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Inflation, human capital and Tobin's q

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ABSTRACT

A strong US postwar low frequency negative correlation exists between inflation and Tobin's q. To explain this, a production-based monetary asset pricing model is formulated with a rising marginal cost of investment, cash-in-advance and human capital based endogenous growth. Higher money supply growth causes higher inflation, lower output growth, and a lower q in the long run. The baseline model simulates well correlations of the US inflation rate and Tobin's q at each frequency of high, business cycle, low, and the "medium term." It also performs well in correlations and volatilities compared to related exogenous growth versions.

1. Introduction

Danthine and Donaldson (1986), Labadie (1989) and Giovannini and Labadie (1991) use monetary general equilibrium endowment economies to take up the challenge of explaining the negative association between firm value and inflation.[1] The importance of such work is that while there is an emerging literature that shows how monetary policy, for example, affects the stock market through sticky wages and inflation targeting (Christiano et al., 2008), less is known of the longer run effect of inflation taxes on the stock market. Here we extend such work to explain the following evidence on low frequency correlation between firm value and inflation.

Fig. 1 illustrates the US quarterly postwar negative correlation between the inflation rate and Tobin's q, one measure of firm value. Fig. 2 brings out the low frequency aspect of this data's negative correlation using a Christiano and Fitzgerald (2003) band pass filter with a 32–100 window. Fig. 3 shows a similar UK low frequency negative correlation.[2] Figs. 1 and 2 have a US quarterly sample period of 1960:1–2007:4; their negative correlation is particularly pronounced starting in the late 1960s when inflation began a long trend upwards. Tobin's q bottoms out around the early 1980s when inflation peaks.

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[1] Fama (1981) begins his study by stating "There is much evidence that common stock returns and inflation have been negatively related during the post-1953 period."

[2] The raw US data for q used in Figs. 1 and 2 come from the Smithers & Co (http://www.smithers.co.uk/) and the inflation rate is based on implicit price deflator; a very similar negative correlation also results using Hall's (2001) Tobin's q estimates. Fig. 3 UK quarterly CPI data are from the Office of National Statistics; Tobin's q data are from the Bank of England, in which the methodology for computing q is described in Price and Schleicher (2005). Note that the low frequency filters here lose the first and last three years of data.

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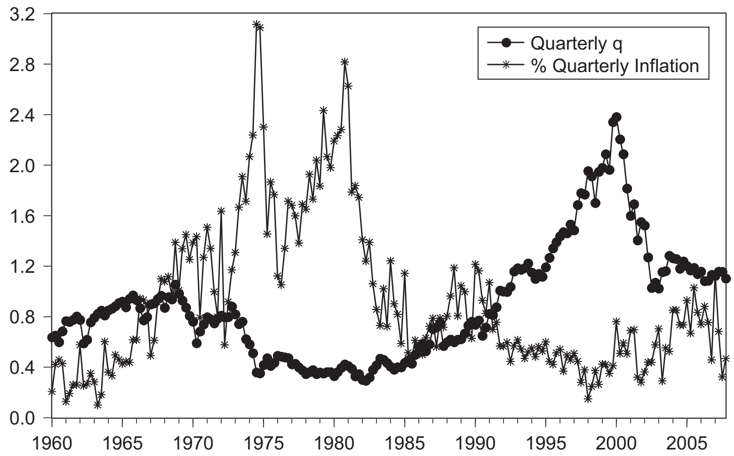

Fig. 1. US q and Inflation: 1960Q1–2007Q4.

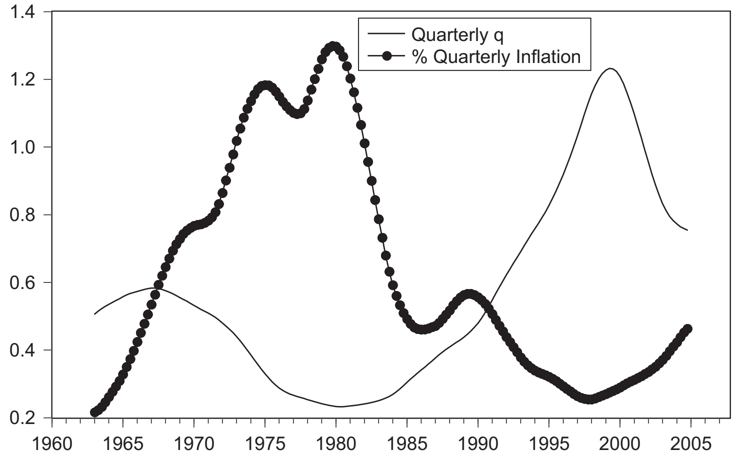

Fig. 2. Low frequency q-inflation relationship: US.

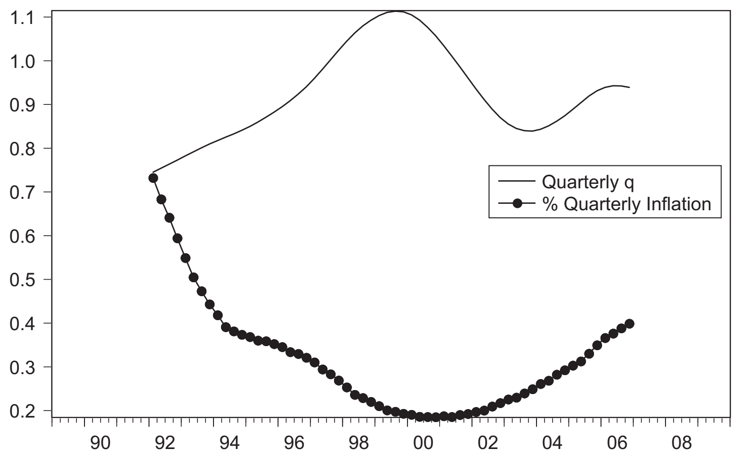

Fig. 3. Low frequency q-inflation relationship: UK.

The subsequent rise of q coincides with an era of further disinflation and high economic growth. Then q reaches an all time high around 2000 shortly after inflation falls to its low around 1%. Since then q has fallen and inflation has risen. Fig. 3 has a quarterly sample period of 1989:1–2009:4. The US and UK correlation coefficients are -0.73 and -0.72 respectively which are statistically significant at the 1% level.

Comin and Gertler (2006) identify low frequency oscillations for several US macroeconomic aggregates, although not Tobin's q and inflation, using a filter with a periodicity of 32–200 quarters. Given 47 years of data in our US sample, the low frequency is defined with a window of 32–100 quarters, instead of 32–200 quarters, using a Christiano–Fitzgerald (2003)

Table 1
Robustness check for low frequency US q-inflation correlation.

Band pass	1960Q1–2007Q4	1965Q1–2007Q4
32–100 qrs	−0.7319**	−0.7646**
32–150 qrs	−0.7377**	−0.7707**

Note: * means statistically significant at the 5% level and ** means statistically significant at the 1% level.

band-pass filter without detrending.[3] Table 1 shows robustness also with respect to the filter window and the data period. A window of 32–150 gives a marginally stronger negative correlation. In the 1965Q1–2007Q4 sample, the correlation is more negative for all windows.[4]

Firm market value has long been associated with output growth,[5] just as the inflation tax has been shown in theory and evidence to reduce the long term output growth rate (Gomme, 1993; Gillman and Kejak, 2005; Gillman et al., 2004). To better explain the inflation-q correlation, this literature plus Figs. 1–3 suggest linking the hypothesis that the inflation tax affects output growth and then firm valuation measures such as Tobin's q at low frequencies. This view requires shifting to a baseline production-based, two-sector, dynamic stochastic general equilibrium (*DSGE*) within an endogenous growth monetary setting. Along the balanced growth path of our economy, an increased money supply growth rate causes higher inflation, a lower output growth rate, via lower human capital utilization rate, a lower marginal cost of physical capital investment, and subsequently a lower q. Standard shocks to sectoral productivities and money supply rates generate simulated cyclic behavior that is compared to correlation and volatility evidence and to simulated performance of related models.[6]

The baseline results of correlations support the q and inflation rate negative link found in low frequency data. These correlations match data also using windows at a high frequency, business cycle, low frequency and a Comin and Gertler (2006) type "medium term" window that combines all these frequencies. The emphasis is on the low frequency window because of the balanced growth path linkage of inflation and q. Here also the model compares well for correlations of q with output growth and with normalized output levels, as well as for inflation with output growth and output levels. The robustness comparison is relative to two related models with exogenous growth.

