CONTRASTING MODELS OF THE EFFECT OF INFLATION ON GROWTH

Max Gillman
Central European University

Michal Kejak
CERGE-EI

Abstract. The paper formulates a nesting model for studying the theoretical literature on inflation and endogenous growth. It analyses different classes of endogenous growth models, with different usage of physical and human capital, with different exchange technologies. First, the paper shows that a broad array of models can all generate significant negative effects of inflation on growth. Second, it shows that these models can be differentiated primarily by the fact whether there is a Tobin-type effect of inflation and also whether the inflation–growth effect becomes weaker as the inflation rate rises, a non-linearity, or stays essentially constant over the range of inflation rates. The paper compares these features of the models to empirical evidence as a way to summarize the efficacy of the models.

Keywords. Balanced-growth-path; Calibration; Cash-in-advance; Endogenous growth; Human capital; Inflation

1. Introduction

There are three main controversies in the literature on the long-run effect of inflation on growth. First is whether models can exhibit a significant negative effect of stationary inflation on the balanced-path growth rate. Second is the nature of the inflation–growth effect across the whole range of the levels of inflation rate. Third is whether the inflation–growth models and evidence can at the same time be consistent with evidence of Tobin-type (1965) effects.

The contribution of the paper is to first bring together for comparison several main approaches to modelling the inflation–growth effect by nesting them within a general model. This shows what factors determine the magnitude of the inflation–growth effect across these different approaches and it yields the following notable result: a robustness for the ability to generate a strong magnitude of the inflation–growth effect. In addition, the paper explains the source of the effect by

0950-0804/05/01 0113–24 JOURNAL OF ECONOMIC SURVEYS Vol. 19, No. 1

showing that the key distinguishing feature of competing approaches is whether inflation acts mainly as a tax on physical capital or on human capital. The outcome of this determines whether the inflation–growth effect accompanies an inverse or positive Tobin (1965) effect. Finally, evidence on the growth and Tobin (1965) effects is brought to bear on these competing models, as a way to lend support to favouring one approach over another.

The paper's approach to explaining this literature is to allow for a mix of physical and human capital in the production of goods and for a mix of exchange means, money and credit, in buying the goods. The general model starts with an exchange technology extended from the standard cash-in-advance genre using microfoundations in such a way that it also encompasses a special case of the shopping time model. The extension specifies the production of credit, which is used as an alternative to money. This helps distinguish the overall inflation–growth effect in terms of its theoretical characteristics over the range of (non-hyperinflation) inflation rates, as well as some additional 'money and banking' facts.

Starting with Ireland's (1994) 'Money and Growth: An Alternative Approach' that compares to transitional inflation–growth effects found in Sidrauski's (1967) 'Money and Growth', the paper sets out a model that puts Ireland's (1994) approach within an aggregate consumption good setting. From this capital-only economy that includes credit, the paper next covers a case of Stockman's (1981) capital-only economy with investment as a 'cash good'; this model with uncertainty added is used in Dotsey and Sarte (2000).[1] The paper then turns to human capital-only models that compare to Gillman *et al.* (1999) and Stokey and Lucas (1989) (section 5.8). Then the paper sets out models with both types of capital that compare to Gomme (1993) and to the capital accumulation process of Chari *et al.* (1996). Finally, an extension to Gomme (1993) is put forth that includes credit, as in Gillman and Kejak (2002, 2005).

Most evidence finds a negative inflation–growth effect. For example, by way of large changes in the inflation rate, Gylfason and Herbertsson (2001) list some 17 studies for which all but one find a significant decrease in the growth rate from increasing the inflation rate from 5 to 50%. More by the way of a marginal increase in the inflation rate, Chari *et al.* (1996) review the empirical results from increasing the inflation rate from 10 to 20%; they report a significant fall in the growth rate within a range of 0.2–0.7%; for example, the growth rate falls from an initial level of 3% at a 10% inflation rate to between 2.8 and 2.3% at a 20% inflation rate. Recent findings, for example, of Barro (2001) compound the evidence of a strongly significant negative effect of inflation on growth.

In addition, evidence suggests that the negative effect is marginally stronger at low inflation rates and marginally weaker as the inflation rate rises. This negative and highly non-linear effect is strongly supported in Judson and Orphanides (1999), Ghosh and Phillips (1998), Khan and Senhadji (2001) and Gillman *et al.* (2003). Some evidence is qualified by findings of a 'threshold' rate of inflation, above which the effect is strongly significant and negative, but below which the effect is insignificant and positive. For industrialized country samples, this threshold level has been tested for and found to be very low, at a 1% inflation rate

(Khan and Senhadji, 2001), although others have assumed (without such testing) higher thresholds of 2.5% (Ghosh and Phillips, 1998) or 10% (Judson and Orphanides, 1999). The 'threshold' for developing country samples has been found through testing to be at 11% (Khan and Senhadji, 2001), below which again the inflation–growth effect is insignificant and positive. However, when using instrumental variables in order to adjust for possible inflation–growth endogeneity bias, the negative non-linear inflation–growth effect has been reinstated at all positive inflation rate levels for both developed and developing country samples (Ghosh and Phillips, 1998; Gillman *et al.*, 2003). This suggests no inconsistency in a modelling approach that focuses only on a negative effect of inflation on growth.[2]

Tobin (1965) evidence includes an inflation-induced decrease in the real interest rate, an increase in the average investment level and decline in the consumption level, normalized by output, and a rise in the aggregate capital-to-effective-labour ratio. Ahmed and Rogers (2000) find a variety of Tobin (1965) long-run evidence for the US, including a decrease in the real interest rate because of permanent inflation increases. Similarly, Rapach (2003) finds that permanent inflation increases lower the long-run real interest rate in 14 out of 14 countries studied. There is also the related evidence in Gillman and Nakov (2004) of inflation Granger-causing increases in the capital-to-effective-labour ratios in the US and UK postwar data, which is consistent with a Tobin (1965) effect of increased capital intensity.

