains in the goods and human capital sectors while ignoring other political economy factors. At the same time the analysis is consistent with a rising time allocation in human capital investment, in accordance with claims of such a trend. A potent policy dimension arises that we leave for future research: Education policy that efficaciously continues to boost human capital productivity may interact with public finance considerations by allowing for a gradual reduction in tax rates that further enhances growth and human capital investment in a virtuous cycle. A research issue left untouched here is whether our simulated postwar rise in the human to physical capital ratio could contribute to explaining structural transformation of economies toward more human capital–intensive sectors, as related, for example, to Kaboski (2009) or Buera and Kaboski (2012).

Appendix A

A. *Average Tax Rates: Figure A1*

Online Appendix B provides details on construction of the data series graphed in figure A1. The data come from the US Bureau of Economics Analysis (BEA), Office of Management and Budget (OMB), and Piketty and Saez (2006, 2007). The lines in figure A1 are as follows: squares, weighted average of personal and corporate tax rates; x's, average personal income tax rate on top 0.5 percent; dashed line, average corporate income tax rate; and triangles, federal government receipts as a percentage of GDP.
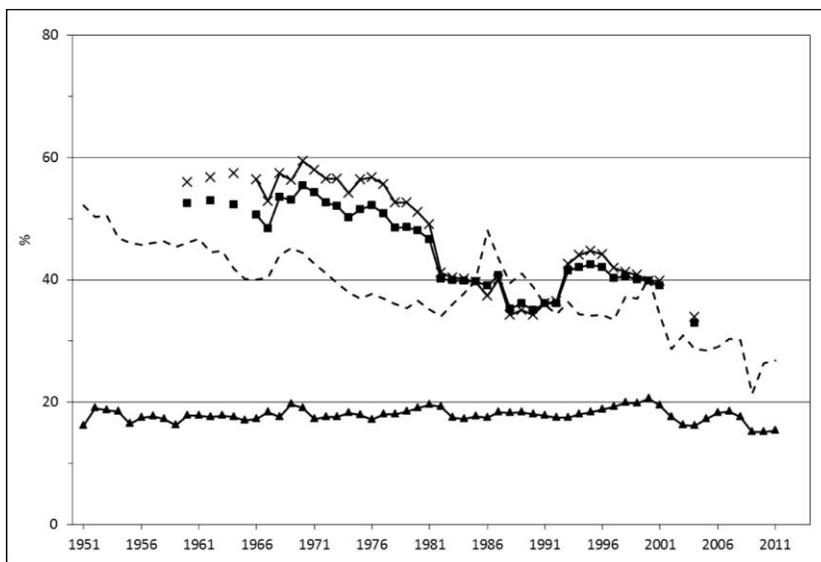

Figure A1.—Average US tax rates, 1951–2011

B. Proof of Proposition 4

Welfare W consists of terms involving c/k and g. Equation (16) and $y = rk$ imply $\tau_k a = \gamma$, and so

$$d(\tau_k a)/dA_E = d\gamma/dA_E = 0.$$

For c/k, it then holds that

$$d(c/k)/dA_E = A_G[d(\tau_k a)/dA_E] = 0.$$

Since

$$g = A_G[1 - \omega_E \tau_k - (1 - \omega_E)\tau_k a] - \delta_K - \rho,$$

it follows that $dg/dA_E = -A_G \omega_E(\partial \tau_k/\partial A_E)$. The derivative $d\tau_k/dA_E$ is found from the fact that $d(\tau_k a)/dA_E = 0$, which implies that

$$[a + \tau_k(\partial a/\partial \tau_k)]d\tau_k + \tau_k(\partial a/\partial A_E)dA_E = 0,$$

and so

$$d\tau_k/dA_E = -\tau_k(\partial a/\partial A_E)/[a(\eta^a_{\tau_k} + 1)],$$

where $\partial a/\partial A_E \le 0$. Thus from equation (15),

$$dW/dA_E = dg/dA_E = A_G \omega_E \tau_k(\partial a/\partial A_E)/[a(1 - |\eta^a_{\tau_k}|)] < 0$$

for $|\eta^a_{\tau_k}| < 1$ and $a \in (0, 1)$.