The baseline model has three features which give rise to a negative relation between q and inflation: (i) a Lucas and Prescott (1971) physical capital adjustment cost with a rising marginal cost of investment,[7] such that along the balanced growth path (BGP) q is a function of the output growth rate (Proposition 3), (ii) human capital investment that endogenizes the balanced growth path equilibrium (*BGP*) growth rate (Lucas, 1988), and (iii) a cash-in-advance inflation tax economy (Lucas, 1980). In a related paper, McGrattan and Prescott (2005) argue that the rise until 2000 of the US stock price to GDP ratio is due to lower taxes on corporate distributions to shareholders. Our paper complements this approach from the monetary side through the inflation tax effects on firm value. McGrattan and Prescott focus on intangible capital; we use human capital which often is partly accounted for within intangible capital.[8] Relative to Jones et al. (2005), their human capital shock here instead arises as an independent shock to productivity in a separate household sector producing human capital, as related to DeJong and Ingram (2001).[9] This gives a slowly adjusting human capital that acts as an alternative approach for example to habit formation as in Boldrin et al. (2001). Closely related to our baseline model is Maffezzoli's (2000) endogenous growth two-country model with human capital, physical capital adjustment costs, and related shocks.

Section 2 sets out the baseline model; Section 3 analyzes the *BGP* equilibrium q, and Section 4 provides the US postwar calibration. Section 5 presents impulse response analysis; Section 6 presents the main cyclical correlation and volatility results; and Section 7 summarizes the paper and suggests extensions.

[3] In a similar spirit to Comin and Gertler (2006), we did not detrend the series in order to keep low frequency components that are of central interest in the paper.

[4] The low frequency q-inflation correlation is robust to the choice of q and inflation series. Alternative shorter series (up to 1999) for q of Hall (2001) and also the use of CPI rate of inflation produce similar estimates for this correlation.

[5] For example, see Hansen and Scheinkman (2009, 2011), and Cagetti et al. (2002).

[6] Related models are Basu (1987) and Gillman and Kejak (2010). An empirical money supply and inflation link at low frequencies continues to be found in samples including the US and UK, for example in Haug and Dewald (2011); this offers support of the channel of money supply shocks affecting inflation, and possibly q, at low frequencies.

[7] Basu (1987) and Hercowitz and Sampson (1991) use the same adjustment cost, but with 100% depreciation, while we relax that assumption; Belo et al. (2010) find empirical support for a rising marginal cost of investment.

[8] Tobin's q is the ratio of stock price index to the capital stock which can be alternatively written as stock price/GDP multiplied by GDP/capital stock. Given that the output/capital ratio is stable, as pointed out by McGrattan and Prescott (2005), the stock price/GDP can also reflect Tobin's q behavior in the long run.

[9] Two-sector models go back to Benhabib et al. (1991) and Greenwood and Hercowitz (1991), and include Gomme and Rupert (2007).

2. The model

2.1. The representative household

The representative household allocates time between leisure (x_t), work in the goods sector (l_{Gt}) at a nominal wage W_t, and work in the human capital investment (l_{Ht}). Households own the human capital (h_t) and augment it through human capital investment. Firms own the physical capital (k_t) and accumulate it through physical investment (i_t).

At time t, households first trade in goods with the cash held in advance, M_t, and then they visit the asset markets to trade in stocks at the ex-dividend prices V_t and in nominal bonds at the price P_t^b. Nominal bonds B_t held at date t pay 1 unit of currency with certainty in the following periods. Money is used to buy goods, and is augmented by the central bank through a stochastic nominal lump-sum transfer N_t, which with market clearing in equilibrium equals $\mu_t M_{t-1}$; μ_t is the stochastic growth rate of money supply.

At date t, the revenues of the household are nominal dividends per ownership share in the goods producer, D_t, factored by the shares z_t, plus wages $W_t l_{Gt} h_t$ and the lump sum transfer N_t. Expenses are investment in bonds, $P_t^b B_{t+1} - B_t$, in cash, $M_t - M_{t-1}$, and in stocks, $V_t(z_{t+1} - z_t)$, plus consumption purchases $P_t c_t$.

The household maximizes the following life time utility function:

$$\text{Max } E_0 \sum_{t=0}^{\infty} \beta^t \{U(c_t) + \psi \Gamma(x_t)\} \tag{1}$$

where $U(\cdot)$ and $\Gamma(\cdot)$ are monotonically increasing and strictly concave functions, with the parameter $\psi \geq 0$, subject to the flow budget constraint facing the household,

$$D_t z_t + W_t l_{Gt} h_t + N_t - (P_t^b B_{t+1} - B_t) - V_t(z_{t+1} - z_t) - (M_t - M_{t-1}) - P_t c_t = 0, \tag{2}$$

time allocation,

$$1 = x_t + l_{Gt} + l_{Ht}, \tag{3}$$

and human capital accumulation and exchange constraints.

Human capital investment is linear in effective labor time $l_{Ht} h_t$ as in Lucas (1988), with a depreciation rate of δ_h and with A_{Ht} the exogenous sectoral productivity shock, giving the accumulation constraint of

$$h_{t+1} = (1 - \delta_h) h_t + A_{Ht} l_{Ht} h_t. \tag{4}$$

The exchange constraint requires money to purchase consumption such that

$$P_t c_t \leq M_{t-1} + N_t. \tag{5}$$

The household first order conditions are found in Appendix A. The standard stochastic discount factor m_{t+1} facing the household is given by

$$m_{t+1} \equiv \frac{\beta E_{t+1}\left[U'(c_{t+2}) \dfrac{c_{t+2}}{c_{t+1}} \dfrac{1}{1+\mu_{t+2}}\right]}{E_t\left[U'(c_{t+1}) \dfrac{c_{t+1}}{c_t} \dfrac{1}{1+\mu_{t+1}}\right]}. \tag{6}$$

Using Eqs. (A.13) and (A.14) in Appendix A, the stock price and bond price equations can be written typically as

$$1 = E_t m_{t+1} \left\{ \frac{v_{t+1} + d_{t+1}}{v_t} \right\} \tag{7}$$

and

$$p_t^b = E_t m_{t+1}, \tag{8}$$

where v_t is the real share price, $v_t \equiv V_t / P_t$ and p_t^b is the real price of bond that satisfies $p_t^b \equiv P_t^b / P_t$.

2.2. The firm's problem

The firm produces output y_t with a Cobb–Douglas production function $A_{Gt} F(k_t, l_{Gt} h_t)$ in physical capital k_t and effective labor $l_{Gt} h_t$, with A_{Gt} the stochastic total factor productivity (*TFP*) at date t, and $\alpha \in (0,1)$, such that

$$y_t = A_{Gt} F(k_t, l_{Gt} h_t) = A_{Gt} k_t^{\alpha} (l_{Gt} h_t)^{1-\alpha}. \tag{9}$$

The firm costs are wages and the nominal physical capital investment $P_t i_t$. With λ_t the shadow price of the consumer's nominal income in Eq. (2), A_{Gt} the stochastic total factor productivity (*TFP*) at date t, and $\alpha \in (0,1)$, the firm solves

$$\underset{\{l_{Gt}\},\{i_t\}}{\text{Max}} E_0 \sum_{t=0}^{\infty} \lambda_t [P_t A_{Gt} k_t^{\alpha} (l_{Gt} h_t)^{1-\alpha} - W_t l_{Gt} h_t - P_t i_t] \tag{10}$$

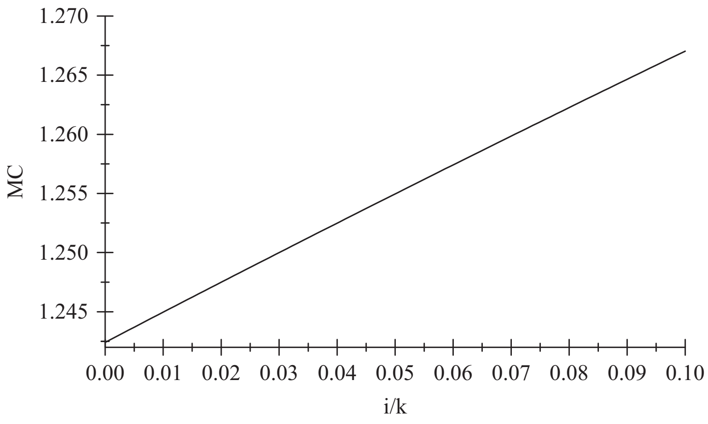

Fig. 4. Marginal cost of investment function, MC_t.

subject to the physical capital accumulation constraint, for $\delta_k \in (0,1)$ and $\theta \in (0,1)$ of

$$k_{t+1} = k_t \left[1 - \delta_k + \frac{\xi_t i_t}{k_t} \right]^{\theta}, \tag{11}$$

as in Basu (1987) and Hercowitz and Sampson (1991). The parameter θ represents the extent of adjustment cost; with $\theta = 1$ there is no adjustment cost. ξ_t represents an investment specific technology (IST) component. We fix this at a steady state level $\overline{\xi}$ for calibrating the long run Tobin's q.[10]

The marginal cost of investment, MC_t, can be expressed by solving for i_t in Eq. (11), and differentiating with respect to next period capital, as

$$MC_t \equiv \frac{\partial i_t}{\partial k_{t+1}} = \frac{1}{\theta \overline{\xi}} \left(\frac{k_{t+1}}{k_t} \right)^{(1-\theta)/\theta} = \frac{1}{\theta \overline{\xi}} \left(1 - \delta_k + \frac{\overline{\xi} i_t}{k_t} \right)^{1-\theta}, \tag{12}$$

which is rising in k_{t+1}, or in i_t. Fig. 4 graphs the MC_t function for a varying i_t/k_t, given $\theta = 0.8$, $\delta_k = 0.03$ and $\overline{\xi} = 1.4$, as in the baseline calibration below. The marginal cost rises as the investment rate rises.