The literature on how to model such evidence extends the traditional Tobin (1965) modification of the Solow exogenous growth model, whereby money is introduced as an alternative to capital. Similar to the original IS-LM model, in the Tobin (1965) model, an increase in the money-supply growth rate, or in the inflation rate, causes investment, capital and output to rise. But the growth rate is exogenous and so is unaffected by the inflation rate. The extensions from the Tobin (1965) framework are classes of the Cass–Koopman neoclassical model that endogenizes the savings rate of the exogenous Solow growth model through utility maximization; this gives the Euler equation results whereby the growth rate equals the marginal product of capital, net of time preference, and (for CES [constant elasticity of substitution] utility) normalized by a utility parameter. Furthermore, the extensions are of the endogenous growth genre as extended to a monetary setting, whereby the rate of return on real money, being based on the inflation rate, can affect the marginal product of capital and the growth rate. In the endogenous growth models, inflation typically causes the growth rate to fall, while the output as a balanced-growth-path ratio relative to different variables can rise or fall, resulting in either Tobin (1965) effects or inverse Tobin (1965) effects. The idea of an inverse Tobin (1965) effect follows from Stockman (1981), whereby the inflation rate increase causes a capital stock decrease.

Therefore, in the endogenous growth models, the inflation rate affects the growth rate because it affects the marginal product of capital, either that of physical capital as in *Ak* models, or that of human capital as in *Ah* models, or that of both physical and human capital in combined capital models. Some models have produced insignificant long-run inflation–growth effects, for example, the *Ak* models of

Ireland (1994) and Dotsey and Sarte (2000) and the physical and human capital model of Chari *et al*. (1996), while at least equally diverse models have produced significant and negative inflation–growth effects, including the *Ak* models of Haslag (1998) and Gillman and Kejak (2004), the *Ah* model of Gillman *et al*. (1999), Gylfason and Herbertsson's (2001) model with money in the goods production function, Gomme's (1993) physical and human capital model and Gillman and Kejak's (2002, 2005) extension of Gomme (1993). Using a nesting as based on *Ak*, *Ah* or a combination of physical and human capital can illustrate the results from most of the models, and hence this is the approach taken here.

Note that these are balanced-path, stationary-state, results. And it is actually the rate of money-supply growth in these models that is exogenous, changes in which 'cause' changes in the stationary inflation rate and simultaneously cause changes in the output growth rate. However, because, for example, long-run evidence finds that money Granger causes inflation (Crowder, 1998), and because some evidence also finds that inflation Granger causes the output growth rate (Gillman and Nakov, 2004; Gillman and Wallace, 2003; Cziraky and Gillman, 2003), this literature on inflation and growth tends to discuss how inflation affects the output growth rate. This convention is also used here. Furthermore, with log-utility as we assume throughout the paper, the nominal interest rate depends on only the money-supply growth rate and the rate of time preference, so that we can calibrate how increases in the nominal interest rate affect the economy in a way equivalent to a money-supply acceleration.

The calibration strategy is to examine the change in the nominal interest rate on the balanced-path growth rate, the real interest rate and the capital-to-labour ratio in the goods sector across the different models. For all models, the growth rate and the real interest rate are fixed at the same values at the optimum, of 3% for the growth rate and 6% for the real interest rate (nine for the gross real interest rate). Given the greater number of degrees of freedom in the more complicated models relative to the simpler models, calibrating them so that they have a common point at the optimum allows for a normalized comparison of how inflation affects the economies as it rises up from its optimal level.

2. The General Monetary Endogenous Growth Economy

The nesting model has three sectors that each uses both physical capital-indexed and human capital-indexed labour: goods production, human capital investment and credit production. The notation, parameter assumptions and production specifications are presented in the Table 1.

Current period utility is of the constant elasticity of substitution form, whereby

$$u(c_t, x_t) = \ln c_t + \alpha \ln x_t \tag{1}$$

The consumer allocates time to labour supplied to the goods producer, to self-production of human capital and to self-production of credit. With an endowment of one unit of time, the time constraint is similar to an adding-up-of-shares constraint:

Table 1. Notation and Assumptions.

Variables		Parameters
Real	l_{Ht}: HC share, HC	$\beta \in [0,1]$
y_t: Goods output	l_{Ft}: Credit share, HC	$\varepsilon \in [0,1]$
c_t: Consumption goods	r_t: Interest rate	$\gamma_1, \gamma_2 \in [0,1]$
x_t: Leisure time	w_t: Effective wage rate	$A_G > 0$
k_t: Physical capital (PC)	Nominal	$A_H > 0$
h_t: Human capital (HC)	M_t: Money stock	$A_F > 0$
i_t: PC investment	P_t: Goods price	$\delta_K \geq 0$
i_{Ht}: HC investment	V_t: Money transfer	$\delta_H \geq 0$
s_{Gt}: Goods share, PC	Definitions	$\alpha > 0$
s_{Ht}: HC share, PC	$m_t \equiv M_t/P_t$	$\sigma \geq -\rho$
s_{Ft}: Credit share, PC	$a_t \equiv m_t/c_t$	$\rho \in (0,1)$
l_{Gt}: Goods share, HC	$d_t \equiv (1-a_t)\,c_t$	$a_2 \in [0,1]$
	Production functions	
Goods	Human capital investment	Credit
$y_t = A_G\,(s_{Gt}k_t)^{\beta}\,(l_{Gt}h_t)^{1-\beta}$	$i_{Ht} = A_H\,(s_{Ht}k_t)^{\varepsilon}\,(l_{Ht}h_t)^{1-\varepsilon}$	$d_t/c_t = A_F(s_{Ft}k_t/c_t)^{\gamma_1}(l_{Ft}h_t/c_t)^{\gamma_2}$