C. Proof of Proposition 5

By taking the total differential of the fraction of tax revenue in output given by $a\tau_k$ with respect to changes in the statutory tax rate and the goods sector productivity, we get

$$d(a\tau_k) = [a + (\partial a/\partial \tau_k)\tau_k]d\tau_k + (\partial a/\partial A_G)dA_G.$$

With this share of tax revenue fixed, $d(a\tau_k) = d\gamma = 0$, and

$$\begin{aligned} d\tau_k &= -(\partial a/\partial A_G)/[a + (\partial a/\partial \tau_k)\tau_k]dA_G \\ &= [\partial(1-a)/\partial A_G]/\{a - [\partial(1-a)/\partial \tau_k]\tau_k\}dA_G \\ &= -\{[\omega_E/(1-\omega_E)](1-a)/A_G\}/\{a - [\omega_E/(1-\omega_E)](1-a)\}dA_G \\ &= -(|\eta^a_{\tau_k}|/A_G)/(1 - |\eta^a_{\tau_k}|)dA_G, \end{aligned}$$

and therefore, $d\tau_k = -\{A_G[(1/|\eta^a_{\tau_k}|) - 1]\}^{-1}dA_G$. Since

$$\partial\{[(1/|\eta^a_{\tau_k}|) - 1]^{-1}\}/\partial|\eta^a_{\tau_k}| = [(1/|\eta^a_{\tau_k}|) - 1]^{-2}/|\eta^a_{\tau_k}|^2$$

and

$$\begin{aligned} \partial|\eta^a_{\tau_k}|/\partial \tau_k &= \partial\{[\omega_E/(1-\omega_E)][(1/a) - 1]\}/\partial \tau_k \\ &= [\omega_E/(1-\omega_E)][\partial(1/a)/\partial \tau_k] \\ &= [\omega_E/(1-\omega_E)](1/a^2)[\partial(1-a)/\partial \tau_k] \\ &= [\omega_E/(1-\omega_E)]^2[(1-a)/\tau_k a^2], \end{aligned}$$

being on the upward-sloping part of the normalized tax revenue curve, that is, $|\eta^a_{\tau_k}| < 1$, implies that both $\partial\{[(1/|\eta^a_{\tau_k}|) - 1]^{-1}\}/\partial|\eta^a_{\tau_k}|$ and $\partial|\eta^a_{\tau_k}|/\partial\tau_k$ are positive. So the size of the tax elasticity of reported income $|\eta^a_{\tau_k}|$ decreases with decreases in the tax rate τ_k and the size of the effect of the goods sector productivity on the magnitude of the tax rate decrease, as captured by the term $\{A_G[(1/|\eta^a_{\tau_k}|) - 1]\}^{-1}$, likewise decreases with decreases in the tax rate τ_k.

D.   *Extended Model Equilibrium Conditions*

Given (k_0, h_0) and subject to the following three constraints, the representative consumer problem is

$$V(k_0, h_0) = \max_{\{c_t, x_t, d_t, k_t, h_t, l_{Gt}, l_{Et}, s_{Gt}, s_{Et}, a_t\}_{t=0}^{\infty}} \int_0^{\infty} (\ln c_t + \alpha \ln x_t) e^{-\rho t} dt,$$

$$\dot{k}_t + \delta_K k_t = a_t[(1 - \tau_k)(s_{Gt} + s_{Et}) r_t k_t + (1 - \tau_l)(l_{Gt} + l_{Et}) w_t h_t]$$
$$+ (1 - a_t)(1 - p_{Et})[(s_{Gt} + s_{Et}) r_t k_t + (l_{Gt} + l_{Et}) w_t h_t]$$
$$+ r_{Et} d_t - c_t + v_t,$$

$$d_t = (l_{Gt} + l_{Et}) w_t h_t + (s_{Gt} + s_{Et}) r_t k_t,$$

$$\dot{h}_t + \delta_H h_t = A_H (l_{Ht} h_t)^{\varepsilon} (s_{Ht} k_t)^{1-\varepsilon}.$$