This MC_t is almost linear in a range that is specified for reasonable growth rates. This standard near-linearity holds for most values of θ, such as for $\theta \in (0.15,1)$, within the growth rate range, while for very low values of θ some concavity is evident. The MC_t schedule undergoes a downward shift in response to an IST progress (higher $\overline{\xi}$). Thus Tobin's q of capital is inversely related to the IST component, $\overline{\xi}$.

In comparison, Belo et al. (2010) use a related adjustment cost function, whereby investment plus their adjustment cost, with the sum denoted by C_t, is given with their parameters of v and a by

$$C_t \equiv i_t + \frac{a}{v} \left(\frac{i_t}{k_t} \right)^{v} k_t,$$

with a marginal cost of $\partial C_t / \partial k_{t+1} = 1 + a(i_t/k_t)^{v-1}$. Like Fig. 4, this gives a rising marginal cost, but one that can be quite convex, mainly through the curvature parameter, v. They find empirical support for significant convexity in their *GMM* estimation; however these interesting results are based on a partial equilibrium model that is not directly comparable to our *DSGE*, endogenous growth setting.

The endogenous growth setting in particular distinguishes our model for example from the *DSGE* model of Christiano et al. (2008). Their comparable equation to our equation (11) is

$$\frac{k_{t+1}}{k_t} = 1 - \delta + \frac{i_t}{k_t} - z \left(\frac{i_t}{k_t} - \eta \right)^2,$$

where z is a parameter and η is the steady state investment to capital ratio. Their adjustment cost is, therefore, zero in the steady state, as in Lucas (1967), while in our specification in Eq. (11), the adjustment cost is positive along the balanced growth path equilibrium.

[10] Such an IST lowers marginal cost of investment and thus the q. We do not bring any stochastic element to ξ_t and fix it at a steady state level for calibration purposes (see also Basu, 2009). In an extended model, such an IST component in the investment technology can be motivated by introducing intangible capital in the form of R&D or organizational capital as in McGrattan and Prescott (2005).

2.3. Forcing processes

The exogenous variables A_{Gt}, A_{Ht}, μ_t follow the processes:

$$A_{Gt}-\overline{A}_G=\rho_G(A_{Gt-1}-\overline{A}_G)+\epsilon_t^G, \tag{13}$$

$$A_{Ht}-\overline{A}_H=\rho_H(A_{Ht-1}-\overline{A}_H)+\epsilon_t^H, \tag{14}$$

$$\mu_t-\overline{\mu}=\rho_\mu(\mu_{t-1}-\overline{\mu})+\epsilon_t^\mu, \tag{15}$$

where ϵ_t^G, $\epsilon_t^H\epsilon_t^\mu$ are white noises with standard deviations σ_G,σ_H and σ_μ respectively. We assume zero contemporaneous covariances between these three shocks. Letters with a bar represent steady state values.

2.4. Characterization of equilibrium

(E.1): Given the processes $\{P_t\}$, $\{W_t\}$, $\{D_t\}$,$\{A_{Ht}\}$,$\{V_t\}$, $\{P_t^b\}$, and $\{N_t\}$, the household maximizes utility in Eq. (1) subject to Eqs. (2)–(5).

(E.2): Given the processes $\{P_t\}$, $\{W_t\}$, $\{A_{Gt}\}$, the goods producer maximizes (10) subject to (11).

(E.3): Spot assets, goods, and money markets clear: $z_t=1$, $B_t=0$, and $N_t=\mu_t M_{t-1}$.

3. Tobin's *q*

The shadow price of physical capital investment normalized by the shadow price of consumption gives a standard expression for Tobin's q.

Proposition 1.

$$q_t \equiv \frac{\omega_t}{P_t\lambda_t}=\frac{1}{\theta\overline{\xi}}\left[1-\delta_k+\frac{\overline{\xi}i_t}{k_t}\right]^{1-\theta}. \tag{16}$$

Proof. This follows directly from the first order condition with respect to physical capital investment, Eq. (A.17) of Appendix A, where the shadow price of consumption $P_t\lambda_t$ is the shadow price of nominal income in Eq. (2) of the household problem multiplied by the nominal price level P_t. □

Corollary 1. *Tobin's q equals the marginal cost of investment, which is rising in* k_{t+1}.

Proof. By Eqs. (16) and (11), $q_t=(1/\theta\overline{\xi})(k_{t+1}/k_t)^{(1-\theta)/\theta}$, which by Eq. (12) is the marginal cost of investment; and $\partial q_t/\partial k_{t+1}>0$. □

As in a standard quadratic q model of investment, the marginal cost of investment here also equals the average q based on the stock market valuation equation (Obstfeld and Rogoff, 1996). In other words,

Proposition 2. *The marginal and average q are the same; in that*

$$q_t=\frac{v_t}{k_{t+1}}.$$

The proof of this proposition is standard, and presented for this model in Appendix B.1.

As investment increases its marginal and average cost rise. And this cost is closely connected to the economy's growth rate. Along the *BGP*, q depends positively on the growth rate, and in turn on the return to capital.

Hereafter, log utility is specified, with $U(c_t)=\ln c_t$ and $\Gamma(x_t)=\ln x_t$. We have the following key proposition.

Proposition 3. *Along the balanced growth path, Tobin's q is a simple rising function of the growth rate and a falling function of the adjustment cost parameter* θ *whereby*

$$q=\frac{1}{\theta\overline{\xi}}(1+g)^{(1-\theta)/\theta}, \tag{17}$$

and this can be expressed through g in terms of either the return on physical or on human capital.

Proof. From Corollary 1, and given that $k_{t+1}/k_t=1+g$ along the *BGP*, then $q=(1/\theta\overline{\xi})(1+g)^{(1-\theta)/\theta}$ and $\partial q/\partial g>0$, and $\partial q/\partial\theta<0$. Further, as shown in Appendix B.2, the balanced growth rate in this economy is given in terms of the physical capital net return $\overline{A}_G F_1-\delta_k$ by

$$1+g=\left[\frac{\beta\theta(1+\overline{A}_G F_1-\delta_k)}{1-\beta(1-\theta)}\right]^\theta, \tag{18}$$

and in terms of the human capital net return of $\overline{A}_H(1-x)-\delta_h$ by

$$1+g=\beta[1+\overline{A}_H(1-x)-\delta_h]. \tag{19}$$

And so

$$q=\frac{1}{\theta}\left[\frac{\beta\theta(1+\overline{A}_GF_1-\delta_k)}{1-\beta(1-\theta)}\right]^{1-\theta}=\frac{1}{\theta\overline{\xi}}(\beta[1+\overline{A}_H(1-x)-\delta_h])^{(1-\theta)/\theta}. \qquad \square \tag{20}$$

A higher *BGP* return on capital, with the return on human and physical capital equal along the *BGP*, causes a higher growth rate and a higher q. A repeated persistent shock, such as an increased money supply during the 1970s, can act to lower the growth rate over an extended period of time, and contribute to an observed low frequency decrease in q. In particular, a repeated, persistent positive money supply rate increase contributes to higher inflation over time, substitution from goods to leisure, a lower human capital utilization rate of $1-x$, and a lower return on both human and physical capital, and in q. With exogenous growth, or without an adjustment cost of physical capital (if $\theta=1$ and $q=1$), there is less such potential interaction between the economy-wide growth rate, the capital return and q that can produce the low frequency inflation and q correlation found in the data.

4. Calibration

In calibrating a standard DSGE growth model, typically only business cycle properties are matched, using exogenous growth models. Endogenous growth also allows examination of the low frequency properties of the simulated model relative to the data. This additional step involves setting the structural parameters to calibrate the growth component of the model, along with low frequency and business cycle aspects. Here, the focus is on the pronounced low frequency fluctuation in two endogenous variables, Tobin's q and inflation.

4.1. Data

Following Baxter and King (1999) and Comin and Gertler (2006), the low frequency component of a series has a periodicity of longer than 32 up to 200 quarters, the business cycle component a periodicity of 6–32 quarters, and the high frequency component a periodicity of 2–6 quarters, given a minimum duration of a cycle as being two quarters. Therefore the low frequency component is identified using a band pass filter to eliminate the periodicity of 2–32 quarters. It is defined with a window of 32 up to 100 quarters given the 47 years of data in our sample.

4.2. Target variables and parameter values

Table 1 presents the data for target variables and the values of these variables in the steady state baseline model. Except for q and leisure x, the data are annual averages of 1960–2007 US data, from the National Income and Product Accounts. The average data value for q is 0.90, using data from Smithers and Co. (2007) as computed with the methodology of Wright (2004). The data for leisure x is 0.50 as in Gomme and Rupert (2007). The average data values of g and π are 3.33% and 4.2%; and the average share in *GDP* of consumption plus government spending is 84% (government expenditure is abstracted from the model and considered as consumption). The calibrated model is close to the target values.