$$1 = x_t + l_{Ft} + l_{Gt} + l_{Ht} \tag{2}$$

Similarly, a share of the capital stock is used potentially in each of the three production functions, and the shares must add to one:

$$1 = s_{Gt} + s_{Ht} + s_{Ft} \tag{3}$$

The physical capital investment equation is standard in its assumption of no costs of adding to the capital stock except the actual capital:

$$k_{t+1} = k_t(1-\delta_K) + i_t \tag{4}$$

The human capital investment technology function follows Becker (1975). The equation for motion of the accumulation is $h_{t+1} = h_t(1-\delta_H) + i_{Ht}$. The investment in human capital i_{Ht} requires effective labour and capital whereby

$$h_{t+1} = h_t(1-\delta_H) + A_H\,(s_{Ht}k_t)^{\varepsilon}\,(l_{Ht}h_t)^{1-\varepsilon} \tag{5}$$

The consumer receives income from the human capital augmented labour for goods production and from the rental of capital to the goods producer; there is also the lump-sum transfer of money V_t from the government that the consumer receives. Given the consumer's endowment of initial money stock M_0, and dividing the income between goods purchases and investment, the equation of motion for the consumer's nominal income, or the income constraint, can be put in terms of the change in the nominal money stock:

$$M_{t+1} - M_t = P_t w_t l_{Gt} h_t + P_t r_t s_{Gt} k_t - P_t c_t - P_t k_{t+1} + P_t k_t(1-\delta_K) + V_t \tag{6}$$

The consumer can buy the consumption good at a price of P_t either using the money (carried over from the end of the last period) or using the credit. The fraction of goods bought with money can vary between zero and one, with $a_t \in [0,1]$, and with $1 - a_t$ being the residual fraction of goods that is bought with credit. A fixed fraction $a_2 \in [0,1]$ of physical capital investment is also bought with money. The rest of the investment is a 'costless' credit good requiring neither money nor credit as is standard in this literature (think of retained earnings). This makes the so-called Clower (1967) constraint

$$M_t = a_t P_t c_t + a_2 P_t i_t, \tag{7}$$

which is that of Stockman (1981) if $a_t = a_2 = 1$.

The consumer's choice of a_t is determined by how much labour the agent decides to spend supplying the alternative to money, this being the credit. Here, the total real credit d_t equals the residual real amount of consumption goods not bought with money, or $d_t \equiv c_t(1 - a_t)$, and is given as

$$d_t = c_t A_F \left(\frac{s_{Ft} k_t}{c_t}\right)^{\gamma_1} \left(\frac{l_{Ft} h_t}{c_t}\right)^{\gamma_2} \tag{8}$$

This exhibits constant returns to scale in its three factors, c_t, $s_{Ft}k_t/c_t$ and $l_{Ft}h_t/c_t$, resulting in an upward sloping marginal cost of credit supply per unit of consumption as long as $\gamma_1 + \gamma_2 < 1$. With $\gamma_1 + \gamma_2 < 1$ the consumption velocity of money, the inverse of a_t, is stationary along the balanced growth path as in the evidence. The Cobb–Douglas case of $\gamma_1 + \gamma_2 = 1$ is problematic because it creates an equilibrium that is not well defined because then both money and credit have a constant marginal cost, and also velocity would not be stationary. Dividing the above equation by c_t and using the definition $d_t \equiv c_t(1 - a_t)$, the share of credit in purchases $(1 - a_t)$ can be written as

$$(1 - a_t) = A_F \left(\frac{s_{Ft} k_t}{c_t}\right)^{\gamma_1} \left(\frac{l_{Ft} h_t}{c_t}\right)^{\gamma_2} \tag{9}$$

In this specification, the effective labour and capital inputs are proportional to total consumption, so that the share of credit use remains constant when consumption is growing only if the effective labour and capital inputs grow at the same rate.

A combined exchange constraint for money and credit, which are perfect substitutes, results by solving for a_t from (9) and substituting this into (7):

$$M_t = P_t \left[c_t - A_F \left(\frac{s_{Ft} k_t}{c_t}\right)^{\gamma_1} \left(\frac{l_{Ft} h_t}{c_t}\right)^{\gamma_2} c_t + a_2 i_t \right] \tag{10}$$

This results in the standard (Lucas, 1980) 'cash-only' Clower (1967) constraint when $a_2 = 0$ and $A_F = 0$, so that credit is prohibitively costly to produce.

