THE DEMAND FOR BANK RESERVES AND OTHER MONETARY AGGREGATES

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The article starts with Haslag's (1998) model of the bank's demand for reserves and reformulates it with a cash-in-advance approach for both financial intermediary and consumer. This gives a demand for a base of cash plus reserves that is not sensitive to who gets the inflation tax transfer. It extends the model to formulate a demand for demand deposits, yielding an M*1-type demand, and then includes exchange credit, yielding an* M*2-type demand. Based on the comparative statics of the model, it provides an interpretation of the evidence on monetary aggregates. This explanation relies on the nominal interest as well as technology factors of the banking sector.* (*JEL* E31, E13, O42)

I. INTRODUCTION

Modeling the monetary aggregates in general equilibrium has been a challenge. There are some examples such as Chari et al. (1996), and Gordon et al. (1998), who present models that are compared to Base money. Ireland (1995) presents one that he relates to M1-A velocity. These models have been employed as ways to explain the actual monetary aggregate time-series evidence. However, McGrattan (1998), for example, argues that the simple linear econometric model in which velocity depends negatively on the nominal interest rate may do just as well or better in explaining the evidence.

The article here takes up the topic by modeling a nesting of the aggregates that uses a set of factors that expands from the nominal interest rate by including the production of banking services. Through this approach the productivity factor of banking enters, as well as a cost to using money, sometimes thought of as a convenience cost. With this general equilibrium model, and its comparative statics, an explanation of velocity is provided that depends in part on the nominal interest rate, similar in spirit to McGrattan (1998). Also using technology factors, we explain U.S. evidence on monetary base velocity, M1 velocity, and M2 velocity, as well as for the ratios of various aggregates. This more extended explanation than previous work highlights the limits to a nominal interest rate story, while revealing a plausible role of technological factors in determining the aggregate mix.

The original literature on the welfare cost of inflation, well-represented by Bailey (1992), assumes no cost to banks in increasing their exchange services as consumers flee from currency during increasing inflations.[1] Similarly, Johnson (1969) and Marty (1969) assume no real costs for banks in producing "inside money."[2] The approach here builds on the

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1. "The presence of the banking system has no real effect whatever but merely alters the nominal rate of inflation necessary to achieve a given real size of the government budget" (Bailey 1992, 234); in the model here the latter statement is true, but not the former because capital is used up in banking activities and because reserve requirements affect the real interest rate when there is a non-Friedman optimum rate of interest.

2. Johnson (1969, 32), for example, writes that "a banking organization could issue non-interest-bearing deposites, assumed to be costless to administer." Marty (1969, 106) discusses demand deposits and assumes that "the cost of setting up and running a bank is zero." Here both authors are focusing on the wealth effects of inside and outside money.

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more recent literature of Gillman (1993), Aiyagari et al. (1998), and Lucas (2000) that assumes resource costs to avoiding the inflation tax by using alternative exchange means. In particular, we specify production functions for banking instruments, both demand deposits (inside money) and credit, that require real resource use. This gives rise to the role of banking productivity factors in explaining the movement of aggregates.[3]

The next section reviews Haslag's (1998) model and shows how it is sensitive to the distribution of the lump sum inflation proceeds. This sensitivity makes tentative the growth effect of inflation with the model. The demand for reserves can be made insensitive to the distribution of the inflation tax transfer by framing it within a model in which the bank must hold money in advance as in the timing of transactions that is pioneered in Lucas (1980). This is done in section III using Haslag's (1998) notation, *Ak* production technology, and full savings intermediation. The resulting real interest rate depends negatively on the nominal interest rate, so inflation negatively affects the growth rate, similar in fashion to the central result of Haslag (1998). A parallel consumer cash-in-advance demand for goods is also added, as in Chari et al. (1996), to give a model of reserves plus currency.

The article then expands the model to give a formulation of the demand for the base plus non–interest-bearing demand deposits, or an aggregate similar to M1.[4] Following a credit production approach used in a series of related works (see Gillman and Kejak 2002; Gillman et al. 2003; Gillman and Nakov 2003), we then add credit, or interest-bearing demand deposits, to give a formulation for an aggregate similar to M2.

II. SENSITIVITY TO LUMP TRANSFERS

In Haslag (1998), all savings funds are costlessly intermediated into investment by the bank. The bank must hold reserves in the form of money. This gives rise to a bank demand for money to meet reserve requirements on the savings deposits. The consumer-agent does not use money, although the lump sum inflation tax is transferred to the agent. Instead the agent simply holds savings deposits at the bank and earns interest as the bank intermediates all investment. The bank's return is lowered by the need to use money for reserves. Further, the timing of the model is such that inflation decreases the real return to depositors, and therefore also the growth rate, through the requirement that reserves be held as money.

The following model gives the reported result in Haslag (1998).[5]

With the gross return on invested capital being $1 + A - \delta$, as in an *Ak* model, with the time t capital stock denoted by k_t, the savings deposits denoted by d_t, the nominal money stock by M_t, the price level by P_t, and the net return paid on deposits denoted by r_t, the nominal profits are given as

$$\Pi_t = P_t(1+A-\delta)k_t + M_{t-1} - P_t(1+r_t)d_t. \quad (1)$$

This is stated as a maximization problem with respect to k_t, M_{t-1}, d_t and subject to two constraints. The constraints (with equality imposed) are that the sum of capital and last periods real balances equals deposits:

$$k_t + M_{t-1}/P_{t-1} = d_t, \quad (2)$$

3. Hicks (1935) seeks a theory of money based on marginal utility, with cash held in advance of purchases, as Lucas (1980) follows. Hicks shunts aside both Keynes's alternative to Fisher's quantity theory as found in his *Treatise* (see Gillman 2002 on flaws in this theory), and considers "Velocities of Circulation" as in Fisher's quantity theory an "evasion." He reasons that money use suggests the existence of a friction and that "we have to look the friction in the face." The "most obvious sort of friction" is "the cost of transfering assets from one form to another." Hicks says that we should consider "every individual in the community as being, on a small scale, a bank. Monetary theory becomes a sort of generalisation of banking theory." In alignment with Hicks, the agent in this article acts as a bank in part, and the bank has costs from creating new instruments, such as demand deposits and credit. But in contrast, here velocity is endogenously determined as a fundamental part of the resulting equilibrium. Hicks's and Lucas's approaches converge with Fisher's.

4. This abstracts from the interest that is earned on some demand deposit accounts included in the U.S. M1 aggregate, because this interest tends to be of nominal amounts compared to the savings accounts included in M2.

5. However, to get this result, three changes were made to the model actually published in Haslag (1998), indicating incidental errors in the published paper: The money stock in the profit equation (1) is in time $t-1$, instead of t as published; and the money stock and the price level in equation (2) are in time $t-1$ instead of time t as published. The actual return in the article as published is that $r_t = (A - \delta)[1 - \gamma_t(1 + g_t)]$, where g_t denotes the balanced-path growth rate; it is independent of the inflation rate.

and that a fraction γ_{t-1}, given in the last period, of time t deposits is held as real money balances in time $t-1$:

$$M_{t-1}/P_{t-1} = \gamma_{t-1} d_t. \tag{3}$$

Assuming zero profit, this yields through simple substitution the return reported by Haslag (1998):

$$1 + r_t = (1 + A - \delta)(1 - \gamma_{t-1}) + \gamma_{t-1}(P_{t-1}/P_t). \tag{4}$$

The result is sensitive to who gets the lump-sum cash transfer from the government. If the transfer instead goes to the bank, the only user of money in the model, then there is no growth effect of inflation. This can be seen in the following way: Let the money supply process be given as in Haslag (1998) as $M_t = M_{t-1} + H_{t-1}$, where H_{t-1} is the lump-sum transfer by the government. With the transfer given to the bank, the profit of equation (1) becomes

$$\Pi_t = P_t(1 + A - \delta)k_t + M_{t-1} + H_{t-1} - P_t(1 + r_t)d_t. \tag{5}$$

Let the balanced growth rate of the economy be denoted by g_t, and the consumer's time preference by ρ, whereby the consumer's problem in Haslag (1998) with log utility gives that $1 + g_t = (1 + r_t)/(1 + \rho)$. With this growth rate in mind, the zero profit equilibrium now gives a rate of return to depositors of

$$1 + r_t = (1 + A - \delta)(1 - \gamma_{t-1}) + \gamma_t(1 + g_t), \tag{6}$$

and there is no inflation tax on the return or on the growth rate.