Table 3 gives the baseline model parameter values. Standard values are chosen for β, α, and ψ. The mean money supply growth rate, μ is chosen to be consistent with the 4.2% annual average inflation rate of the data, whereby it is the sum of the latter and the growth rate g. The human capital technology parameters $\overline{A}_H$ and δ_h are set so as to target the annual average GDP growth rate; the human capital utilization rate of $1-x$ is based on Eqs. (4) and (19). The physical capital depreciation rate is set at 0.03 in line with Benk et al. (2009). The remaining two investment technology parameters θ and $\overline{\xi}$ are fixed at 0.8 and 1.4 respectively as part of targeting the long run average q at 0.9.[11]

4.3. Shock process parameters

Table 4 reports the baseline values of the shock processes. The three forcing processes described in (13)–(15) involve six parameters, namely three autocorrelation parameters, ρ_G, ρ_H, ρ_μ, and three standard deviation parameters, σ_G, σ_H, σ_μ. The money supply parameters ρ_μ and σ_μ are 0.74 and 0.004, as estimated from an $AR(1)$ regression of quarterly seasonally adjusted currency supply growth from the Federal Reserve Bank of St. Louis database for 1960–2007.

For the other two shocks, the closest paper may be Maffezzoli (2000) who employs similar stochastic goods and human capital technologies, although Maffezzoli has an international focus, plus human capital spillover and the use of both

[11] Multiple combinations of θ and $\overline{\xi}$ can be consistent with a q of 0.9. Given the incomplete depreciation of capital in our setting, our estimate of θ is reasonable; changing θ and $\overline{\xi}$ in the vicinity of the baseline values does not affect the key calibration results.

physical and human capital in the Cobb–Douglas production of human capital. As in Maffezzoli, ρ_G and ρ_H are both set at 0.96. Because of the presence of an additional monetary shock in our model, we choose a smaller standard deviation than Maffezzoli. We set $\sigma_G = \sigma_H = 0.001$ with a view to target the standard deviations of growth and inflation.

5. Impulse response analysis

There are two types of structural shocks, real productivity shocks, one in each of the goods and human capital investment sectors, and a money supply growth rate shock. Both productivity shocks tend to induce a negative correlation between inflation and q as well as between inflation and growth, but the human capital sector shock has such an effect that is an order of magnitude stronger than that of the goods sector. The monetary shock induces a Tobin (1965) type effect of an increase in physical capital accumulation that initially weakens the negative q-inflation correlation, but then marginally strengthens this correlation over an extended period.

To see these effects, the impulse responses to orthogonalized shocks to A_G, A_H and μ are shown as based on the log-linearization of the full equation system (A.19)–(A.25) that is given in Appendix A. In Figs. 5–7, the notation is "*iy*" $\equiv i/y$, "*kh*" $\equiv k/h$, "*lg*" $\equiv l_G$, "*lh*" $\equiv l_h$, "*infl*" $\equiv \pi$.

Fig. 5 shows that a positive productivity shock in the goods sector makes agents substitute away from human capital investment time and leisure towards labor. This effort shocks upwards the physical capital investment rate (iy), with a consequent gradual increase in the physical capital to human capital ratio (kh). The output growth rate (g) falls as the physical capital investment rate rises via diminishing marginal return to capital. The greater productivity also raises the real wage and lowers the relative price of output, causing the inflation rate (infl) to be initially shocked downwards. q initially rises, as the investment rate and the labor in the goods sector are shocked upwards, as can be seen in Eq. (21) below, which is derived simply by using the average product of capital y_t/k_t and Eq. (16):

$$q_t = \frac{1}{\overline{\xi}\theta}\left[1-\delta_k+\frac{\overline{\xi} i_t}{y_t}A_{Gt}\left(\frac{k_t}{h_t}\right)^{\alpha-1} l_{Gt}^{1-\alpha}\right]^{1-\theta}. \tag{21}$$

However as k_t/h_t gradually rises, this pushes q down. As k_t/h_t begins to fall, the investment rate i_t/y_t falls below its baseline and so does q. Meanwhile the inflation rate rises over time, moving in negative correlation to the q effects.

Fig. 6 shows that a positive shock to A_H causes agents to switch from leisure and labor in goods production towards human capital investment time, causing the growth rate to rise. The physical investment rate declines as the consumer shifts towards human capital investment and a lower k_t/h_t. A lower l_G and investment rate shock q downwards, again as

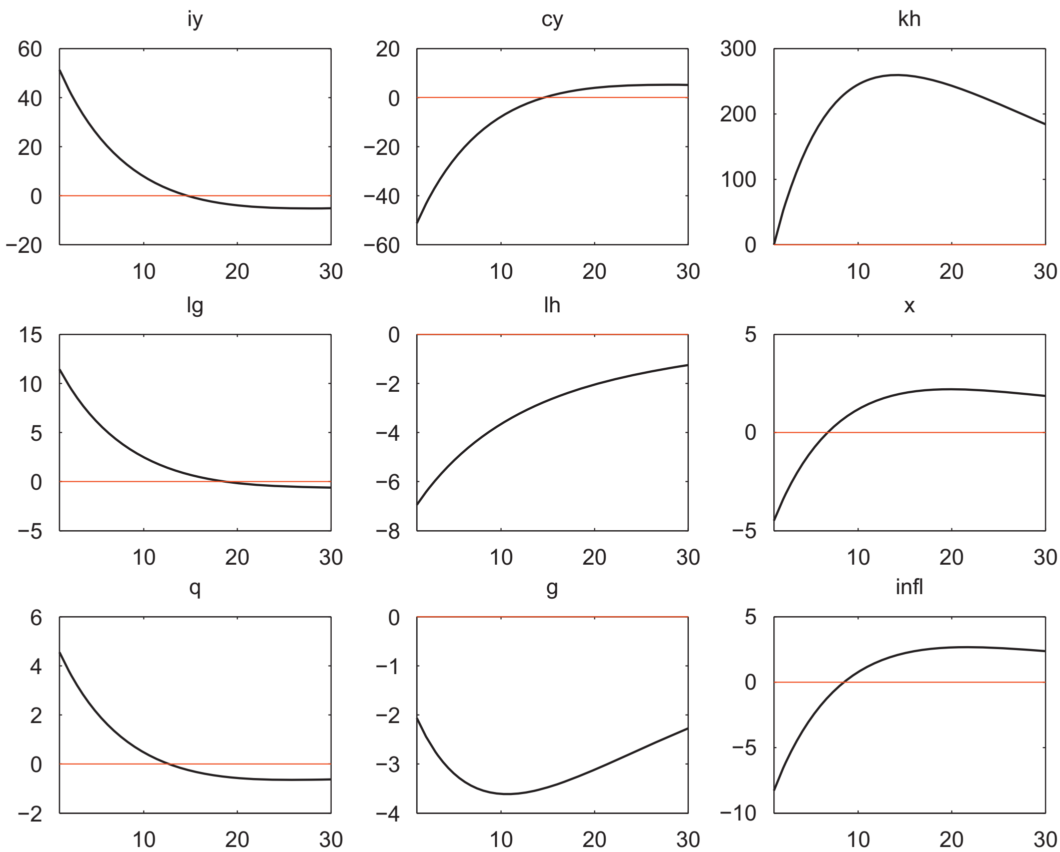

Fig. 5. Impulse responses in percent of standard error of an orthogonalized shock to ϵ_t^G.