The goods producer maximizes profit subject to the CRS (constant returns to scale) production technology, with the following first-order conditions, and zero profit in equilibrium:

$$w_t = (1 - \beta) A_G (s_{Gt} k_t)^{\beta} (l_{Gt} h_t)^{-\beta}; \tag{11}$$

$$r_t = \beta A_G (s_{Gt} k_t)^{\beta - 1} (l_{Gt} h_t)^{1 - \beta} \tag{12}$$

The government supplies nominal money through the lump-sum transfer V_t at a steady rate σ, whereby

$$M_{t+1} = M_t + V_t \equiv M_t(1+\sigma) \tag{13}$$

This money supply process is used without alteration in all models of the paper.

With social resources being that output is divided between consumption and investment, the social resource constraint can be found to be

$$y_t = c_t + i_t \tag{14}$$

This resource constraint holds for all of the calibrated models below.

The consumer maximizes the preference-discounted stream of utility in (1) subject to the constraints (5), (6) and (10), with respect to c_t, x_t, M_{t+1}, k_{t+1}, h_{t+1}, s_{Gt}, l_{Gt}, s_{Ft} and l_{Ft}. The first-order conditions are presented in Appendix. The stationary variables on the balanced growth path are the shares l_G, l_H, l_F, x, s_G, s_H and s_F, while the variables that grow at the rate g are c_t, $m_t = M_t/P_t$, k_t, h_t, i_t, i_{Ht}. Equilibrium can be characterized by the marginal rate of substitution between goods and leisure, the balanced-path growth rate, the equivalence between the returns on physical and human capital, and the marginal condition between credit and money use, made known by Baumol (1952).

The goods–leisure marginal rate of substitution is

$$\frac{\alpha x}{c_t} = \frac{1 + aR + w\left(\frac{l_F h}{c}\right) + r\left(\frac{s_F k}{c}\right)}{wh_t} = \frac{1 + aR + (1-a)(\gamma_1 + \gamma_2)R}{wh_t} \tag{15}$$

This rate equals the ratio of the shadow price of goods to that of leisure. The goods shadow cost is one plus the shadow cost of exchange, $aR + w(l_F h/c) + r(s_F k/c)$ with $w(l_F h/c) + r(s_F k/c)$ being a real resource cost of inflation avoidance through credit activity. Or the shadow cost can be written equivalently as the weighted average of cash and credit $aR + (1-a)(\gamma_1 + \gamma_2)R$.

The balanced-path growth rate can be expressed by

$$1 + g = \frac{1 + \frac{r}{1 + a_2 R} - \delta_K}{1 + \rho} = \frac{1 + (1-\varepsilon)A_H\left(\frac{l_H h}{s_H k}\right)^{-\varepsilon}(1-x) - \delta_H}{1+\rho} \tag{16}$$

The growth rate is decreased because of the a_2 factor if $R > 0$, where the need to use money to buy investment goods acts as a tax, as in Stockman (1981). And here, given that $\delta_K = \delta_H$, the return on physical capital $r/(1 + a_2 R)$ is equal to the return on human capital $(1-\varepsilon)A_H(l_H h/s_H k)^{-\varepsilon}(1-x)$. An increase in leisure works directly to bring down the human capital return.

The linkage between R, σ and π along the BGP (balanced growth path) in all of the models is first through the Fisher equation,

$$1 + R \equiv (1+\pi)\left(1 + \frac{r}{(1 + a_2 R)} - \delta_K\right), \tag{17}$$

that can be derived by introducing government bonds.[3] Second, using the Fisher equation plus Clower (1967) constraint (7) and the growth rate (16), the nominal interest rate and money growth rate are related by

$$1 + R = (1 + \sigma)(1 + \rho) \tag{18}$$

Changes in the nominal interest rate are directly caused by changes in the money-supply growth rate. And in response to an increase in σ, and in R, it is important to realize that the gross real interest rate r falls, even as π rises. The change in r is emphasized throughout the paper as part of the Tobin (1965) effect and is simultaneous with an increasing capital-to-effective-labour ratio across sectors as a result of a higher σ.

The factor input ratios in the goods and human capital sectors are given by

$$\frac{r}{w} = \left(\frac{\beta}{1-\beta}\right)\left(\frac{l_G h_t}{s_G k_t}\right) = \left(\frac{\varepsilon}{1-\varepsilon}\right)\left(\frac{l_H h_t}{s_H k_t}\right) \tag{19}$$

The credit sector input equilibrium is determined by the Baumol-type (1952) conditions:

$$R = \frac{w}{\gamma_2 A_F \left(\frac{l_F h_t}{c_t}\right)^{\gamma_2 - 1}\left(\frac{s_F k_t}{c_t}\right)^{\gamma_1}}; \tag{20}$$

$$R = \frac{r}{\gamma_1 A_F \left(\frac{l_F h_t}{c_t}\right)^{\gamma_2}\left(\frac{s_F k_t}{c_t}\right)^{\gamma_1 - 1}} \tag{21}$$

Each of the above two exchange conditions set the marginal cost of money, R, equal to the marginal factor cost divided by the marginal factor product in producing credit. With this general equilibrium setting for the Baumol (1952) condition, combined with the existence of an explicit credit sector, these conditions are nothing more than a standard microeconomic sectoral condition whereby the marginal cost of output equals the factor price divided by its marginal product. This implies, in equalizing the marginal costs of different exchange means as in the original Baumol (1952) model, that the marginal cost of credit is the nominal interest rate; it is verified with a decentralized formalization for an explicit credit sector, and an explicit price of the credit service, that the price of credit is the nominal interest rate in Gillman and Kejak (2004) and Gillman (2000).