Alternatively let the profit function be given as equation (5). Then assume that the stock and reserve constraints, equations (2) and (3), are all in terms of current period variables, as in a standard cash-in-advance economy where here the reserve constraint now would look like a Clower (1967) type of constraint. Substituting in M_t for $M_{t-1} + H_{t-1}$, then, the model is exactly as in Chari et al. (1996). This gives the result, also found in Einarsson and Marquis (2001), that

$$1 + r_t = (1 + A - \delta)(1 - \gamma_t) + \gamma_t. \tag{7}$$

The return is lowered because reserves are idle, but there is no inflation tax.

III. MODELS OF MONETARY AGGREGATES

Monetary Base

The financial intermediary has a demand for nominal money, denoted by M_t^r, as created by the need for reserves, with the reserve ratio denoted by $\gamma \in [0, 1]$. But here, as in Chari et al. (1996), the reserve constraint is considered as the bank's Clower (1967) constraint and structured accordingly in a fashion parallel to the consumer's, being that

$$M_t^r = \gamma P_t d_t. \tag{8}$$

In addition, the asset constraint adds together the current period real money stock with the current period capital stock to get the current period real deposits. In real terms this is written as

$$k_t + M_t^r/P_t = d_t. \tag{9}$$

Unlike Chari et al. (1996), the bank has to set aside cash in advance of the next period's accounting of the reserve requirement to meet any increase in its reserve requirements. The bank has revenue from its return on investment and costs from payment of interest to depositors, and from any increase in money holdings for reserves.

The technology for the output of goods, as in Haslag (1998), is an Ak production function, making the current period profit function:

$$\Pi_t^r = P_t(1 + A - \delta)k_t + M_t^r - M_{t+1}^r - P_t d_t(1 + r_t). \tag{10}$$

The profit maximization problem is dynamic because of the way money enters the bank's profit function in two different periods, the same dynamic feature of the consumer problem. The competitive bank discounts the nominal profit stream by the nominal rate of interest, and maximizes the time 0 discounted stream, denoted by $\hat{\Pi}_0^r$, with respect to the real capital stock, k_t, the real deposits, d_t, and the money stock used for reserves, denoted by M_{t+1}^r, and subject to the Clower (1967) type of reserve and asset stock constraints of equations (8) and (9):

$$(11)\quad \underset{d_t, M^r_{t+1}, k_t}{Max}\ \hat{\Pi}^r_0 = \sum_{t=0}^{\infty}\prod_{i=1}^{t}(1/[1+R_i])^t\{[P_t(1+A-\delta)k_t + M^r_t - M^r_{t+1} - P_t(1+r_t)d_t] + \lambda_t[P_t d_t - M^r_t - P_t k_t] + \mu_t[M^r_t - \gamma P_t d_t]\}.$$

Assuming a constant money supply growth rate, so that the nominal interest rate is constant over time, the first-order conditions imply that the rate of return is

$$(12)\quad 1 + r = (1 + A - \delta)(1 - \gamma) - \gamma R.$$

Using the Fisher equation of nominal interest rates (presented in [17]), with equation (12) shows that there is a negative effect of inflation on the return. Combined with the consumer's problem and the derivation of the balanced-growth rate as depending on the real interest rate, inflation therefore causes a negative effect on the balanced-path growth rate.

The bank does not receive any lump-sum transfer from the government; the consumer-agent receives it all. However, the distribution only affects how much profit the intermediary makes. Because the profit is transferred to the consumer, just as is the lump-sum transfer of inflation proceeds, the distribution of the inflation proceeds between the bank and the consumer can be changed without affecting the allocation of resources in the economy. For example, if the intermediary gets part of the inflation proceeds transfer, by an amount at time t equal to $M^r_{t+1} - M^r_t$, then in equilibrium the money terms cancel from the profit function, and $\Pi^r_t/(P_t k_t) = R[\gamma/(1-\gamma)]$. At the Friedman optimum, this profit is zero.[6]

Consider a consumer problem as in Haslag (1998) except that now the consumer uses cash, as in Lucas (1980). The problem then includes the setting aside of the consumer's cash in advance of trading in the next period, denoted by M^c_{t+1}, and the receipt of the lump-sum government transfer of inflation proceeds, denoted by H_t.

The consumer's Clower (1967) constraint is

$$(13)\quad M^c_t = P_t c_t.$$

The consumer also makes real (time) deposits, denoted by d_t, with the real return, denoted by r_t, as the form of all savings and wholly intermediated through banks, as in Haslag (1998). This involves choosing the next period deposits d_{t+1} and receiving as real income $(1 + r_t)d_t$. The nominal current period profit of the intermediation bank, Π^r_t, is received by the consumer each period as a lump-sum income source. This makes the consumer current period budget constraint of income minus expenditures as in the following:

$$(14)\quad P_t(1 + r_t)d_t + H_t + \Pi^r_t + M^c_t - M^c_{t+1} - P_t c_t - P_t d_{t+1} = 0.$$

The problem is to maximize the time preference discounted stream of current period utility, where $\beta \equiv 1/(1+\rho)$ denotes the discount factor, subject to the income and Clower (1967) constraints:

$$(15)\quad \underset{c_t, d_{t+1}, M^c_{t+1}}{Max}\ L = \sum_{t=0}^{\infty}\beta^t\{u(c_t) + \lambda_t[P_t(1+r_t)d_t + H_t + \Pi^r_t + M^c_t - M^c_{t+1} - P_t c_t - P_t d_{t+1}] + \mu_t[M^c_t - P_t c_t]\}.$$

The first-order conditions are

$$(16)\quad u_{c_t} = \lambda_t P_t(1 + \mu_t/\lambda_t),$$

$$(17)\quad \lambda_t/(\lambda_{t+1}\beta) = (1 + r_{t+1})(1 + \pi_{t+1}) \equiv (1 + R_{t+1}),$$ [7]

$$(18)\quad \lambda_t/(\lambda_{t+1}\beta) = 1 + \mu_{t+1}/\lambda_{t+1}.$$

These imply that

$$(19)\quad u_{c_t} = \lambda_t P_t(1 + R_t),$$

6. See Bailey (1992) for an early discussion of intermediary earnings during inflation. If current period non-negative profit is required for the bank intermediary to exist, then a transfer to the bank as in the above-described transfer scheme, with $\Pi^r_t/(P_t k_t) = R_t[\gamma/(1-\gamma)]$, would satisfy this at all inflation rates.

7. Including the market for nominal bonds as in Lucas and Stokey (1987) would give R_t as the price of the bonds and would explicitly derive the Fisher equation.

so that the nominal interest rate is the shadow exchange cost of buying a unit's worth of consumption. Using this latter equation to form an Euler equation, then along the balanced-growth equilibrium with log utility it follows that the growth rate of consumption, where $1 + g_{t+1} = c_{t+1}/c_t$, is constant and given by

$$(20) \qquad 1 + g = (1 + r)/(1 + \rho).$$

The demand for money is given by the Clower (1967) constraint, $M_t^c = P_t c_t$. This standard Lucas (1980) demand function can be thought of as a demand for "currency," in this, the simplest version of the model.

The total demand for money is the sum of the bank's and the consumer's, and this is set equal to the total money supply as a condition of market clearing in equilibrium:

$$(21) \qquad M_t^r + M_t^c \equiv M_t^b.$$

The total money supply equation is that this period's money base, denoted by M_t^b, plus the lump-sum transfer equal next period's base supply of money:

$$(22) \qquad M_t^b + H_t = M_{t+1}^b.$$

Assume that the money supply growth rate is constant at σ, where $\sigma \equiv H_t/M_t^b$.