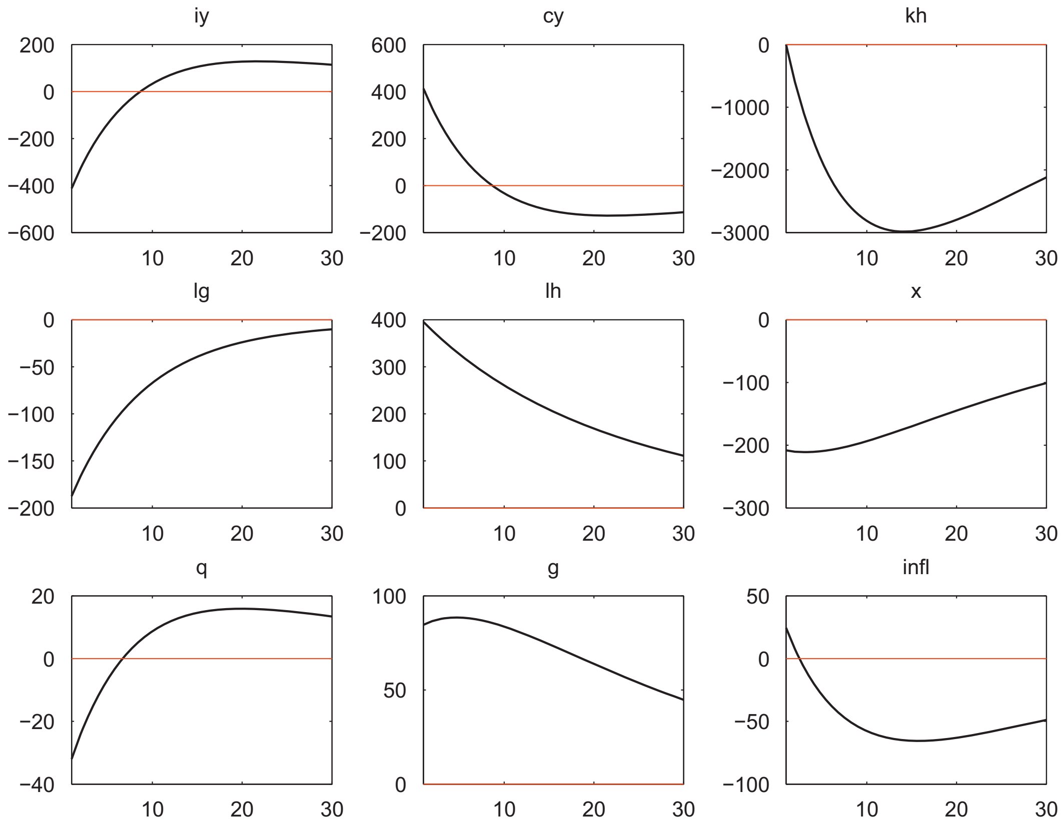

Fig. 6. Impulse responses in percent of standard error of an orthogonalized shock to ϵ_t^H.

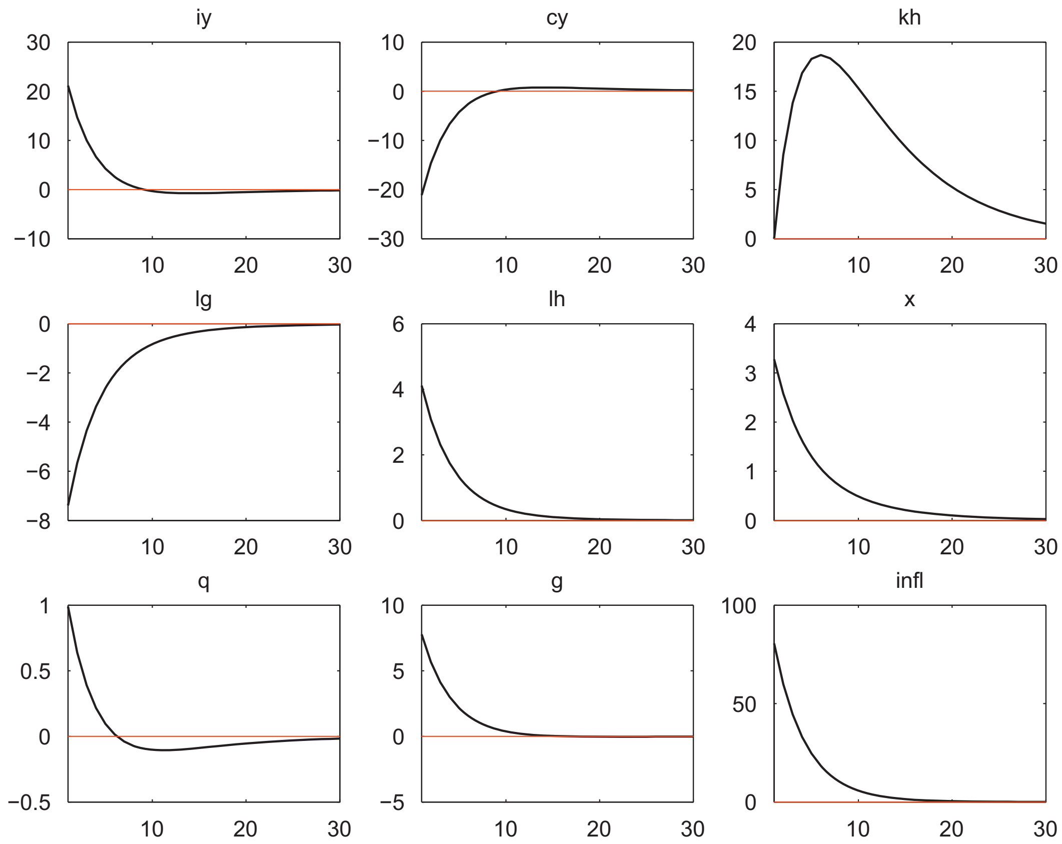

Fig. 7. Impulse responses in percent of standard error of an orthogonalized shock to ϵ_t^μ.

Table 2
Values of the growth model target variables: actual and model.

Target variables, 1960–2007	Data	Model
GDP growth (g) (%)	3.33	3.26
Rate of inflation (π) (%)	4.2	4.03
c/y	0.84	0.79
i/y	0.13	0.21
q	0.90	0.90
Leisure (x)	0.50	0.52

Table 3
Baseline structural parameter values.

β	α	δ_k	δ_h	ψ	$\overline{\xi}$	θ	$\overline{A}_G$	$\overline{A}_H$	μ
0.96	0.36	0.03	0.024	1.84	1.4	0.8	1.2	0.21	0.075

Table 4
Baseline second moment parameter values.

ρ_G	ρ_H	ρ_μ	σ_G	σ_H	σ_μ
0.96	0.96	0.74	0.001	0.001	0.004

in Eq. (21). Inflation falls over time as the increased human capital time leaves less time for goods production, causing a higher wage rate and lower relative price of output. Over time q and inflation move in opposite directions, giving rise to a stronger negative correlation as compared to the A_G shock above.

In Fig. 7, a positive monetary shock raises the inflation rate, thereby inducing substitution from goods to leisure and human capital investment, which are not subject to the inflation tax. The initial rise in the investment rate (iy) corresponds to the gradual rise in the physical capital to human capital ratio (kh), and a rise in q. There is a positive Tobin (1965) effect of inflation on the investment rate that raises q and contributes to positive correlation between q and inflation.

6. Results

The baseline model simulations are compared to the data and to the simulations of two related exogenous growth models. Data sources are detailed in Appendix C. The two exogenous growth models are special cases of our baseline endogenous growth model. First, human capital h_t is assumed to grow at an exogenous balanced growth rate, g, as set at 3.33% in Table 2. Then two variants are examined. First, assuming a fixed labor supply ($\bar{l}_G$) defines a comparative exogenous growth model which we call Exog1. The second variant that we call Exog2 still assumes human capital growing at the rate g, but in addition allows for normal labor-leisure choice as in the baseline model. Note that since the human capital production technology (4) is dropped from both versions of the exogenous growth models, there are only two shock processes, A_{Gt} and μ_t, kept at the same level as in Table 4 for comparability.

Table 5 evaluates the performance of these growth models in matching the correlations between relevant macrovariables at different frequencies: (i) high frequency of 2–6 quarters, (ii) business cycles frequency of 6–32 quarters, (iii) low frequency of 32–100 quarters, (iv) and the medium term of 2–100 quarters.[12] Given that the low frequency correlation between q and inflation is of central interest in our paper, the baseline endogenous model predicts the sign and magnitude of the low frequency correlation remarkably well. On the other hand, the exogenous growth models mispredict the sign of this key correlation. The reason for the stark failure of the exogenous growth model in this regard would appear to be the missing human capital transmission mechanism which is reflected in the persistent inverse comovement between q and inflation in response to a shock to A_H (see, Fig. 6). Table 5 shows also that the baseline model does well in simulating the low and medium term frequency correlation of inflation with the normalized output level and with the output growth rate.

In Table 6, the q and output growth correlation is close to data at the medium term frequency. It does comparatively well in the medium term for the correlation between inflation and the employment (l_G). Other financial variables in Table 6

[12] For the raw data, the same symmetric weight frequency filter of Christiano and Fitzgerald (2003) is used to compute the second moments of all relevant variables at different frequencies. None of the series were detrended except the labor hours which shows a clear downward trend during our sample period.

Table 5
Matching key correlations.

Variable	High freq. 2–6 qrs	Bus cyc. 6–32 qrs	Low freq. 32–100 qrs	Med term 2–100 qrs
corr(q_t,π_t)				
Data	−0.076	−0.245**	−0.732**	−0.461**
Endo	−0.028*	−0.121*	−0.751*	−0.338*
Exog1	0.819	0.702	0.124	0.588
Exog2	0.944	0.876	0.361	0.791
corr$\left(\frac{y_t}{y_{t-1}},\pi_t\right)$				
Data	0.029	−0.459**	−0.434**	−0.278**
Endo	0.459	0.102	−0.725	−0.263*
Exog1	0.965	0.886	0.091	0.636
Exog2	0.989	0.945	0.358	0.850
corr(y_t/h_t,π_t)				
Data	−0.042	0.213**	0.550**	0.280**
Endo	−0.254	0.075	0.760	0.368*
Exog1	−0.085	−0.005	0.149	0.067
Exog2	−0.110	0.017	0.237	0.123
corr(c_t/h_t,π_t)				
Data	−0.050	−0.093	0.083	0.023
Endo	−0.495	0.094	0.800	0.408
Exog1	−0.901	−0.681	0.057	−0.321
Exog2	−0.971	−0.854	−0.007	−0.588
corr(i_t/h_t,π_t)				
Data	−0.070	0.100	−0.354**	−0.093
Endo	−0.004	0.013	0.080	0.031
Exog1	0.820	0.722	0.353	0.632
Exog2	0.946	0.902	0.627	0.855

Note: For the data * and ** mean statistically significant at the 5% and 1% levels respectively. For the models $*$ means it is within the 95% confidence band with the data. The confidence interval is computed by using Fisher's z transform.