When time subscripts are dropped, the first-order conditions with respective multipliers of λ, χ, and μ are

$$0 = (1/c) e^{-\rho t} - \lambda, \tag{A1}$$

$$0 = (\alpha/x) e^{-\rho t} - \mu \varepsilon A_H (l_H h)^{\varepsilon-1} (s_H k)^{1-\varepsilon}, \tag{A2}$$

$$0 = \lambda r_E - \chi, \tag{A3}$$

$$\dot{\lambda} = -\lambda[a(1 - \tau_k) r(s_G + s_E) + (1 - a)(1 - p_E) r(s_G + s_E) - \delta_K]$$
$$-\mu(1 - \varepsilon) A_H (l_H h)^{\varepsilon} (s_H k)^{-\varepsilon} (1 - s_G - s_E) - \chi r(s_G + s_E), \tag{A4}$$

$$\dot{\mu} = -\lambda[a(1 - \tau_l) + (1 - a)(1 - p_E)] w(l_G + l_E)$$
$$-\mu[\varepsilon A_H (l_H h)^{\varepsilon-1} (s_H k)^{1-\varepsilon} l_H - \delta_H] - \chi w(l_G + l_E), \tag{A5}$$

$$0 = \lambda[a(1 - \tau_l) + (1 - a)(1 - p_E)] wh - \mu \varepsilon A_H^{\varepsilon-1} (l_H h)(s_H k)^{1-\varepsilon} h + \chi wh, \tag{A6}$$

$$0 = \lambda[a(1 - \tau_k) + (1 - a)(1 - p_E)] rk - \mu(1 - \varepsilon) A_H^{\varepsilon} (l_H h)(s_H k)^{-\varepsilon} k + \chi rk, \tag{A7}$$

$$0 = [(1 - \tau_l) - (1 - p_E)] w(l_G + l_E) h + [(1 - \tau_k) - (1 - p_E)] r(s_G + s_E) k. \tag{A8}$$

From (A8) we get

$$p_E = \tau_l \frac{wh(l_G + l_E)}{wh(l_G + l_E) + rk(s_G + s_E)} + \tau_k \frac{rk(s_G + s_E)}{wh(l_G + l_E) + rk(s_G + s_E)}.$$

Assuming $\tau_l = \tau_k = \tau$, then $p_E = \tau$. Then from (A4) with the use of (A3) and (A7), we get $-\dot{\lambda}/\lambda = r(1 - \tau + r_E) - \delta_K$, and from (A5) with the use of (A3) and (A6), it follows that

$$-\dot{\mu}/\mu = \varepsilon A_H (l_H h)^{\varepsilon-1} (s_H k)^{1-\varepsilon} (1 - x) - \delta_H.$$

Using $-\dot{\lambda}/\lambda$ and the derivative of the log of (A1) with respect to time, $\dot{c}/c = r(1 - \tau + r_E) - \delta_K - \rho$, and along the BGP, $g = r(1 - \tau + r_E) - \delta_K - \rho$ and variables c, k, and h grow at the rate g.

E.   Growth Accounting

The best reference for our growth accounting is Baier et al. (2006), which uses a new and more comprehensive data set on the growth of output, physical capital, and human capital and inputs this in a Lucas (1988) type production function with human capital (as in our economy) to construct the human capital data. Our approach is an extension of this with the added human capital investment sector, related to McGrattan and Prescott (2010). Using the data on physical and human capital growth in the data set from Baier et al., we apply their growth accounting procedure to find the TFP growth and the factor productivity growth in each of the two sectors.