M1

Now consider an extension in which the consumer suffers a nominal cost of using money that is proportional to the amount of cash used to make purchases. This can be thought of as the convenience cost of using money. This can be related to the average amount stolen in robberies by pickpockets, lost by carelessness, and spent on protection against crime and carelessness. It can also be Karni's (1974) time costs or Baumol's (1952) shoe-leather costs. These costs can be affected by the availability of bank locations, and now ATM locations.[8] Let this amount be given by ϕM_t^c, with $\phi \in [0, 1]$. Second, assume that a second bank exists, a bank that supplies only non–interest-bearing deposits, denoted by $M_t^{dd,s}$, that can be used in exchange. This money can be thought of demand deposits as in the United States or as a debit card as is more common in Europe.[9] The bank charges a nominal fee of P_t^{dd} per unit of real deposits, so that it receives from the consumer total such receipts equal to $P_t^{dd}(M_t^{dd,s}/P_t)$; and the bank produces these non–interest-bearing deposits through a production process. The consumer receives from the deposit bank its nominal profit, denoted by Π_t^{dd}, the profit from the intermediation bank, and the lump-sum inflation tax transfer from the government. The consumer's demand for the real non–interest-bearing deposits is denoted by M_t^{dd}/P_t. Also, the consumer invests in capital that is rented out by the demand deposit bank at the rate of r_t, with this capital denoted by k_t^{dd}. This makes the bank similar to a "mutual" customer-owned bank, and its capital does not get intermediated through the savings deposit bank. The depreciation rate on this capital is assumed to be zero, so that the consumer invests in this capital each period by the amount of $k_{t+1}^{dd} - k_t^{dd}$.

The consumer chooses what fraction of purchases to be made with cash, denoted by $a_t^c \in [0, 1]$, and what fraction to be made with noninterest demand deposits, $a_t^{dd} \in [0, 1]$; where

$$(23) \qquad a_t^c + a_t^{dd} = 1.$$

The Clower (1967) constraints become

$$(24) \qquad M_t^c = a_t^c P_t c_t;$$

$$(25) \qquad M_t^{dd} = (1 - a_t^c) P_t c_t.$$

8. We are indebted to Bob Lucas for originally suggesting the concept of the cost from crime and to Rowena Pecchenino for comments on this. Note that these costs are on the consumer side of the problem, while costs of alternative instruments for exchange are on the banking firm side of the problem. The so-called shopping time costs (Lucas 2000) actually compare better to the bank firm costs in this problem, as is shown in note 12. Karni's and Baumol's costs are a story more about the costs on the consumer side. The diffusion of ATMs plausibly affects both banking productivity and the consumer's cost of using money.

9. In Russia, after losing confidence in the bank sector during its collapse in 1997, people are again starting to use banks to hold cash. "'I'm used to carrying all my cash with me, but with a [debit] card it's easier,' said Denis Tafintsev, a 25-year-old warehouse manager. 'If you lose your card you don't lose your money.'" ("Retail Banking Grows in Russia", *Wall Street Journal Europe*, 28 May 2003, p. M1).

The consumer problem now is

$$(26)\qquad \underset{c_t,d_{t+1},k^{dd}_{t+1},M^c_{t+1},M^{dd}_{t+1},a^c_t}{Max} L = \sum_{t=0}^{\infty}\beta^t\{u(c_t)+\lambda_t[P_t(1+r_t)d_t+H_t + \Pi^r_t+\Pi^{dd}_t+M^c_t+M^{dd}_t-M^c_{t+1} - \phi M^c_t - M^{dd}_{t+1}-(P^{dd}_t/P_t)M^{dd}_t - P_tc_t - P_td_{t+1}-P_tk^{dd}_{t+1} + P_tk^{dd}_t(1+r_t)]+\mu^c_t[M^c_t-a^c_tP_tc_t] + \mu^{dd}_t[M^{dd}_t-(1-a^c_t)P_tc_t]\}.$$

The first-order condition with respect to a^c_t gives that $\mu^{dd}_t=\mu^c_t$. In combination with the first-order conditions with respect to the two money stocks, M^c_{t+1} and M^{dd}_{t+1}, this implies that the interior solution satisfies

$$(27)\qquad P^{dd}_t/P_t=\phi.$$

Note that the shadow cost of buying goods with cash is given by the marginal condition

$$(28)\qquad u_{c_t}=\lambda_tP_t(1+R_t+\phi),$$

so that the shadow exchange cost now is equal to $R_t+\phi$ instead of only R_t, as in the previous subsection.

The demands for the cash and for the demand deposits are given by the Clower (1967) constraints in equilibrium, where the a^c_t variable is determined by finding the equilibrium bank supply of demand deposits and setting this equal to the demand for demand deposits.

The original bank, the capital intermediation bank, has the same problem as stated previously. Now consider the specification for the production function of the new bank. This bank uses real resources in the process of producing demand deposits and so is costly, unlike the intermediation bank. With an $\hat{A}K$ type production function for the non–interest-bearing demand-deposit bank, it can be shown that the equilibrium would not be well defined. If the $\hat{A}$ parameter equals ϕ, then there is no unique equilibrium; and if $\hat{A}$ equals any other value, there is an equilibrium either with no demand for cash or with no demand for credit. A unique equilibrium is satisfied by specifying a diminishing returns technology whereby there is a margin at which the fixed ϕ is equal to the variable marginal cost of producing the demand deposits. Initially assume that the new demand deposit bank faces the following production function that is diminishing in its capital input. Denoting the shift parameter by $\hat{A}_{dd}$ and the capital input by k^{dd}_t, and with $\alpha\in(0,1)$, let the function be specified as

$$(29)\qquad M^{dd,s}_t/P_t=\hat{A}_{dd}(k^{dd}_t)^{\alpha}.$$

The demand deposit bank gets revenue from "printing" new demand deposits, $M^{dd}_{t+1}-M^{dd}_t$, and from the fee the consumer pays for the services, and on the cost side rents capital from the consumer at the market real interest rate of r_t. The current period profit, Π^{dd}_t, is given as the revenue minus the costs,

$$(30)\qquad \Pi^{dd}_t=(P^{dd}_t/P_t)M^{dd}_t-P_tr_tk^{dd}_t + M^{dd}_{t+1}-M^{dd}_t.$$

With a constant money supply growth rate, the nominal interest rate is constant at R and the deposit bank faces the following dynamic profit maximization problem:

$$\underset{d_t,M^{dd}_{t+1},k_t}{Max}\ \widehat{\Pi}^r_0=\sum_{t=0}^{\infty}\Pi^t_{i=1}(1/[1+R_i])^t\{[(P^{dd}_t/P_t) \times M^{dd}_t-P_tr_tk^{dd}_t+M^{dd}_{t+1}-M^{dd}_t] + \lambda_t[P_t\hat{A}_{dd}(k^{dd}_t)^{\alpha}-M^{dd}_t]\}.$$

The first-order conditions imply that

$$(31)\qquad R+(P^{dd}_t/P_t)=r_t/[\hat{A}_{dd}\alpha(k^{dd}_t)^{\alpha-1}],$$

which, when combined with the consumer's equilibrium condition (27), gives that

$$(32)\qquad R+\phi=r_t/[\hat{A}_{dd}\alpha(k^{dd}_t)^{\alpha-1}].$$

This equation sets the marginal cost of demand deposits to the marginal cost of capital divided by the marginal product of capital in producing demand deposits, a standard microeconomic pricing condition.[10]

10. This result extends the traditional literature, such as Marty (1969, 105), who postulates that "if bank money were the only money, competitively produced bank money not subject to outside constraints will result in equality of the price of money with its cost of production. Since these costs are zero, the price of money would in equilibrium be zero." Here with positive production costs, the marginal cost in equilibrium is equal to the cost of the substitute, cash, which is $R+\phi$. This can be zero only at the Friedman optimum of $R=0$ combined with the case that $\phi=0$, in which case there will be no demand for demand deposits.