The Baumol (1952) conditions determine the equilibrium demand for money and its interest elasticity. For example, when only money is used, such as in the standard Lucas (1980) cash-in-advance model, then $m = c$, and the interest elasticity of money demand is simply the interest elasticity of consumption. Gillman (1993) shows in a related economy that this type of model gives a very low magnitude of the interest elasticity of money, while when credit is produced to avoid the inflation, the interest elasticity rises in magnitude by several-fold. Gillman and Kejak (2002, 2005) show that the higher is the interest elasticity in magnitude the lower is the inflation–growth effect in magnitude, along with the Tobin-type (1965) effects. And a substantially rising interest elasticity as inflation increases produces a highly non-linear inflation–growth profile as is similar to evidence.

3. Physical Capital Only Models

3.1 *Ireland (1994)*

Ireland (1994) uses only physical capital in an aggregate production function with a constant marginal product of capital: the Ak model. In addition, the consumer avoids inflation through a sector that provides credit for buying goods instead of using money. The credit is produced using only goods and is used across a continuum of stores, selling a continuum of goods, with a different monotonically changing cost of credit at each store. As the inflation rate goes up, credit is used at more stores, with each marginally added store having a somewhat higher cost of producing the credit. This has the effect in aggregate of establishing a rising marginal cost of credit as more credit is used to avoid inflation.

Such a continuum of stores, with a continuum of goods and each with a different cost of credit that can be used to buy the goods, is also found in Gillman (1993), Gillman (2000), Aiyagari *et al.* (1998) and Erosa and Ventura (2000). The credit supply in these models is therefore very similar except that they use time, rather than goods or capital as in Ireland (1994), to provide the credit. To illustrate Ireland's (1994) model in a way compatible with the standard neoclassical growth and business-cycle paradigm, consider using a single aggregate consumption good as in the section 2 model. Here, the production function for credit explicitly has an increasing marginal cost, rather than this resulting in aggregate from a continuum of stores. And because goods are costlessly convertible into capital in the Ireland (1994) economy, here it can be assumed that capital is used (rather than goods) in the production of the credit.

Assume the following special case of the section 2 economy. Let there be a zero, instead of one, time endowment. Set the utility value of leisure to zero; $\alpha = 0$. Assume there is no human capital investment, including that $h_0 = 0$ and $A_H = 0$. Also with only physical capital being used, $\beta = 1$, goods output is CRS, and this gives the Ak function. Also, here the money is used only for consumption goods, so that $a_2 = 0$. For the credit production, let $\gamma_2 = 0$ so that there is only capital used with diminishing returns and an increasing marginal cost (Table 2).

Table 2. Assumptions for Special Case: Ireland (1994) Economy.

Parameters	Production Functions
$\alpha = h_0 = A_H = \gamma_2 = a_2 = 0$; $\beta = 1$	$y_t = A_G s_{Gt} k_t$; $d_t = A_F \left(\frac{s_{Ft} k_t}{c_t}\right)^{\gamma_1} c_t$

The credit production function then is given by $(1-a_t)c_t = A_F(s_{Ft}k_t/c_t)^{\gamma_1}c_t$, and the shares of capital add to one, $s_{Gt}+s_{Ft}=1$. The Clower (1967) constraint (7) with $a_2=0$ is $M_t=a_tP_tc_t$. The Clower (1967) constraint can be combined with the credit production function to make the combined exchange constraint (10) now as given by $M_t = P_tc_t - P_tA_F(s_{Ft}k_t)^{\gamma_1}c_t^{1-\gamma_1}$. In the above equation, when $s_{Ft}k_t=0$ so that no credit is produced, the standard 'cash-only' Clower (1967) constraint results in which $a_t=1$. Note also that the amount of resources that the consumer willingly uses in credit production to avoid inflation is the capital $s_{Ft}k_t$. The rental value of this capital is the amount that corresponds precisely to Lucas (2000) measure of the welfare cost of inflation. However, where that cost was the value of the shopping time spent, here the welfare cost is the rental value of the capital used in credit production. Solving for credit capital $s_{Ft}k_t$ from the last equation gives $s_{Ft}k_{Ft}=[(1-(m_t/c_t))/A_F]^{1/\gamma_1}c_t$. Analogous to the shopping time model, the credit capital falls with increases in m_t and with decreases in c_t.

With $\alpha=h_0=A_H=\gamma_2=a_2=0$ and $\beta=1$, the consumer maximizes the preference-discounted stream of utility in (1) subject to the constraints (6) and (10), with respect to c_t, M_{t+1}, k_{t+1} and s_{Gt}. The real interest rate from the firm problem is $r=A_G$. The balanced-growth rate is constant, as given by $1+g=(1+A_G-\delta_K)/(1+\rho)$.

The shadow price of goods is $1+R-R(1-\gamma_1)A_F(s_Fk_t/c_t)^{\gamma_1}$, showing that credit use decreases the shadow exchange cost of goods below R as it would be with only money. The single Baumol (1952) condition, comparable to (21), is $R=A_G/[\gamma_1A_F(s_{Ft}k_t/c_t)^{\gamma_1-1}]$. This condition implies that capital in credit production, relative to consumption, rises as the nominal interest rate rises. This gives the diversion of capital from goods production when the inflation rate rises, one of Ireland's (1994) main results.[4]

The 'great ratios' can be found to equal as $c_t/y_t=(\rho/A_G)(1+A_G-\delta_K)/\left[(1+\rho)+[A_G-\rho(1-\delta_K)](R\gamma_1A_F/A_G)^{1/(1-\gamma_1)}\right]$, $i_t/y_t=1-c_t/y_t$, $c_t/k_t=[\rho/(1+\rho)](1+A_G-\delta_K)/[1+A_G(R\gamma_1A_F/A_G)^{1/(1-\gamma_1)}]$ and $y_t/k_t=c_t/k_t+(A_G-\rho(1-\delta_K))/(1+\rho)$. When the nominal interest rate R rises, c_t/y_t, c_t/k_t and y_t/k_t fall, while i_t/y_t rises, similar to Tobin (1965). However, the real interest rate r is constant while the Gillman and Nakov (2003) evidence indicates that r falls with increases in inflation in the long run.