The aim is to get postwar estimates for growth in our model's productivity parameters, A_G and A_H. In the model these are assumed constant for any given BGP. However, we consider the move from one BGP to another, while ignoring transition dynamics, by allowing these to be time varying and so seek to estimate from the data $\dot{A}_G/A_G$ and $\dot{A}_H/A_H$. Using the function $F(\cdot)$ to rewrite in shorthand the production function of the goods sector, $y_{Gt} = A_{Gt}F[s_{Gt}k_t, l_{Gt}h_t]$, and otherwise using the same notation, the parameter A_{Gt} represents the level of technology, TFP, at time t, whereby

$$\dot{y}_t/y_t = (\dot{A}_{Gt}/A_{Gt}) + (F_K k_t F)(\dot{k}_t/k_t) + (F_H h_t/F)(\dot{h}_t/h_t)$$

uses the variables in per-worker terms. This implies that

$$\dot{A}_{Gt}/A_{Gt} = (\dot{y}_t/y_t) - (1 - \beta)(\dot{k}_t/k_t) - \beta(\dot{h}_t/h_t),$$

where $1 - \beta = A_{Gt}F_K k_t/F$ is capital's share of income. For the human capital sector, and ignoring the small magnitude of capital in the evasion sector, similarly rewrite $i_{Ht} = A_{Ht}G[(1 - s_{Gt})k_t, (1 - l_{Gt})h_t]$, with $i_{Ht} = \dot{h}_t + \delta_H h_t$. Expressed in growth rates in per-worker terms, this implies

$$\dot{y}_{Gt}/y_{Gt} = (\dot{A}_{Gt}/A_{Gt}) + (A_{Gt}F_K s_{Gt}k_t/y_{Gt})(s_{Gt}\dot{k}_t/s_{Gt}k_t)$$
$$+ (A_{Gt}F_H h_t/h_t)(l_{Gt}\dot{h}_t/l_{Gt}h_t)$$

and

$$\dot{i}_{Ht}/i_{Ht} = (\dot{A}_{Ht}/A_{Ht}) + [A_{Ht}G_K(1 - s_{Gt})k_t/i_{Ht}]\{[(1 - s_{Gt})k_t]/(1 - s_G)k_t\}$$
$$+ [A_{Ht}G_H(1 - l_{Gt})h_t/i_{Ht}]\{[(1 - \dot{l}_{Gt})h_t]/(1 - l_{Gt})h_t\}.$$

Assuming competition, CRS production, constant shares of capitals across sectors, s_G and l_G, and letting $1 - \beta$ and $1 - \varepsilon$ denote capital's shares of income in each goods and human capital sector, respectively, then

$$\dot{y}_{Gt}/y_{Gt} = (\dot{A}_{Gt}/A_{Gt}) + (1 - \beta)(\dot{k}_t/k_t) + \beta(\dot{h}_t/h_t),$$
$$\dot{i}_{Ht}/i_{Ht} = (\dot{A}_{Ht}/A_{Ht}) + (1 - \varepsilon)(\dot{k}_t/k_t) + \varepsilon(\dot{h}_t/h_t),$$
$$i_{Ht}/h_t = (\dot{h}_t/h_t) + \delta_H.$$

TABLE A1
US Productivity Estimates, 1890–2000

	Pre–World War II					
	1890	1900	1910	1920	1930	1940
$\dot{A}_{Gt}/A_{Gt}$	.00401	.00629	.01281	−.00833	.00243	.00557
$\dot{A}_{Ht}/A_{Ht}$	−.00522	.02713	.01925	.08448	.10628	.13727
	Post–World War II					
	1950	1960	1970	1980	1990	2000
$\dot{A}_{Gt}/A_{Gt}$	.02370	−.00221	.00743	−.00054	.00302	.013921
$\dot{A}_{Ht}/A_{Ht}$	.03001	.03840	.07663	.00347	.04298	.02993

Together it results that $(\dot{i}_H/h) - (i_H \dot{h}/h^2) = (\ddot{h}/h) - (\dot{h}^2/h^2)$. Multiplying this last equation through by h/i_H, it results that

$$\dot{i}_{Ht}/i_{Ht} = [(\ddot{h}_t/\dot{h}_t) + \delta_H]/[1 + \delta_H(h_t/\dot{h}_t)].$$