Solving for the equilibrium capital stock,

$$(33) \qquad k_t^{dd} = [\hat{A}_{dd}\alpha(R+\phi)/r_t]^{1/(1-\alpha)},$$

and substituting this into equation (29) gives the supply of demand deposits as

$$(34) \quad M_t^{dd,s}/P_t = \hat{A}_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r_t]^{\alpha/(1-\alpha)}.$$

As the cost of using money $R+\phi$ falls due to a nominal interest falling toward $R=0$, there is still production due to cost ϕ. If in addition ϕ goes to zero, the capital used in produced non-interest-bearing deposits, and the output also goes to zero, and then the consumer uses only cash.

Here $M_t^{dd,s}/P_t = M_t^{dd}/P_t$ and the M1 aggregate can be represented as follows:

$$(35) \qquad M_t^c + M_t^{dd} \equiv M1_t.$$

The problem with this specification is that in the equilibrium, with a positive growth rate g_t, the ratio of M_t^c/M_t^{dd} is increasing toward infinity. Although there may be some trend in this ratio empirically, it should be explainable by changes in other exogenous factors that determine the ratio; with constant exogenous factors, theoretically the trend should be stable on the balanced growth path. To see that the ratio is not stable, equations (24) and (25) imply that $M_t^c/M_t^{dd} = a_t^c/(1-a_t^c)$. The solution for a_t^c is found by setting equal the supply and demand from equations (25) and (34), giving that $a_t^c = 1 - [(\hat{A}_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r_t]^{\alpha/(1-\alpha)})/c_t]$, with $r_t = (1+A-\delta)(1-\gamma)-\gamma R - 1$ by equation (12). This implies that $a_t^c/(1-a_t^c) = \{1 - [(\hat{A}_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r_t]^{\alpha/(1-\alpha)})/c_t]\}/\{\hat{A}_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r_t]^{\alpha/(1-\alpha)}/c_t\}$, or $a_t^c/(1-a_t^c) = \{c_t/(\hat{A}_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r_t]^{\alpha/(1-\alpha)})\} - 1$. By inspection it is clear that with c_t rising when there is positive growth on the equilibrium path, and with the real interest rate being stable given that there is a stationary inflation rate, the ratio $a_t^c/(1-a_t^c)$ also rises toward infinity toward a cash-only solution with no demand deposits.

An alternative production function that gives a stationary ratio of M_t^c/M_t^{dd} is one that includes an externality that affects the shift parameter $\hat{A}_{dd}$. In particular let $\hat{A}_{dd} = A_{dd}c_t^{1-\alpha}$, so that the production function is CRS in terms of capital and goods consumption:

$$(36) \qquad M_t^{dd,s}/P_t = A_{dd}c_t{}^{1-\alpha}(k_t^{dd})^{\alpha}.$$

This function is a type of positive externality in which the goods output is complementary to the bank's output; see also Romer (1986). It has the property that the share of goods bought with demand deposits, a_t^{dd}, is a function of the capital to goods ratio; by equations (23), (25), and (36),

$$(37) \qquad a_t^{dd} = A_{dd}(k_t^{dd}/c_t)^{\alpha}.$$

This means that the bank takes the aggregate consumption as given and demands capital and produces demand deposits in proportion to the aggregate consumption. Substituting the alternative production function into the profit maximization problem of equation (30), with $\hat{A}_{dd} = A_{dd}c_t^{1-\alpha}$, the solution is

$$(38) \qquad k_t^{dd}/c_t = [A_{dd}\alpha(R+\phi)/r_t]^{1/(1-\alpha)}.$$

From equations (37) and (38), the solution for the equilibrium share of demand deposits is

$$(39) \quad a_t^{dd} = A_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r_t]^{\alpha/(1-\alpha)}.$$ [11]

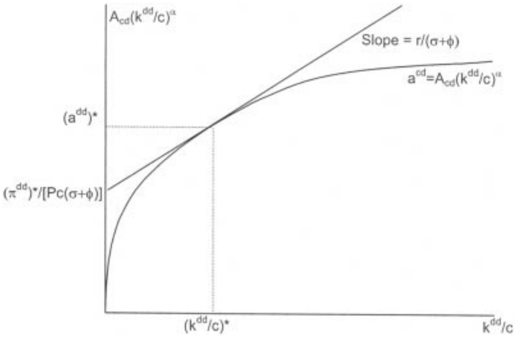
Figure 1 graphs the equilibrium for the demand deposit bank. To graph this, the current period profit function was solved along the balanced growth path. The dynamic nature of the bank problem brings the growth rate into the equilibrium profit function, which is substituted in for using equation (20). The resulting profit solution can be written as

$$(40) \ \Pi_t^{dd}/[P_t c_t(\sigma+\phi)] = M_t^{dd}/(P_t c_t) - r(k_t^{dd}/c_t)/(\sigma+\phi),$$

which is graphed as the straight line in Figure 1.

11. Note that if $a_t^{dd} = 1$, and so $a_t^c = 0$, there would be no consumer demand for cash. The monetary equilibrium would still have well-defined nominal prices as long as $\gamma > 0$, so that there was a reserves demand for cash by the intermediation bank. This could then be characterized solely as a legal restrictions demand for money. At $a_t^c = 0$, and $\gamma = 0$, and with a positive supply of money, prices may not be well defined.

FIGURE 1
Equilibrium in the Demand Deposit Bank Sector



With the production function of equation (36), the balanced-growth path exists and the ratio M_t^c/M_t^{dd} is stationary along it. Stationarity of M_t^c/M_t^{dd} follows directly, where it is shown that $M_t^c/M_t^{dd} = a_t^c/(1-a_t^c)$. By equation (23) this can be written as $M_t^c/M_t^{dd} = (1-a_t^{dd})/a_t^{dd}$ and by inspection of equation (39) can be seen to be stationary.

M2

The model can be expanded to its full form by allowing the agent the choice of using costly credit to make purchases, or "exchange credit," along with cash or non–interest-bearing demand deposits. Here the credit is like a credit card, such as the American Express card, rather than a debit card. The agent must pay a fee for this service that is proportional to the amount of the exchange credit; this is like the percentage fee paid by stores using the American Express card (without a rollover debt feature). Denoting the time t nominal amount of exchange credit demanded by the consumer by M_t^{cd}, and the nominal fee by P_t^{cd}, the consumer's expenditure on such fees is given by $(P_t^{cd}/P_t)M_t^{cd}$. The consumer again owns the exchange credit bank, receives the nominal profit, denoted by Π_t^{cd}, and rents nondepreciating capital k_t^{cd} and invests in the capital each period by an amount $k_{t+1}^{cd} - k_t^{cd}$. The consumer must pay off the debt incurred using the exchange credit at the end of the period. But this credit saves the agent from having to set aside money in advance of trading, and so allows avoidance of the inflation tax. Now with three types of exchange, let the share of consumption good purchases made by cash and by non-interest-bearing demand deposits remain notated by a_t^c and a_t^{dd}, and the share of consumption good purchases made by exchange credit by a_t^{cd}, where the shares sum to one:

$$a_t^c + a_t^{dd} + a_t^{cd} = 1. \tag{41}$$

This adds a third Clower (1967) constraint to the consumer's problem, allowing the three constraints to be written as

$$M_t^c = P_t c_t a_t^c, \tag{42}$$

$$M_t^{dd} = P_t c_t a_t^{dd}, \tag{43}$$

$$M_t^{cd} = P_t c_t (1 - a_t^c - a_t^{dd}). \tag{44}$$

The consumer problem now buys goods with cash or demand deposits as before, but also has a debit of $a_t^{cd} P_t c_t$ for credit purchases, and has a debit of $(P_t^{cd}/P_t)M_t^{cd}$ due to the credit fee.