The solution for the share of money usage is $a=1-[(R\gamma_1/A_G)^{\gamma_1/(1-\gamma_1)}A_F^{1/(1-\gamma_1)}]$. Because both a and c_t/k_t fall with an increase in R, it can be seen that the money-to-capital ratio also falls with an increase in the nominal interest rate, in that $m_t/k_t=ac_t/k_t$. This indicates substitution from real money to capital when inflation rises, again as in Tobin (1965).

Ireland (1994) demonstrates how the increase in capital coming from the diversion of capital into banking decreases the growth rate along the transition, but not in the stationary state at the limiting end of the transition. In the model above also there is no long-run growth effect of inflation, contrary to evidence.

However, in this model and in Ireland's (1994) are some of the empirically supported Tobin (1965) effects.

3.2 *Stockman/Dotsey and Sarte*

A special case of the Stockman (1981) economy, also used in Dotsey and Sarte (2000), results by assuming only a goods sector, with no human capital and no credit production, and by assuming a constant marginal product of capital, whereby $y = A_G k_G$. This is the same Ak production function as in the last section except that now $s_G = 1$. With only money used in exchange, $a_t = 1$. Also as in Stockman (1981), let investment be purchased with money as well as goods, so that $a_t = a_2 = 1$. By using the Ak function, this puts the Stockman model in an endogenous growth setting.

The assumptions are summarized in Table 3.

With $\alpha = h_0 = A_H = A_F = 0$ and $a_t = a_2 = s_G = \beta = 1$, the consumer maximizes the preference-discounted stream of utility in (1) subject to the constraints (6) and (10), with respect to c_t, M_{t+1} and k_{t+1}. The balanced-path solution has a growth rate of $1 + g = [1 + (A_G/(1 + R)) - \delta_K]/(1 + \rho)$, so that an increase in the nominal interest rate lowers the growth rate. The after-inflation-tax marginal product of capital $A_G/(1 + R)$ falls because investment must be purchased with money. The shadow price of goods is $1 + R$, and the rest of the solution is $c_t/y_t = 1 - ((g + \delta_K)/A_G)$, $i_t/y_t = (g + \delta_K)/A_G$ and $m_t/k_t = A_G$.

An increase in the nominal interest rate causes the growth rate to fall and c/y to rise and i/y to fall, which as Stockman (1981) noted is similar to an 'inverse' Tobin (1965) effect; r is constant. Furthermore, with m/k constant, there is no substitution between money and capital as in Ireland's (1994) and Tobin's (1965) model. In fact, there is no possibility to avoid the inflation tax as there is only one good produced, one means of exchange and one input to utility.

However, the calibration along the balanced-growth path, given in Table 3 and graphed in Figure 1, shows that a significant negative growth effect can result robustly in this model. Note that Figure 1, as well as the subsequent Figure 2, graphs the nominal interest rate against the growth rate, rather than the inflation

Table 3. Assumptions for Special Case: Stockman (1981) Economy.

Assumptions		Production Functions
$\alpha = h_0 = A_H = A_F = 0$; $a_t = a_2 = s_G = \beta = 1$		$y = A_G k_G$
	Calibration	
Parameters		Variables
$\rho = \delta_k = 0.03,\ A_G = 0.0909$		$R = 0,\ g = 0.03,\ r = 0.0909$
Δ Nominal interest rate	Δ Growth rate	Δ Real interest rate
0.00→0.10	−0.0080	0
0.10→0.20	−0.0067	0