From $\dot{A}_{Gt}/A_{Gt}$ above, the TFP growth rate in the goods sector is

$$\dot{A}_{Gt}/A_{Gt} = (\dot{y}_{Gt}/y_{Gt}) - (1 - \beta)(\dot{k}_t/k_t) - \beta(\dot{h}_t/h_t).$$

From the two equations above in $\dot{i}_{Ht}/i_{Ht}$, the TFP growth rate in the human capital sector is

$$\dot{A}_{Ht}/A_{Ht} = [(\ddot{h}_t/\dot{h}_t) + \delta_H]/[1 + \delta_H(h_t/\dot{h}_t)] - (1 - \varepsilon)(\dot{k}_t/k_t) - \varepsilon(\dot{h}_t/h_t).$$

Using 10-year intervals of data, we construct a discrete form of the series for gross investment, with $i_{H,t} = h_t - h_{t-1}(1 - \delta_H^{10})$ and $g_{I,t} = (i_{H,t}/i_{H,t-1}) - 1$. The baseline calibration is an annual $\delta_H = 7$ percent such that the decade depreciation rate $\delta_{H,10} = 51.6$ percent satisfies $1 - \delta_{H,10} = (1 - \delta_{H,H})^{10}$.

Using the data set from Baier et al. (2006) and the above methodology within our calibrated economy, table A1 shows the computed US growth rate of productivity increases for the goods sector and the human capital sector for each decade from 1890 to 2000. These estimates indicate that the growth rate in the human capital investment sector exceeds that of the goods sector in every decade of the twentieth century; this gives unbalanced results similar to what McGrattan and Prescott (2010) exploit in a related context. Here the pre–World War II results indicate that the post–World War II results are not abnormal relative to the longer period, making the postwar results more plausible in this sense.

References

Agell, J., and M. Persson. 2001. "On the Analytics of the Dynamic Laffer Curve." *J. Monetary Econ.* 48 (October): 397–414.

Allingham, M. G., and A. Sandmo. 1972. "Income Tax Evasion: A Theoretical Analysis." *J. Public Econ.* 1:323–38.

Baier, S., G. Dwyer, and R. Tamura. 2006. "How Important Are Capital and Total Factor Productivity for Economic Growth?" *Econ. Inquiry* 44 (January): 23–49.

Bailey, M. 1956. "The Welfare Cost of Inflationary Finance." *J.P.E.* 64:93–110.

Baumol, W. J. 1952. "The Transactions Demand for Cash: An Inventory Theoretic Approach." *Q.J.E.* 66:545–56.

Beaudry, P., M. Doms, and E. Lewis. 2010. "Should the Personal Computer Be Considered a Technological Revolution? Evidence from U.S. Metropolitan Areas." *J.P.E.* 118 (5): 988–1036.

Beaudry, P., and P. Francois. 2010. "Managerial Skills Acquisition and the Theory of Economic Development." *Rev. Econ. Studies* 77 (1): 90–126.

Becker, G. S. 1965. "A Theory of the Allocation of Time." *Econ. J.* 75 (September): 493–517.

———. 1968. "Crime and Punishment: An Economic Approach." *J.P.E.* 76:169–85.

Benk, S., M. Gillman, and M. Kejak. 2010. "A Banking Explanation of the US Velocity of Money: 1919–2004." *J. Econ. Dynamics and Control* 34 (4): 765–79.

Berger, A., and D. Humphrey. 1997. "Efficiency of Financial Institutions: International Survey and Directions for Future Research." *European J. Operational Res.* 98 (2): 175–212.

Bowen, Howard R. 1987. *The Costs of Higher Education.* San Francisco: Jossey-Bass.

Buera, F. J., and J. Kaboski. 2012. "The Rise of the Service Economy." *A.E.R.* 102 (October): 2540–69.

Buera, F. J., and Yongseok Shin. 2013. "Financial Frictions and the Persistence of History: A Quantitative Exploration." *J.P.E.* 121 (2): 221–72.

Cagan, P. 1956. "The Monetary Dynamics of Hyperinflation." In *Studies in the Quantity Theory of Money*, edited by Milton Friedman. Chicago: Univ. Chicago Press.