This makes the consumer problem

$$\begin{aligned}(45)\quad & \underset{c_t, d_{t+1}, k_{t+1}^{dd}, k_{t+1}^{cd}, M_{t+1}^c, M_{t+1}^{dd}, M_t^{cd}, a_t^c, a_t^{dd}}{Max} L \\ &= \sum_{t=0}^{\infty} \beta^t \{u(c_t) + \lambda_t [P_t(1+r_t)d_t + H_t \\ &\quad + \Pi_t^r + \Pi_t^{dd} + \Pi_t^{cd} + M_t^c + M_t^{dd} \\ &\quad - M_{t+1}^c - P_t k_{t+1}^{dd} + P_t k_t^{dd}(1+r_t) \\ &\quad - P_t k_{t+1}^{cd} + P_t k_t^{cd}(1+r_t) - \phi M_t^c \\ &\quad - M_{t+1}^{dd} - (P_t^{dd}/P_t)M_t^{dd} \\ &\quad - (P_t^{cd}/P_t)M_t^{cd} - P_t c_t - P_t d_{t+1}] \\ &\quad + \mu_t^c [M_t^c - a_t^c P_t c_t] + \mu_t^{dd}[M_t^{dd} - a_t^{dd} P_t c_t] \\ &\quad + \mu_t^{cd}[M_t^{cd} - (1 - a_t^c - a_t^{dd})P_t c_t]\}.\end{aligned}$$

The first-order conditions imply that the interior solution satisfies

$$P_t^{dd}/P_t = \phi \tag{46}$$

$$P_t^{cd}/P_t = R + \phi, \tag{47}$$

and the shadow cost goods is again, as in the last section, given by

$$u_{c_t} = \lambda_t P_t (1 + R_t + \phi). \tag{48}$$

Denote the name for the exchange credit banking firm as Amex. Amex is assumed to supply the exchange credit, denoted by $M_t^{cd,s}$, using only capital, denoted by k_t^{cd}, in a diminishing returns fashion similar to the technology for the demand deposit bank. Although this technology could be given as $(M_t^{cd,s}/P_t) = \hat{A}_{cd}(k_t^{cd})^{\theta}$, where $\hat{A}_{cd} > 0$ and $\theta \in (0,1)$, for a general diminishing returns case, the problem would arise that the equilibrium share of the Amex credit would trend down toward zero if there was a positive growth rate g_t, making infeasible the existence of a balanced-growth path. Therefore consider a technology similar to equation (36), which gives a stable share of exchange credit in purchases. In particular, let the function be specified with a complementary goods externality that affects the shift parameter $\hat{A}_{cd}$, whereby $\hat{A}_{cd} = A_{cd}c^{1-\theta}$, so that

(49) $$M_t^{cd,s}/P_t = A_{cd}c_t^{1-\theta}(k_t^{cd})^{\theta}.$$

The profit maximization problem is static and given by

(50) $$\underset{k_t^{cd}}{Max}\,\Pi_t^{cd} = P_t^{cd}A_{cd}c_t^{1-\theta}(k_t^{cd})^{\theta} - P_t r_t k_t^{cd}.$$

The equilibrium conditions of the consumer and Amex bank imply that

(51) $$R_t + \phi = P_t^{cd}/P_t = r_t/[A_{cd}\theta(k_t^{cd}/c_t)^{\theta-1}];$$

(52) $$k_t^{cd}/c_t = [A_{cd}\theta(R_t + \phi)/r_t]^{1/(1-\theta).}$$

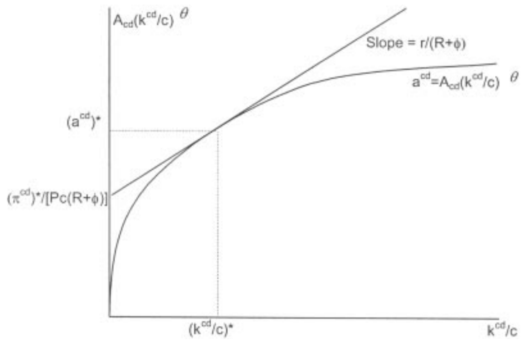
This means that as the nominal interest rate rises, the Amex bank expands credit supply and k_t^{cd}/c_t rises in equilibrium. *It means that the marginal costs of exchange are equated to* $R + \phi$ *across all of the different forms of exchange*, being cash, demand deposits, or credit. This equalization of the marginal costs of the various means of exchange, the basis of Baumol's (1952) equilibrium, is one of the most important features of the general equilibrium.

Equating the supply and demand for the Amex credit, from equations (44) and (49), and using equation (51), the share of exchange credit can be found to be

(53) $$a_t^{cd} = A_{cd}^{1/(1-\theta)}[\theta(R_t + \phi)/r_t]^{\theta/(1-\theta)},$$

also rising as the nominal interest rate goes up. Note that by substituting equation (53) into equation (41), so that $1 - a_t^{cd} = a_t^c + a_t^{dd}$, and

FIGURE 2
Equilibrium in the Credit Bank Sector



then substituting in equation (39), the solution for a_t^c is found.[12]

Figure 2 illustrates the equilibrium for the credit bank. At the Friedman optimum of $R = 0$, some credit would still be provided as long as $\phi > 0$. This use of credit at $R = 0$ contrasts to zero such use of credit in Gillman (1993), Ireland (1994), and Gillman and Kejak (2002).

The money market clearing condition here is that the demand for the exchange credit equals the supply of the exchange credit. This can also be further aggregated to

(54) $$M_t^c + M_t^{dd} + M_t^{cd} \equiv M2_t,$$

and can be considered an aggregate like M2. It includes the monetary base, demand deposits, plus the exchange credit that allows funds to collect interest during the period, as do certificates of deposit, and is then paid off

12. Alternatively, the exchange credit sector can be kept implicit by having the consumer engage in "self-production" of the exchange credit. This can be done by constraining the consumer's problem by the technology constraint (49), combining this constraint with equation (44), solving for a_t^{cd}, and using this to substitute in for a_t^{cd} in the consumer problem (45), with the consumer now choosing k_t^{cd} instead of a_t^{cd}. This approach would make the revised Clower contraint (44) equal to $M_t^{cd} = P_t A_{cd}c^{1-\theta}(k_t^{cd})^{\theta}$. Setting $\gamma = 0$ and $\phi = 0$, then $M_t^c/(P_t c_t) = 1 - a_t^{cd} = a_t^c$, and only this one Clower constraint would be necessary. Now solve this constraint for k_t^{cd}, and it would take a form exactly analogous to a special case of the McCallum and Goodfriend (1987) shopping time constraint, but in capital instead of time, that depends on real money balances and goods in the same direction: $k_t^{cd} = c_t[1 - (M_t^c/P_t)/c_t]^{1/\theta}(1/A_{cd})^{1/\theta}$; with $\partial k_t^{cd}/\partial(M_t^c/P_t) < 0$, and $\partial k_t^{cd}/\partial c_t > 0$ (see Walsh 1998, on shopping time models).

with "money market mutual funds" invested in short-term government securities. So it is a mixed set of non–interest-bearing aggregates that suffer the inflation tax and are traditionally thought of as money-like in nature, and of the Amex credit and money market accounts that avoid the inflation tax, unlike "money."

IV. CHANGES IN AGGREGATES OVER TIME

The model of M2 can be used to analyze how subsets of aggregates change according to changes in exogenous factors. In particular the focus is on changes in the money supply growth rate, σ, or more simply in the nominal rate of interest because this is given by $R = \sigma + \rho + \rho\sigma$. Also the focus is on changes in the banking productivity parameters A_{dd} and A_{cd}, and the banking cost parameter ϕ. Comparative statics of these factors are then applied to explain the actual profiles of the velocity of monetary aggregates, and the profiles of their ratios.