Table 6
Matching other variable correlations.

Variable	High freq. 2–6 qrs	Bus cyc. 6–32 qrs	Low freq. 32–100 qrs	Med term 2–100 qrs
corr(l_{Gt},π_t)				
Data	0.126	0.140	0.580**	0.362*
Endo	−0.303	−0.163	0.416*	0.009
Exog1	0	0	0	0
Exog2	−0.947	−0.908	−0.913	−0.892
corr(c_t/h_t,i_t/h_t)				
Data	0.066	0.809**	0.739**	0.711**
Endo	−0.001	0.043	0.219	0.112
Exog1	−0.544	−0.186	0.786	0.247
Exog2	−0.862	−0.686	0.532	−0.343
corr(i_t/h_t,y_t/h_t)				
Data	0.728**	0.856**	0.063	0.561**
Endo	0.862	0.714	0.426	0.500*
Exog1	0.478	0.605	0.887	0.678
Exog2	0.202	0.354	0.780	0.468*
corr(c_t/h_t,y_t/h_t)				
Data	0.446**	0.835**	0.317**	0.641**
Endo	0.505*	0.730	0.976	0.917
Exog1	0.477*	0.669	0.982	0.879
Exog2	0.322	0.438	0.945	0.670*
corr(q_t,y_t/y_{t-1})				
Data	−0.022	0.351**	0.119	0.128*
Endo	−0.716	−0.413	0.684	0.219*
Exog1	0.926	0.875	0.304	0.749
Exog2	0.977	0.957	0.573	0.917
corr(R_{mt},π_t)				
Data	−0.086	−0.204**	0.396**	−0.059
Endo	−0.208	−0.314*	−0.834	−0.508
Exog1	0.244	0.093	−0.526	−0.080
Exog2	0.490	0.222	−0.609	−0.025
corr(R_t,π_t)				
Data	0.065	0.644**	0.659**	0.572**
Endo	0.994	0.970	0.617*	0.890
Exog1	0.999	0.999	0.997	0.998
Exog2	0.999	0.999	0.999	0.999

Note: Same as Table 5.

Table 7
Sensitivity analysis: low frequency correlation between q and inflation.

US (UK) data		$-0.73(-0.72)$
Model: Calibration		Model low freq. corr.
σ_H	σ_G	
0.001	0.001	-0.75
0.002	0.001	-0.88
0.003	0.001	-0.92
0.001	0.002	-0.77
0.001	0.003	-0.78

Note: Other parameters as in Table 4.

Table 8
Matching the volatilities.

Variable	High freq. 2–6 qrs	Bus cyc. 6–32 qrs	Low freq. 32–100 qrs	Med term 2–100 qrs
c_t/y_t				
Data	0.002	0.005	0.013	0.012
Endo	0.002*	0.005*	0.005	0.010*
Exog1	0.001	0.001	0.001	0.002
Exog2	0.001	0.001	0.001	0.002
i_t/y_t				
Data	0.002	0.007	0.010	0.011
Endo	0.002	0.005	0.005	0.010*
Exog1	0.002	0.001	0.001	0.002
Exog2	0.001	0.001	0.001	0.002
$\frac{y_t}{y_{t-1}}$				
Data	0.006	0.005	0.001	0.008
Endo	0.001	0.001	0.003	0.003
Exog1	8.1e−05	1e−03	1e−03	2e−03
Exog2	8e−04	1e−03	7e−05	1e−03
l_{Gt}				
Data	0.012	0.031	0.053	0.053
Endo	0.001	0.002	0.002	0.003
Exog1	0	0	0	0
Exog2	9e−04	1e−03	1e−03	2e−3
π_t				
Data	0.002	0.002	0.003	0.004
Endo	0.002*	0.003	0.003*	0.005*
Exog1	0.002*	0.003	0.002	0.004*
Exog2	0.002*	0.003	0.002	0.004*
q_t				
Data	0.045	0.121	0.287	0.280
Endo	0.0002	0.0004	0.001	0.001
Exog1	7e−05	1e−03	8e−04	1e−03
Exog2	6e−04	1e−03	5e−04	1e−03
R_{mt}				
Data	0.020	0.014	0.007	0.025
Endo	0.003	0.001	0.003	0.003
Exog1	1e−05	2e−03	2e−03	3e−03
Exog2	5e−04	0.0001	0.0001	0.002
R_t				
Data	0.005	0.001	0.001	0.002
Endo	0.001	0.002	0.001*	0.003
Exog1	0.001	0.002	0.001*	0.003
Exog2	0.001	0.002	0.001*	0.003

Note: * means within 95% confidence band of the sample standard deviation.

are the stock return, R_{mt}, and the nominal interest rate R_t.[13] The baseline model does well in the inflation and stock return correlation at the business cycle frequency and in the inflation and nominal interest rate correlation at the low frequency (as in a long run Fisher equation).

6.1. *Sensitivity of low frequency q-inflation correlation*

Table 7 reports the sensitivity of our central correlation with respect to variations in the standard deviation of the two productivity shocks. The first three rows show that the low frequency correlation between q and inflation is sensitive to a change in the standard deviation of the productivity shock to human capital. The next two rows show in contrast marginal sensitivity to changes in the productivity shock to the goods sector. The q-inflation correlation is more sensitive to a change in σ_H than σ_G. This is consistent with how the impulse responses in Figs. 5 and 6 are an order of magnitude higher for the human capital shock than for the goods sector shock. This in turn highlights the role of human capital shock in determining the low frequency correlation between q and inflation. Given that the central object of our paper is to understand the low frequency relation between q and inflation, we consider this as a success of our endogenous growth model in predicting the cross correlation properties of q-inflation data.

6.2. *Volatility*

Table 8 reports the simulated and actual data standard deviations of key macroeconomic variables at the same four frequencies. Consumption and investment ratios are simulated well in the baseline model in the medium term; the growth volatility is fairly close in the low and medium term for the baseline model. In the low frequency, the model's stock market return R_{Mt} is half of that in the data, but still orders of magnitude better than in the exogenous growth models.[14] The q volatility is not well matched in any of the three models, but still is best in the baseline model at low frequency. Inflation and the nominal interest rate are similarly well simulated in all three models. For the inflation rate, the baseline endogenous growth model slightly overpredicts the inflation. By construction the volatility of labor supply is zero in the exogenous growth model with fixed labor supply while the volatility of output growth rate is zero also for both versions of exogenous growth models.

7. Conclusion

The paper contributes to an explanation of the empirical low frequency negative correlation between Tobin's q and inflation. It employs a *DSGE* endogenous growth model that identifies plausible fundamentals. The importance of this study is that it shows long run inflation tax effects that can complement the study of shorter run effects such as in Christiano et al. (2008). To do this the paper develops a production based model which included closed-form expressions for Tobin's q that give intuition on how the money supply growth causes the linkage between inflation, q and human capital utilization along the balanced growth path. The impulse response analysis helps reveal a novel transmission mechanism of the productivity and monetary shocks through a "human capital channel". The simulation results then provide an explanation in particular for the observed low frequency negative correlation between q and inflation. Comparison of the baseline to alternative models including exogenous growth variants highlights the success of the baseline in this respect, while showing an ability to capture more of the q and stock return volatility at all examined frequencies. This suggests how the human capital sector and its productivity shock is a key to the overall results.

Extension to include elements of Christiano et al. (2008) could be introduced to try to explain better higher frequency correlations as well. Habit persistence as in Boldrin et al. (2001) could be used within the baseline model to see if additional volatility of q results. A possible extension in terms of money demand is to make the interest elasticity of money demand higher for any given rate of money supply growth so as to cause a higher and more volatile velocity and inflation, and potentially a more volatile q; this can be accomplished by extending the standard cash-only cash-in-advance constraint by allowing in addition an exchange substitute to money as in Gillman and Kejak (2010). In a different extension of the standard cash-in-advance constraint, a greater impact of the inflation on the investment rate and potentially on q might result if exchange is required for both consumption and investment purchases, rather than just consumption; Gillman and Kejak (2011) include investment in the exchange constraint in a related model to our baseline and show this allows the model not only to explain the negative effect of inflation on growth, and the Tobin (1965) effect of inflation towards a higher physical capital to human capital ratio, but also a negative effect of inflation on the investment rate as is consistent with some evidence. Another banking type extension could be the introduction of financial intermediation of savings and investment, which could allow for bank crisis effects (Gillman, 2011) through a stochastic bank productivity factor that may also cause higher q volatility. Finally the ability to explain q through the current model's

[13] The equation for the stock return, R_{mt+1}, is given by $((1-\theta)(\overline{\xi}\theta)^{\theta/(1-\theta)}q_{t+1}^{1/(1-\theta)}+1+MPK_{t+1}-\delta_k)/q_t$. Appendix B.3 outlines the derivation of the stock return equation (21). The nominal interest rate (R_t) is $(1/P_t^b-1)$.