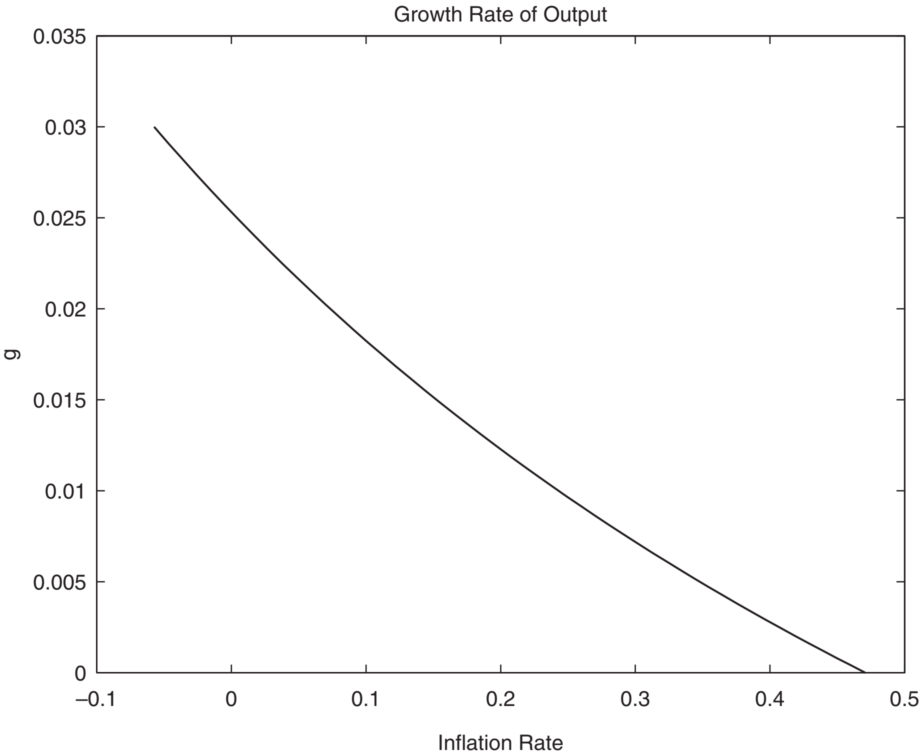

Figure 1. Model 3.2 Inflation and Growth Calibration.

rate, because the analytic solution for R vs. g is simple while that solution for π and g is quite complex.

The inflation–growth effect, of -0.67% in Table 3, falls within the Chari *et al.* (1996) range. But over the whole range of inflation rates, there is only a marginal non-linearity resulting in a counter-empirical negative growth rate as inflation gets above 50%.

4. Human Capital Only Models

Two models using only human capital are reviewed here. Both use a linear production of goods using only human capital-indexed labour. The difference is in the nature of the human capital investment function. This function can be 'costless' in the sense that a certain amount of output can be costlessly transformed into human capital; this is the analogue to the standard physical capital investment accumulation (4). Or the human capital investment can be 'costly', as in Becker (1975) and Lucas (1988), whereby labour time and possibly physical capital inputs with diminishing returns to each input are transformed into human

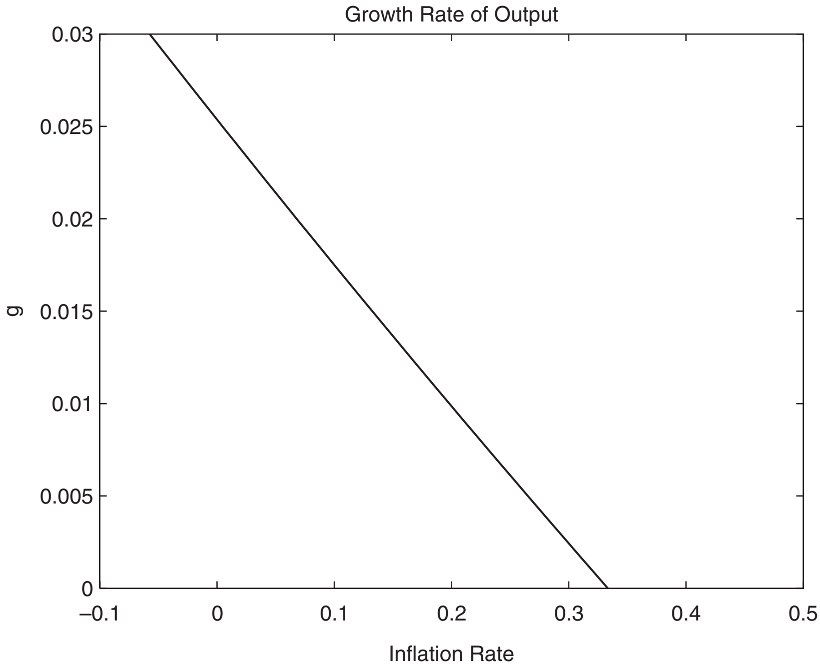

Figure 2. Model 4.1 Inflation and Growth Calibration.

capital; King and Rebelo (1990) describe this as the analogue of costly physical capital investment, such as the 'adjustment cost' in Lucas (1967).

Table 4 summarizes the specification of costly human capital model 4.1. And here, define the gross marginal product of capital as $\widetilde{r} \equiv A_H(1-x)$. With $\alpha = k_0 = A_F = a_2 = \beta = \varepsilon = 0$, the consumer maximizes the preference-discounted stream of utility in (1) subject to (5), (6) and (10), with respect to c_t, x_t, M_{t+1}, h_{t+1} and l_{Gt}. The real wage rate from the firm problem is given by $w = A_G$, while the growth rate is expressed as $1+g = (1 + A_H(1-x) - \delta_H)/(1+\rho)$. The negative effect of inflation comes through its induced decrease in leisure time and the resulting change in the marginal product of human capital. The marginal rate of substitution between goods and leisure is $x/(\alpha c_t) = (1+R)/(A_G h_t)$, and the closed-form solution of the economy is $c_t/h_t = \rho A_G(1 + A_H - \delta_H)/[A_H(1+\rho[1+\alpha(1+R)])]$, $x = \rho\alpha(1+R)(1+A_H - \delta_H)/[A_H(1+\rho[1+\alpha(1+R)])]$ and $1+g = (1+A_H-\delta_H)/\,[1+\rho[1+\alpha(1+R)]]$.

Table 4 summarizes a significant negative inflation–growth effect, of 0.81%. Instead of the baseline value of $\alpha = 3$ and $A_H = 0.1836$, this growth effect becomes smaller in magnitude, 0.56% when α is set equal to 2, and A_H recalibrated to 0.1527. This shows sensitivity but still robustness in generating a large magnitude. But as Figure 2 shows that the inflation–growth profile is almost linear, and the

Table 4. Assumptions for Special Case: 4.1 Economy.