The explanation of the aggregates relies on changes in productivity that result from changes in U.S. bank law. This approach can be formalized by adding a proportional tax to the credit firm's proceeds from selling the credit, denoted by τ, whereby the price received by the firm is $P_{cd}(1-\tau)$. Now assume that the tax proceeds are destroyed, as regulations sometimes are modeled. Then the equilibrium is such that in equation (51) the productivity factor is factored by $(1-\tau)$. An increase in regulations makes τ bigger, and effective (net) productivity smaller, whereas deregulation causes τ to decrease and effective (net) productivity to increase. The same regulations can likewise affect A_{dd}. Now consider the following brief review of major U.S. deregulatory laws in banking to indicate how and when the effective productivity factor might shift.

Financial Deregulation and Increases in Bank Productivity

Significant U.S. financial deregulation manifested with the Depository Institutions Deregulation and Monetary Control Act of 1980, the Garn-St. Germain Financial Modernization Act of 1982, the Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994, and the Gramm-Leach-Bliley Act of 1999. The 1980 law phased out interest ceilings and allowed banks to pay more interest on deposits. The 1982 law allowed banks to offer money market accounts to compete with mutual funds. The 1994 act allowed national bank branching and consolidation:

> Congress passed significant reform legislation in the 1990s. In 1994, the Riegle-Neal Interstate Banking and Branching Efficiency Act repealed the McFadden Act of 1927 and Douglas Amendments of 1970, which had curtailed interstate banking. In particular, the McFadden Act, seeking to level the playing field between national and state banks with respect to branching, had effectively prohibited interstate branch banking. Starting in 1997, banks were allowed to own and operate branches in different states. This immediately triggered a dramatic increase in mergers and acquisitions. The banking system began to consolidate and for the first time form true national banking institutions, such as Bank of America, formed via the merger of BankAmerica and NationsBank. (Guzman 2003).

The 1999 law permitted mergers between banks, brokerage houses, and insurance companies, "allowing banking organizations to merge with other types of financial institutions under a financial holding company structure" (Hoenig 2000).

Comparative Statics and Comparison to the Evidence

The income velocity of money is defined as income divided by a particular monetary aggregate. The income in the economy comes from the goods production function; this makes it equal to $(A-\delta)k_t$, which equals $(A-\delta)(1-\gamma)d_t$. The velocity of the monetary aggregates can then be defined as $(A-\delta)(1-\gamma)d_t/M_t^b$.

PROPOSITION 1. *Given $g = 0$, and along the balanced growth path, the base money velocity rises with the nominal interest rate, or*

$$\partial[(A-\delta)(1-\gamma)d_t/M_t^b]/\partial R > 0.$$

Proof. The solution for the base velocity is

$$(A-\delta)(1-\gamma)d_t/M_t^b = [(A-\delta)(1-\gamma)(d_t/c_t)] / [1-a^{dd}-a^{cd}+\gamma(d_t/c_t)],$$

where $a^{dd} = A_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r]^{\alpha/(1-\alpha)}$, $a^{cd} = A_{cd}^{1/(1-\theta)}[\theta(R+\phi)/r]^{\theta/(1-\theta)}$, $r = (A-\delta)(1-\gamma) - \gamma(1+R)$, $(1+g) = (1+r)/(1+\rho)$, and $d_t/c_t = [1+\phi(1-a^{dd}-a^{cd}) + g(k_t^{dd}+k_t^{cd})/c_t] / [(A-\delta)(1-\gamma)-g-\gamma]$. At $g = 0$, $d_t/c_t = [1+\phi(1-a^{dd}-a^{cd})]/[(A-\delta)(1-\gamma)-\gamma]$

FIGURE 3
U.S. Base Velocity and Nominal Interest Rates: 1959–2003

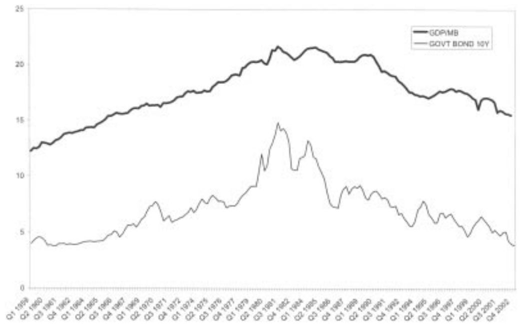
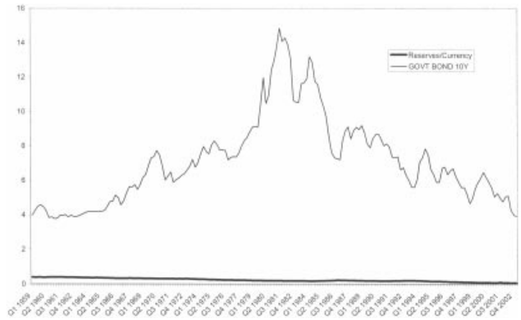


FIGURE 4
U.S. Reserves to Currency Ratio and Interest Rates: 1959–2003



and $(A-\delta)(1-\gamma)d_t/M_t^b = (A-\delta)(1-\gamma)/\{[(A-\delta)(1-\gamma)+\gamma]/([1/(1-a^{dd}-a^{cd})]+\phi)+\gamma\}$.

Substituting in for a^{dd}, a^{cd}, it can be seen that $\partial[(A-\delta)(1-\gamma)d_t/M_t^b]/\partial R > 0$.

Note that the solution of d_t/c_t requires substituting into the budget constraint of the problem in equation (45), using equations (10), (30), (41), (42), (50). ■

Figure 3 shows the post–1959 U.S. base money velocity and the 10-year bond, U.S. Treasury, interest rate. McGrattan (1998) presents such a graph and argues, in her comment on Gordon et al. (1998), that the nominal interest rate goes a long way to explaining base money velocity.[13] And this is the implication of the result of Proposition 1. The difference from McGrattan (1998) is that she uses a simple linear econometric equation, as found in Meltzer (1963) and Lucas (1988), to argue that the nominal interest rate has a direct effect on velocity. Here the velocity is derived analytically to make the point from the general equilibrium perspective.[14]

Comparative statics for the other factors, A_{cd}, A_{dd}, and ϕ, are ambiguous in general because of the d_t/c_t factor, but holding d_t/c_t constant then all three factor have a positive effect on base velocity. This positive direction of the effect of these factors is also readily apparent in calibrations. Although these other factors do not provide any obvious help in interpreting base velocity empirical evidence, they do provide an explanation as based on the model of the evidence on the ratio of reserves to currency.

Figure 4 shows the post-1959 U.S. reserves/currency ratio against the long-term interest rate. There is a marked trend down, with a flattening out period during the 1980s, and a rather more pronounced downward direction after 1994. In the model, M_t^r/M_t^c is the notation for the reserves to currency ratio and is given $M_t^r/M_t^c = \gamma d_t/c_t/(1-a^{dd}-a^{cd})$. With d_t/c_t held constant, the reserves to currency ratio rises with increases in each R, ϕ, A_{dd}, and A_{cd}. Because the U.S. reserves/currency trend is downward and the effect of the nominal interest is upward in the 1959–81 period, it appears that the nominal interest plays no role in explaining this ratio. In contrast, the hypothesis of a downward trend in the cost of using money, ϕ, serves well to explain the evidence.

M1 velocity is defined by $(A-\delta)(1-\gamma)d_t/M1_t = [(A-\delta)(1-\gamma)(d_t/c_t)]/(1-a_t^{cd})$. With d_t/c_t held constant, along the balanced growth path, M1 velocity rises with the nominal interest rate because a_t^{cd} rises. Similarly, an increase in A_{cd} and ϕ cause M1 velocity to go up.

Figure 5 shows the U.S. M1 velocity and the 10-year U.S. Treasury interest rate from 1959 to 2003. The rise in velocity from 1959 to 1981 is consistent with the rise in the nominal interest rate. While still following changes in

13. McGrattan (1998) argues that the long-term rate is better to use than the short-term rate that Gordon et al. (1998) use. "Low frequency movements in velocity are well-explained by low frequency movements in observed interest rates."