[14] The underperformance in matching the business cycle volatility of stock return is not surprising given the staggering failure of the standard real business cycle models in matching this (Gomme et al., 2011).

shocks could be illustrated further by backing out the implied shocks of the model over time using data series, such as in the shock identification used by Nolan and Thoenissen (2010).

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Appendix A. Equilibrium conditions

Define the Lagrange multipliers associated with the consumer's flow budget constraint (2) as λ_t, the human capital technology (4) as η_t and the cash-in-advance constraint (5) as γ_t. The consumer's first order conditions are

$$c_t : \beta^t U'(c_t) - P_t(\lambda_t + \gamma_t) = 0, \tag{A.1}$$

$$M_t : -\lambda_t + E_t\{\lambda_{t+1} + \gamma_{t+1}\} = 0, \tag{A.2}$$

$$z_{t+1} : -\lambda_t V_t + E_t \lambda_{t+1}\{V_{t+1} + D_{t+1}\} = 0, \tag{A.3}$$

$$B_{t+1:} - P_t^b \lambda_t + E_t \lambda_{t+1} = 0, \tag{A.4}$$

$$h_{t+1} : -\eta_t + E_t \lambda_{t+1} l_{Gt+1} W_{t+1} + E_t \eta_{t+1}(1 - \delta_h + A_{Ht+1} l_{ht+1}) = 0, \tag{A.5}$$

$$l_{Gt} \cdot - \psi \Gamma'(1 - l_{Gt} - l_{Ht})\beta^t + \lambda_t W_t h_t = 0, \tag{A.6}$$

$$l_{Ht} : -\psi \Gamma'(1 - l_{Gt} - l_{Ht})\beta^t + A_{Ht}\eta_t h_t = 0. \tag{A.7}$$

Using (A.1) and (A..2)

$$\lambda_t = \beta^{t+1} E_t \frac{U'(c_{t+1})}{P_{t+1}}, \tag{A.8}$$

which upon substitution in (A.3) and (A.4) yields

$$V_t E_t \left[\frac{U'(c_{t+1})}{P_{t+1}}\right] = \beta E_t \left[E_{t+1}\left[\frac{U'(c_{t+2})}{P_{t+2}}\right]\{V_{t+1} + D_{t+1}\}\right], \tag{A.9}$$

$$P_t^b E_t \left[\frac{U'(c_{t+1})}{P_{t+1}}\right] = \beta E_t \left[E_{t+1}\left[\frac{U'(c_{t+2})}{P_{t+2}}\right]\right]. \tag{A.10}$$

A binding cash in advance constraint means that (5) reduces to

$$\frac{M_t}{P_t} = c_t, \tag{A.11}$$

which implies that

$$\frac{P_t}{P_{t+1}} = \frac{c_{t+1}}{c_t} \frac{1}{1 + \mu_{t+1}}. \tag{A.12}$$

Upon substitution into (A.9) and (A.10) it results that

$$v_t E_t \left[U'(c_{t+1}) \frac{c_{t+1}}{c_t} \frac{1}{1 + \mu_{t+1}}\right] = \beta E_t \left[E_{t+1}\left[U'(c_{t+2}) \frac{c_{t+2}}{c_{t+1}} \frac{1}{1 + \mu_{t+2}}\right]\{v_{t+1} + d_{t+1}\}\right] \tag{A.13}$$

and

$$p_t^b E_t \left[U'(c_{t+1}) \frac{c_{t+1}}{c_t} \frac{1}{1 + \mu_{t+1}}\right] = \beta E_t \left[E_{t+1}\left[U'(c_{t+2}) \frac{c_{t+2}}{c_{t+1}} \frac{1}{1 + \mu_{t+2}}\right]\right], \tag{A.14}$$

where v_t is the real share price (V_t/P_t), $p_t^b = P_t^b/P_t$, and w_t denotes the real wage (W_t/P_t).

Using (A.11) and (6), one obtains the following compact expression for m_{t+1}:

$$\frac{\lambda_{t+1} P_{t+1}}{\lambda_t P_t} = m_{t+1}. \tag{A.15}$$

For the goods producer, define ω_t as the Lagrange multiplier associated with the adjustment cost technology (11). The firms' first order conditions are

$$l^f_{Gt} : \frac{W_t}{P_t} = A_{Gt} F_2(k_t, l^f_{Gt} h_t), \tag{A.16}$$

$$i_t : \lambda_t P_t = \theta \omega_t \left(1-\delta_k + \frac{i_t}{k_t}\right)^{\theta-1}, \tag{A.17}$$

$$k_{t+1} : -\omega_t + E_t(P_{t+1}\lambda_{t+1} A_{Gt+1} F_{1t+1}) + E_t \omega_{t+1}\left[\left\{1-\delta_k + \frac{\overline{\xi} i_{t+1}}{k_{t+1}}\right\}^{\theta} + \theta\left\{1-\delta_k+\frac{\overline{\xi} i_{t+1}}{k_{t+1}}\right\}^{\theta-1}\left\{\frac{\overline{\xi} i_{t+1}}{k_{t+1}}\right\}\right] = 0 \tag{A.18}$$

The model can then be summarized by the following equations.

Tobin's q equation:

$$q_t = E_t m_{t+1}\left[\alpha \frac{y_{t+1}}{k_{t+1}} + \frac{1-\delta_k}{\overline{\xi}} + (1-\theta)(\overline{\xi}\theta)^{\theta/(1-\theta)} q_{t+1}^{1/(1-\theta)}\right]. \tag{A.19}$$

l_G equation:

$$\frac{A_{Gt}}{A_{Ht}} \cdot l_{Gt}^{-\alpha} \cdot \left[\frac{k_t}{h_t}\right]^{\alpha} = E_t\left[m_{t+1} A_{Gt+1} l_{Gt+1}^{1-\alpha}\left(\frac{k_{t+1}}{h_{t+1}}\right)^{\alpha}\right] + E_t\left[m_{t+1} l_{Gt+1}^{-\alpha}\left(\frac{k_{t+1}}{h_{t+1}}\right)^{\alpha}(1-\delta_h + A_{Ht+1} l_{ht+1})\frac{A_{Gt+1}}{A_{Ht+1}}\right]. \tag{A.20}$$

x equation:

$$\frac{\psi}{x_t} - (1-\alpha)\beta E_t\left[\frac{1}{1+\mu_{t+1}} A_{Gt} l_{Gt}^{-\alpha}\left(\frac{k_t}{h_t}\right)^{\alpha-1}\left(\frac{c_t}{k_t}\right)^{-1}\right] = 0. \tag{A.21}$$

k/h equation:

$$\frac{k_{t+1}}{h_{t+1}} = \frac{\{(1-\delta_k)(k_t/h_t) + A_{Gt} l_{Gt}^{1-\alpha}(k_t/h_t)^{\alpha} - (c_t/k_t).(k_t/h_t)\}^{\theta}\left(\frac{k_t}{h_t}\right)^{1-\theta}}{1-\delta_h + A_{Ht}(1-l_{Gt}-x_t)}. \tag{A.22}$$

Output growth equation:

$$\frac{y_{t+1}}{y_t} = \left[\frac{A_{Gt+1}}{A_{Gt}}\right]\left[\frac{k_{t+1}/h_{t+1}}{k_t/h_t}\right]^{\alpha}\{A_{Ht} l_{Ht} + 1-\delta_h\} \cdot \left[\frac{l_{Gt+1}}{l_{Gt}}\right]^{1-\alpha}. \tag{A.23}$$

Inflation equation:

$$\frac{P_{t+1}}{P_t} = \frac{1+\mu_{t+1}}{\{(c_{t+1}/k_{t+1})/(c_t/k_t)\}\{(k_{t+1}/h_{t+1})/(k_t/h_t\}\{A_{Ht} l_{Ht} + 1-\delta_h\}}. \tag{A.24}$$

The discount factor equation:

$$m_{t+1} = \beta \frac{\{1+(1+\rho)\overline{\mu} - \rho\mu_{t+1}\}}{\{1+(1+\rho)\overline{\mu}-\rho\mu_t\}} \frac{(c_t/k_t)}{(c_{t+1}/k_{t+1})} \cdot \frac{(k_t/h_t)}{(k_{t+1}/h_{t+1})} \cdot \frac{1}{1-\delta_h + A_{Ht} l_{Ht}}. \tag{A.25}$$

Eq. (A.19) follows from (A.18), (16) and (A.15). Eq. (A.20) follows from (A.5)–(A.8), (A.15) and (A.16). Eq. (A.21) follows from (A.6), (A.8) and (A.16). Eq. (A.22) follows by combining (4), (9) and (11). The growth equation (A.23) follows from (4) and (9). To obtain the inflation equation (A.24) rewrite the cash-in-advance constraint (5) using (4) as

$$\frac{P_{t+1}}{P_t} = \frac{(1+\mu_{t+1})(A_{Ht} l_{Ht} + 1-\delta_h)^{-1}}{\frac{c_{t+1}}{k_{t+1}} \frac{k_{t+1}}{h_{t+1}}\left(\frac{c_t}{k_t}\frac{k_t}{h_t}\right)^{-1}}.$$

For Eq. (A.25), use the log utility specification and Eq. (4) to rewrite this as

$$m_{t+1} = \frac{\beta \frac{c_t}{k_t}\frac{k_t}{h_t}}{\frac{c_{t+1}}{k_{t+1}}\frac{k_{t+1}}{h_{t+1}}} \cdot \frac{E_{t+1}\left[\frac{1}{1+\mu_{t+2}}\right]}{E_t\left[\frac{1}{1+\mu_{t+2}}\right]} \cdot \frac{1}{1-\delta_h + A_{Ht} l_{Ht}}.$$

Next take a first order approximation around the steady state and use the forcing process for money supply growth in Eq. (15) to get the expression in Eq. (A.25).