Chen, B. L. 2003. "Tax Evasion in a Model of Endogenous Growth." *Rev. Econ. Dynamics* 6 (2): 381–403.

Clark, J. A. 1984. "Estimation of Economies of Scale in Banking Using a Generalized Functional Form." *J. Money, Credit, and Banking* 16 (1): 53–68.

Cloyne, J. 2013. "Discretionary Tax Changes and the Macroeconomy: New Narrative Evidence from the United Kingdom." *A.E.R.* 103 (June): 1507–28.

Dhami, S., and A. Al-Nowaihi. 2010. "Optimal Taxation in the Presence of Tax Evasion: Expected Utility versus Prospect Theory." *J. Econ. Behavior and Org.* 75 (2): 313–37.

Eckstein, Zvi, and Leonardo Leiderman. 1992. "Seigniorage and the Welfare Cost of Inflation: Evidence from an Intertemporal Model of Money and Consumption." *J. Monetary Econ.* 29 (June): 389–410.

Ehrlich, I. 1973. "Participation in Illegitimate Activities: A Theoretical and Empirical Investigation." *J.P.E.* 81 (3): 521–65.

———. 1996. "Crime, Punishment, and the Market for Offenses." *J. Econ. Perspectives* 10 (1): 43–67.

Ehrlich, I., and F. Lui. 1999. "Bureaucratic Corruption and Endogenous Economic Growth." *J.P.E.* 107, no. 6, pt. 2 (December): S270–S293.

Feldstein, M. 1995. "The Effect of Marginal Tax Rates on Taxable Income: A Panel Study of the 1986 Tax Reform Act." *J.P.E.* 103:551–72.

Fullerton, D., and M. Karayannis. 1994. "Tax Evasion and the Allocation of Capital." *J. Public Econ.* 55:257–78.

Gillman, M., and M. Kejak. 2005. "Inflation and Balanced-Path Growth with Alternative Payment Mechanisms." *Econ. J.* 115 (500): 247–70.

———. 2011. "Inflation, Investment and Growth: A Banking Approach." *Economica* 78 (310): 260–82.

Gomme, P., and P. Rupert. 2007. "Theory, Measurement and Calibration of Macroeconomic Models." *J. Monetary Econ.* 54 (2): 460–97.

Guryan, J. 2009. "The Race between Education and Technology: A Review Article." *J. Human Capital* 3 (2): 177–96.

Hall, Robert E., and Charles I. Jones. 1999. "Why Do Some Countries Produce So Much More Output per Worker than Others?" *Q.J.E.* 114 (1): 83–116.

Hancock, D. 1985. "The Financial Firm: Production with Monetary and Nonmonetary Goods." *J.P.E.* 93 (5): 859–80.

Jones, L. E., R. E. Manuelli, H. E. Siu, and E. Stacchetti. 2005. "Fluctuations in Convex Models of Endogenous Growth, I: Growth Effects." *Rev. Econ. Dynamics* 8 (October): 780–804.

Kaboski, Joe. 2009. "Education, Sectoral Composition, and Growth." *Rev. Econ. Dynamics* 12 (January): 168–82.

King, R. G., and S. Rebelo. 1990. "Public Policy and Economic Growth: Deriving Neoclassical Implications." *J.P.E.* 98, no. 2, pt. 2 (April): S126–S150.

Klenow, Peter J., and Andrés Rodríguez-Clare. 1997. "The Neoclassical Revival in Growth Economics: Has It Gone Too Far?" In *NBER Macroeconomics Annual 1997*, edited by Benjamin S. Bernanke and Julio J. Rotemberg, 73–103. Cambridge, MA: MIT Press.

Layard, P. R. G., and A. A. Walters. 1978. *Microeconomic Theory*. New York: McGraw-Hill.

Lucas, R. E., Jr. 1988. "On the Mechanics of Economic Development." *J. Monetary Econ.* 22:3–42.

———. 2000. "Inflation and Welfare." *Econometrica* 68 (March): 247–74.

Mark, N. C., and D. Sul. 2003. "Cointegration Vector Estimation by Panel DOLS and Long-Run Money Demand." *Oxford Bull. Econ. and Statis.* 65 (December): 655–80.