14. Note that Gillman and Otto (2002) take the time series approach of Meltzer (1963) and Lucas (1988) while including a data series on the productivity in banking to capture changes in productivity. They find cointegration of money demand with the productivity series, but without it the money demand appears to be instable. Or as Parry (2000) asserts, "Once deposit interest rates began to vary with market rates, the demands for M1 and M2—the primary guides to monetary policy—became unstable."

FIGURE 5
U.S. M1 Velocity and Nominal Interest Rates: 1959–2003

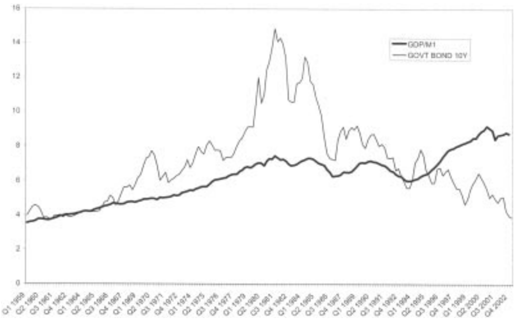
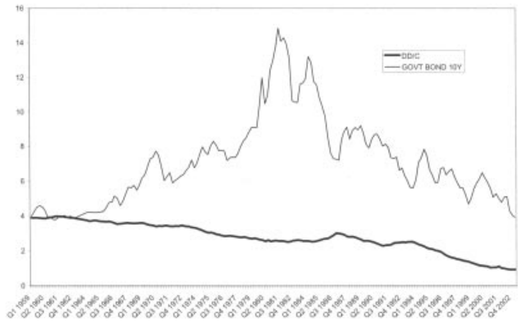


FIGURE 6
U.S. Demand Deposits to Currency Ratio and Interest Rates: 1959–2003



the nominal interest rate in the 1980s, M1 velocity appears to level off rather than fall during this period by as much as would be expected from the decrease in the nominal interest rate. Deregulation of the 1980s, and an associated increase in A_{cd} presents an explanation of the leveling off of velocity in the 1980s. The striking trend upward in velocity after 1994 is consistent with an accelerated increase in A_{cd} that can be from the deregulation of interstate branching that led to national branching and the diffusion of ATMs, as well as the banking consolidation because of the 1999 act. Thus the two factors of the nominal interest rates and the banking productivity each play a distinct role in this explanation.[15]

A way to see further into the M1 velocity profile is to look at the ratio of its components, currency and demand deposits. Analytically the demand deposit to currency ratio in the model is M^{dd}/M^c.

PROPOSITION 2. *The demand deposit to currency ratio, M_t^{dd}/M_t^c, rises with increases in each R, ϕ, A_{dd}, and A_{cd}.*

Proof. From equations (39), (41), (42), (43), (44), and (53), $M_t^{dd}/M_t^c = (A_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r]^{\alpha/(1-\alpha)})/(1 - A_{dd}^{1/(1-\alpha)}[\alpha(R+\phi)/r]^{\alpha/(1-\alpha)} - A_{cd}^{1/(1-\theta)}[\theta(R+\phi)/r_t]^{\theta/(1-\theta)})$, and $\partial(M_t^{dd}/M_t^c)/\partial R > 0$, $\partial(M_t^{dd}/M_t^c)/\partial A_{cd} > 0$, $\partial(M_t^{dd}/M_t^c)/\partial A_{dd} > 0$, and $\partial(M_t^{dd}/M_t^c)/\partial\phi > 0$. ■

15. Ireland (1995) compares U.S. M1-A velocity with six-month Treasury bill interest rates. He explains velocity as following a continuous upward trend due to financial innovation.

Figure 6 shows the U.S. demand deposit to currency ratio and the 10-year U.S. Treasury interest rate for the same 1959–2003 period. In a first look, the ratio simply trends down. But looking more closely shows a simple trend down, from 1959 to 1981, that levels off in the 1980s, as with M1 velocity, and then moves down steadily post-1994 at an accelerated rate compared to the earlier period.

A downward trend in ϕ well explains the downward trend in the demand deposit to currency ratio in a way the nominal interest rate's pre-1981 upward trend and a possible upward trend in A_{cd} and A_{dd} cannot. However, the role of A_{cd} and A_{dd} again emerges as the only way to explain the leveling off of the trend in demand deposits to currency in the 1980s, when there was financial deregulation and a surge in A_{cd} and A_{dd}. Furthermore, the accelerated downward trend in the ratio after 1994 is consistent with an accelerated decrease in ϕ because of the ATM diffusion.[16]

Now consider the velocity of the broader aggregate M2. In the model, M2 velocity is defined by $(A-\delta)(1-\gamma)d_t/M2_t$. This is given by $(A-\delta)(1-\gamma)d/M2_t = (A-\delta)(1-\gamma)d_t/[c_t(a_t^c + a_t^{dd} + a_t^{cd})] = (A-\delta)(1-\gamma)(d_t/c_t)$. The comparative statics of the M2 velocity are therefore as the comparative statics of the ratio of savings to consumption. The effects of R, A_{cd}, A_{dd}, and ϕ are ambiguous in general,

16. Note that stable deposit to currency ratios were reported by Cagan (1956) for the hyperinflations he studied (an exception was post–WW II Hungary that Cagan suggests is due to data problems). This indicates a small role of the nominal interest rate in causing changes in this ratio and is consistent with the small role given here to the nominal interest rate in explaining the U.S. ratio's postwar movement.

FIGURE 7
U.S. M2 Velocity and Nominal Interest Rates: 1959–2003

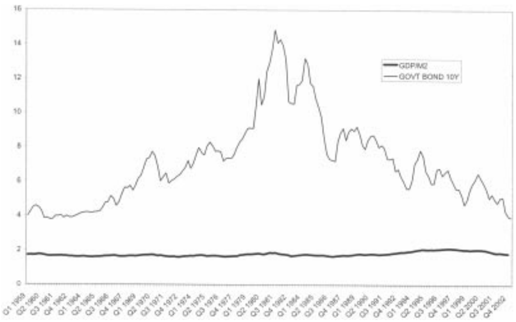
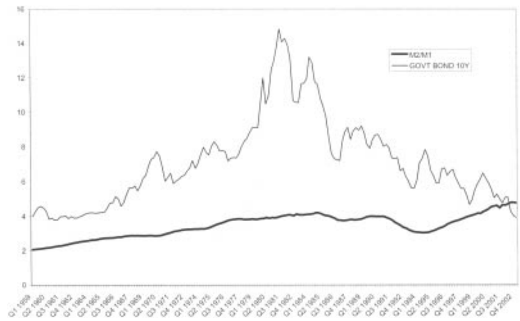


FIGURE 8
U.S. Ratio of M2 to M1 and Interest Rates: 1959–2003



although with $g = 0$, it is true as shown that $\partial(d_t/c_t)/\partial R > 0$. But the (d_t/c_t) factor does not appear to play any significant role in the explanation of base or M1 velocity. Figure 7 indeed shows that U.S. M2 velocity has been remarkably constant relative to the 10-year U.S. Treasury bond rate. Thus the explanation from the model is that the magnitude of changes in (d_t/c_t), because of the factors considered here, is small. It is easy to confirm this with calibrations, although this exercise is not reported. However, one aspect of this is worth noting. With a relatively unchanging d_t/c_t as the explanation for a stable M2 velocity, it is internally consistent with the previous analysis that the comparative statics of A_{cd}, A_{dd}, and ϕ, with d_t/c_t held constant, can be used to explain base and M1 velocity.

Breaking down the components of M2 is more revealing. Consider the ratio of M2 to M1. In the model this is given by $M2_t/M1_t = 1/[1 - A_{cd}^{1/(1-\theta)}\ [\theta(R+\phi)/r]^{\theta/(1-\theta)}]$.