The balanced growth equilibrium solution then follows. Based on (4), (18), the resource and time constraints of Eqs. (A.20) and (A.21), the steady state can be represented as

$$1+g = 1-\delta_h + \overline{A}_H l_h = \left(1-\delta_k + \frac{i}{k}\right)^{\theta}, \tag{A.26}$$

$$1+g=\beta(1-\delta_h+\overline{A}_H(1-x)), \tag{A.27}$$

$$\frac{c}{k}+\frac{i}{k}=\frac{y}{k}=\overline{A}_G\left(\frac{l_G h}{k}\right)^{1-\alpha}, \tag{A.28}$$

$$\frac{c}{k}\frac{\psi}{x}=\frac{(1-\alpha)\beta}{(1+\overline{\mu})}\frac{y}{k}\frac{1}{l_g}, \tag{A.29}$$

$$\beta\theta\left(\alpha\frac{y}{k}+1-\delta_k\right)=[1-\beta(1-\theta)](1+g)^{1/\theta}, \tag{A.30}$$

$$1=x+l_G+l_H. \tag{A.31}$$

Equating the $1+g$ terms in the first equality of (A.26) and (A.27), and using Eq. (A.31), yields a linear relationship between l_G in terms of x as follows:

$$(1-\delta_h)(1-\beta)=\overline{A}_H[l_G-(1-\beta)(1-x)]. \tag{A.32}$$

From Eq. (A.29) and the first part of Eq. (A.28), obtain y/k in terms of i/k and x/l_G. Substituting this into (A.30), and then writing i/k in terms of g from (A.26), yields a further expression for g in terms of x/I_G. Finally replace l_G by its representation in terms of x, and g in terms of x from Eq. (A.27), to get an equation solely in x:

$$\frac{\theta(1-\beta)}{\overline{A}_H}(1-\delta_k)(1-\alpha)(1-\delta_h+\overline{A}_H(1-x))\psi-\frac{\theta(1-\delta_k)(1-\alpha)\beta}{1+\overline{\mu}}x+\frac{(1-\beta(1-\theta))(1-\alpha)}{1+\overline{\mu}}\beta^{1/\theta}(1-\delta_H+\overline{A}_H(1-x))^{1/\theta}x$$
$$-\frac{(1-\beta+\beta\theta(1-\alpha))(1-\beta)}{\overline{A}_H}\psi\beta^{1/\theta-1}(1-\delta_h+\overline{A}_H(1-x))^{1+1/\theta}=0. \tag{A.33}$$

Once x is solved from (A.33), l_G can be solved from (A.32). The remaining endogenous variables are just functions of l_G and x and can be computed.

Appendix B. Proofs

B.1. Proposition 3

Divide (A.13) by k_{t+1}

$$\left(\frac{v_t}{k_{t+1}}\right)=E_t m_{t+1}\left\{\left(\frac{v_{t+1}}{k_{t+2}}\right).(k_{t+2}/k_{t+1})+(d_{t+1}/k_{t+1})\right\}.$$

Noting that $[A_{Gt}F(k_t,l_{Gt}h_t)-(W_t/P_t)l_{Gt}h_t-i_t]=d_t$,

$$\left(\frac{v_t}{k_{t+1}}\right)=E_t m_{t+1}\left\{\left(\frac{v_{t+1}}{k_{t+2}}\right).\frac{k_{t+2}}{k_{t+1}}+A_{Gt+1}F_{1t+1}-\frac{i^k_{t+1}}{k_{t+1}}\right\}.$$

Now use the adjustment cost equation (11) to rewrite the above as

$$\left(\frac{v_t}{k_{t+1}}\right)=E_t m_{t+1}\left\{A_{Gt+1}F_{1t+1}+(1-\delta_k)\overline{\xi}^{-1}+\left(\frac{v_{t+1}}{k_{t+2}}\right)\left(\frac{k_{t+2}}{k_{t+1}}\right)-\frac{1}{\overline{\xi}}\left(\frac{k_{t+2}}{k_{t+1}}\right)^{1/\theta}\right\}. \tag{B.1}$$

Using the definition of q_t (21) rewrite this again as

$$\left(\frac{v_t}{k_{t+1}}\right)=E_t m_{t+1}\left\{A_{Gt+1}F_{1t+1}+(1-\delta_k)\overline{\xi}^{-1}+\left(\frac{v_{t+1}}{k_{t+2}}\right)(\theta\overline{\xi}q_{t+1})^{\theta/(1-\theta)}-\left(\theta\overline{\xi}q_{t+1}\right)^{1/(1-\theta)}\right\}. \tag{B.2}$$

Next verify that (B.2) collapses to (A.19) if $q_t=v_t/k_{t+1}$.

B.2. Proposition 4

Note that from Eqs. (6) and (A.8), along the *BGP*,

$$m_{t+1}=\frac{P_{t+1}\lambda_{t+1}}{P_t\lambda_t}=\frac{\beta}{1+g}. \tag{B.3}$$

Using (16), (A.19) and (B.3), and imposing the balanced growth condition, $i_t/k_t=i_{t+1}/k_{t+1}$ one obtains that

$$\left(1-\delta_k+\frac{i_t}{k_t}\right)^{1-\theta}=\frac{\beta\theta}{1+g}\bar{A}_G F_1+\frac{\beta}{1+g}\left[(1-\theta)\left(1-\delta_k+\frac{i_t}{k_t}\right)+\theta(1-\delta_k)\right]. \tag{B.4}$$

Use the adjustment cost function (11) to write

$$\frac{i_t}{k_t} = (1+g)^{1/\theta} - 1 + \delta, \tag{B.5}$$

which after plugging into Eq. (B.4) yields the proposition result of Eq. (18). Also it is straightforward to verify from Eq. (A.20) the standard result as in such Lucas (1988) human capital models with leisure that $1+g = \beta[1-\delta_h + A_H(1-x)]$.

B.3. Stock return equation

The real stock return, denoted by R_{mt}, is typically defined by

$$R_{mt+1} = \frac{v_{t+1} + d_{t+1}}{v_t}. \tag{B.6}$$

Rewrite (B.6) as

$$R_{mt+1} = \frac{q_{t+1} + (d_{t+1}/k_{t+1})(k_{t+1}/k_{t+2})}{q_t} \cdot \left(\frac{k_{t+2}}{k_{t+1}}\right).$$

Noting that $d_{t+1} = (\alpha y_{t+1}/k_{t+1}) + 1 - \delta_k - (k_{t+2}/k_{t+1})^{1/\theta}$, and using (16) the above reduces to

$$R_{mt+1} = \frac{(1-\theta)(\overline{\xi}\theta)^{\theta/(1-\theta)} q_{t+1}^{1/(1-\theta)} + 1 + MPK_{t+1} - \delta_k}{q_t}$$

The adjustment cost parameter θ drives a wedge between stock return and gross marginal product of capital. In the absence of adjustment cost, with $q_t = 1$, the stock return equals to the gross marginal product of capital, $1 + MPK_{t+1} - \delta_k$.

Appendix C. Data sources

Data for GDP (y), personal consumption expenditure (c), gross private domestic investment (I), currency ($\overline{M}$), the 3-month T-bill rate (R_t) are from the Federal Reserve Bank, St. Louis database. The implicit price deflator series is from the Bureau of Economic Analysis database. The time to goods production (l_G) is the average weekly hours divided by 5 working days. The stock return (R_{mt}) is from Robert Shiller's online databank, with monthly series converted to quarterly. In the absence of a suitable measure for the stock of human capital h_t, we construct this as a geometric trend $(1+g)^t$ where g is the model's quarterly balanced growth rate. Similar procedure for proxying the human capital by such a trend is followed in Benk et al. (2009). The quarterly series for US q is from Smithers & Co. and the annual series for q is from Hall (2001). The UK q is from the Bank of England sources as detailed in footnote 2. The UK CPI is from Office of National Statistics.

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