McGrattan, Ellen R. 2012. "Capital Taxation during the U.S. Great Depression." *Q.J.E.* 127 (3): 1515–50.

McGrattan, Ellen R., and Edward C. Prescott. 2010. "Unmeasured Investment and the Puzzling US Boom in the 1990s." *American Econ. J.: Macroeconomics* 2 (October): 88–123.

Mertens, Karel, and Morten O. Ravn. 2013. "The Dynamic Effects of Personal and Corporate Income Tax Changes in the United States." *A.E.R.* 103 (June): 1212–47.

———. Forthcoming. "A Reconciliation of SVAR and Narrative Estimates of Tax Multipliers." *J. Monetary Econ.*

Moffitt, R., and M. Wilhelm. 2000. "Taxation and the Labor Supply Decisions of the Affluent." In *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, edited by Joel Slemrod. Cambridge, MA: Harvard Univ. Press.

Pecorino, P. 1995. "Inflation, Human Capital Accumulation and Long-Run Growth." *J. Macroeconomics* 17 (3): 533–42.

Piketty, T., and E. Saez. 2006. "The Evolution of Top Incomes: A Historical and International Perspective." *A.E.R.* 96 (May): 200–205.

———. 2007. "How Progressive Is the U.S. Federal Tax System? A Historical and International Perspective." *J. Econ. Perspectives* 21 (Winter): 3–24.

Ragan, K. S. 2013. "Taxes and Time Use: Fiscal Policy in a Household Production Model." *American Econ. J.: Macroeconomics* 5 (1): 168–92.

Ramey, V. A., and N. Francis. 2009. "A Century of Work and Leisure." *American Econ. J.: Macroeconomics* 1 (July): 189–224.

Romer, Christina D., and David H. Romer. 2010. "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks." *A.E.R.* 100 (3): 763–801.

Roubini, N., and X. Sala-i-Martin. 1995. "A Growth Model of Inflation, Tax Evasion, and Financial Repression." *J. Monetary Econ.* 35 (2): 275–301.

Saez, E. 2012. "Inequality and Taxation: Evidence from Data, Experiments, and Tax Policy Variation." Award Abstract no. 1156240 (April 8), Nat. Sci. Found., Washington, DC.

Saez, E., J. Slemrod, and S. H. Giertz. 2012. "The Elasticity of Taxable Income with Respect to Marginal Tax Rates: A Critical Review." *J. Econ. Literature* 50 (1): 3–50.

Schneider, F., and D. Enste. 2000. "Shadow Economies: Size, Causes, and Consequences." *J. Econ. Literature* 38 (1): 77–114.

Sealey, C. W., and J. T. Lindley. 1977. "Inputs, Outputs, and a Theory of Production and Cost at Depository Financial Institutions." *J. Finance* 32 (4): 1251–66.

Slemrod, J. 2001. "A General Model of the Behavioral Response to Taxation." *Internat. Tax and Public Finance* 8 (2): 119–28.

Slemrod, J., and C. Weber. 2012. "Evidence of the Invisible: Toward a Credibility Revolution in the Empirical Analysis of Tax Evasion and the Informal Economy." *Internat. Tax and Public Finance* 19 (February): 25–53.

Thornton, D. 2012. "The U.S. Deficit/Debt Problem: A Longer Run Perspective." *Fed. Reserve Bank St. Louis Rev.* 94 (November/December): 441–56.

Trabandt, M., and H. Uhlig. 2011. "The Laffer Curve Revisited." *J. Monetary Econ.* 58 (4): 305–27.

Wall Street Journal. 2013. "UBS France Is Targeted in Tax-Evasion Probe." European ed., June 3.

Waud, R. N. 1988. "Tax Aversion, Optimal Tax Rates, and Indexation." Working Paper no. 2643, NBER, Cambridge, MA.

Yitzhaki, S. 1974. "Income Tax Evasion: A Theoretical Analysis." *J. Public Econ.* 3 (2): 201–2.