PROPOSITION 3. *Along the balanced growth path, the ratio $M2_t/M1_t$ rises with an increase in the nominal interest rate, or $\partial(M2_t/M1_t)/\partial R > 0$.*

Proof. $\partial([1 - A_{cd}^{1/(1-\theta)}[\theta(R+\phi)/r]^{\theta/(1-\theta)}])/\partial R > 0$. ■

The other comparative statics with respect to A_{cd} and ϕ are ambiguous because of the d_t/c_t factor; holding d_t/c_t constant, the ratio $M2_t/M1_t$ rises with each of these. Now consider Figure 8, which shows the U.S. ratio of M2 to M1 from 1959 to 2003, along with the 10-year U.S. Treasury bond rate. Proposition 3 provides a way to explain the upward trend in M2/M1 from 1959 to 1981, and perhaps the fall in M2/M1 from 1990 to 1994. The leveling off of M2/M1 in the 1980s can be explained by financial deregulation and increases in A_{cd}; note that the downward change in R during this period, and a downward trend in ϕ during this period cannot explain the leveling off of M2/M1, because these factors work to make the ratio go down. The trend upward after 1994 again can be explained by upward increases in A_{cd} because of national branching being allowed, ATM diffusion, and consolidation.

V. DISCUSSION

The demand for bank reserves that Haslag (1998) put forth helps pave the way for modeling the demand for a range of monetary aggregates. The model as revised here acts as a missing link that ties together conventional money demand functions from the cash-in-advance approach with an analog to the monetary aggregates widely studied, by adding a bank's demand for cash reserve. An inflation tax on the deposit rate of return results because, as in the cash-in-advance economies, the intermediation bank must in effect put aside cash-in-advance to meet the demands of the reserve requirement. This is similar to Stockman (1981) in which the Clower (1967) constraint is applied to all investment; here however, the intermediation bank's Clower (1967) constraint applies only to the reserve fraction of the investment.

On the basis of the intermediation bank's demand for reserves plus the imposition of a standard Clower (1967) constraint on the consumer's purchase of goods, the demand for an aggregate similar to the monetary base, reserves plus currency (cash), is constructed whereby the inflation rate can affect the real return to intermediated investment under an *AK* technology because of the need to hold cash reserves. This model is extended to include non–interest-bearing deposits, unlike previous work, and in a way that gives an aggregate analogous to *M*1. The model further is extended to include exchange credit, to give an aggregate analogous to *M*2. In this fully extended model comparative statics are presented for base, M1, and M2 velocity and the ratios of demand deposits to reserves, demand deposits to currency, and M2/M1. With these analytics the empirical evidence on both the velocities and various ratios of the aggregates are explained in an internally consistent way. This requires more than only the nominal interest rate. In addition the productivity of the credit bank sector plays a critical role in explaining aggregate movement during the financial deregulation era. The convenience cost of using money has a unique role in explaining the trends in the reserve to currency and in the demand deposit to currency ratios.

The models here enable the consumer to choose the least expensive source of exchange means. As a result, the Clower (1967) constraints are not "exogenously" imposed on the consumer but rather left as a consumer choice to bind certain fractions of purchases to particular exchange means only to the extent that the particular exchange means is efficient for the consumer to use. This consumer choice among alternative means of exchange might be seen as ameliorating the strength of the criticism of the "deep" models of money that the Clower (1967) constraint is exogenously imposed, or even as offering an alternative approach to the search for deep models.[17]

Note that the model of the exchange credit sets the quantity of credit that is produced equal to the value of the output of the consumption good that is being bought on credit. Aiyagari et al. (1998) instead model credit as a service that is produced, and then enters as an input into a production function for credit goods. The credit goods production is Leontieff in its inputs of the credit service and of the value of the consumption goods being bought with the credit. This Leontieff technology in equilibrium implies as a special case the condition that the credit services output equals the value of the consumption goods being bought with the credit.[18] In this article, as in the continuum-of-stores approach in Gillman (1993), Ireland (1994), and Erosa and Ventura (2000), there are no credit or cash goods per se, only the consumption good that can be bought with cash or credit. This, in a sense, can be thought of as collapsing the Aiyagari et al. (1998) type of credit goods and credit services into a single technology called credit, whereby the equilibrium condition that is implied by the special case of the Leontieff technology of Aiyagari, et al. (1998) is implicitly applied. The advantage of the model here over the continuum-of-stores approach is that here the velocity can be solved more simply.

The model's implications for growth are that inflation lowers growth because it lowers the real interest rate, a result supported in Ahmed and Rogers (2000). However, this feature combined with an *Ak* goods production technology cannot account for the substitution from effective labor to capital, as induced by inflation, that Chari et al. (1996) describe and that Gillman and Nakov (2003) further elaborate; Gillman and Nakov (2003) find evidence in support of this substitution for the postwar U.S. and U.K. data. Thus, although the *Ak* model provides easier analytic tractability, a goods production function with both labor and capital as in Gomme (1993) and Gillman and Kejak (2002) also can account for a negative effect of inflation on growth (see also Jones and Manuelli 1995). Because this approach also involves the inflation-induced labor to capital substitution, it may be useful to nest the models of monetary aggregates within the Gomme (1993) framework.[19]

Gillman and Kejak (2002) go partly in this direction by extending Gomme (1993) so as to

17. See Bullard Smith (2001), and Azariadis et al. (2000), for example, for an alternative approach to modeling "inside" money, based on a three-period model. They apply this to analyze the optimality of restricting inside money; Gillman (2000) analyzes the optimality of such restrictions in a model similar to the paper here.

18. The case is that $q = 1$ in Aiyagari et al. (1998) model, using their notation.

19. Changes in the real interest rate in the *Ak* model presented here occur only through changes in the inflation rate and are discussed in this fashion. In a model with labor and capital, the real interest rate could move endogenously with velocity. At business cycle frequencies, this simultaneity may be interesting to investigate.

include credit, as in section III of this article. One advantage of having monetary aggregates more fully embedded in the King and Rebelo (1990) type of endogenous growth model is that this provides the leisure channel by which to substitute away from inflation and so make the inflation tax less burdensome to the individual consumer. As Gillman and Otto (2002) show, the Gillman and Kejak (2002) model with leisure and the credit substitute in addition creates an interest elasticity of money demand that rises significantly in magnitude with inflation. This feature also exists in our model, and this is the central feature of the Cagan (1956) model. Or, as Martin Bailey (1992) put it, "Cagan's principal conclusion, indeed, is that the demand for real cash balances ... has a higher and higher elasticity at higher and higher rates of inflation." Mark and Sul (2002) report recent international panel evidence in support of the Cagan (1956) money demand function. Only with such an elasticity, within the general equilibrium money demand function, are Gillman et al. (2003) able to explain international evidence on inflation and growth.[20]

The current *Ak* model of this article implies that an increase in the nominal interest rate causes the same degree of a growth rate decrease, no matter what the level of the nominal interest rate. But rather than this linear relation, the evidence shows a high degree of nonlinearity, with stronger negative inflation effects at low inflation rate levels. An inflation-induced rising interest elasticity makes substitution toward leisure less and toward credit more, making the decrease in the growth rate less. Therefore, as inflation rises, the additional leisure and credit channels help explain both effective-labor-to-capital substitution and a rising interest elasticity that leads to a falling magnitude of the marginal decrease in the growth rate.

20. Paal and Smith (2000) offer an overlapping generations model in which low inflation can cause a positive effect on growth, while higher inflation causes a negative level. This is supported in the panel evidence of Ghosh and Phillips (1998), Khan and Senhadji (2000), Judson and Orphanides (1996), and Gillman et al. (2003) in which a threshold level of inflation is found after which the inflation-growth effect is negative. However the positive effect at low inflation rates is found to be insignificant in these works. Gillman et al. (2003) show that using instrumental variables, the effect of inflation on growth is negative for all positive levels of inflation, across both OECD and APEC regions, as well as in the full sample; Ghosh and Phillips (1998) also find this for a full sample.

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