Parameters		Production Functions
$\alpha = k_0 = A_F = a_2 = \beta = \varepsilon = 0$; $a_t = 1$		$y_t = A_G l_{Gt} h_t$; $i_{Ht} = A_H(1 - x_t - l_{Gt})h_t$
	I. Baseline calibration	
Parameters		Variables
$\rho = \delta_H = 0.03$, $\alpha = 3$, $A_G = 0.1836, A_H = 0.1836$		$R = 0, g = 0.03, \widetilde{r} = 0.0909$
	II. Calibration	
$\alpha = 2, A_G = 0.1836, A_H = 0.1527$		$R = 0, g = 0.03, \widetilde{r} = 0.0909$
	I	II
Δ Nominal interest rate	Δ Growth rate	Δ Growth rate
0.00→0.10	−0.0082	−0.0056
0.10→0.20	−0.0081	−0.0056

growth rate becomes negative at a relatively low inflation rate, while in the long-run evidence, the growth rate stays positive for all inflation rates.

Note how this compares to a similar but non-nested model 4.2. Continue to assume that $\beta = \varepsilon = 0$ and that $A_F = 0$, so that only human capital is used in production and there is no credit available. But now assume also that $A_H = 0$, so that human capital investment does not take place through a production process. Instead, in a slight deviation from the section 2 model, assume that goods output can be costlessly turned into human capital, as compared to assuming that goods output can be turned into physical capital in the section 2 model. With $\widetilde{i}_{Ht}$ denoting these goods that become human capital, the social resource constraint becomes $y_t = c_t + \widetilde{i}_{Ht}$. Table 5 summarizes the specification.

The human capital accumulation equation is $h_{t+1} = h_t(1 - \delta_H) + \widetilde{i}_{Ht}$. This accumulation equation is the approach taken in Chari *et al.* (1996), although there, physical capital also is used. With no time in human capital accumulation, the time constraint simplifies even further from that of the last subsection to $1 = x_t + l_{Gt}$.

Table 5. Assumptions for Special Case: 4.2 Economy.

Assumptions		Production Functions
$A_F = A_H = a_2 = \beta = 0$; $a_t = 1$		$y_t = A_G l_G h_t$
	Calibration	
Parameters		Variables
$\rho = \delta_H = 0.03,\ A_G = 0.1836$		$R = 0,\ g = 0.03,\ r = 0.0909$

With $A_F = A_H = a_2 = \beta = 0$, the consumer maximizes the preference-discounted stream of utility in (1) subject to (5), (6) and (10), with respect to c_t, x_t, M_{t+1}, h_{t+1} and $\widetilde{i}_{Ht}$. The first-order conditions imply that the solution for the growth rate in terms of leisure is $1 + g = (1 + A_G(1 - x) - \delta_H)/(1 + \rho)$, where $x = \alpha\rho(1 + R)(1 + A_G - \delta_H)/(A_G(1 + \rho[1 + \alpha(1 + R)]))$ and $c_t/h_t = \rho[1 + A_G - \delta_H)/(1 + \rho[1 + \alpha](1 + R)])$. This makes the growth rate equal $1 + g = (1 + A_G - \delta_H)/(1 + \rho[1 + \alpha(1 + R)])$. The growth rate is identical to the previous model of section 4.1 if $A_G = A_H$, with the same calibration.

The singularity of the two models can also be viewed as implying that both models have two sectors, although the latter model has a simple technology for the human capital sector that is usually viewed as being a one-sector model only in goods production. And it means that the non-nested model here can be made equivalent to the nested model of section 4.1.

5. Models with Physical and Human Capital

In a model with both physical and human capital, a standard Clower (1967) constraint, and with human capital as the source of endogenous growth, the inflation effect on growth depends on the nature of the human capital investment function. The differences are shown by examining a model with a simple accumulation equation (Chari *et al.*, 1996) vs. one with a Becker (1965)–King and Rebelo (1990) human capital investment function.

5.1 *Simple Human Capital Accumulation*

The simple human capital accumulation equation, in which say $\widetilde{i}_{ht} = h_{t+1} - h_t(1 - \delta_H)$, sidesteps the traditional literature on human capital in which time is involved in human capital accumulation (Schultz, 1964; Becker, 1975). An approach sympathetic with this simple accumulation equation, but still fully nested within the Becker (1975) human capital investment function, is to assume that the production function for the human capital investment uses only

Table 6. Calibration for Section 5.1 Economy.

Assumptions			Production Functions
$A_F = a_2 = 0$; $a_t = A_H = \varepsilon = 1$			$y_t = A_G(s_{Gt}k_t)^{\beta}(l_{Gt}h_t)^{1-\beta}$; $i_{Ht} = s_{Ht}k_t$
	Baseline calibration		
Parameters			Variables
$\rho = \delta_H = \delta_K = 0.03$, $\beta = 0.4$, $A_G = 0.0877$			$R = 0, g = 0.03, r = 0.0909$; $s_Gk/l_Gh = 0.0184$
Δ Nominal interest rate	Δ Growth rate	Δ Real interest rate	Δ Capital–labour ratio
0.00→0.10	−0.00281	−0.00298	0.01167
0.00→0.20	−0.00258	−0.00273	0.01167

capital and no labour. Here, instead of assuming that $\widetilde{i}_{ht}$ is transformed goods output, assume instead that the Becker (1975) human capital function has the form of (5) but assumes that $\varepsilon = 0$ and that A_H equals 1, so that only physical capital is used to produce the human capital. Because goods output can be costlessly transformed into physical capital in these models, the use of physical capital instead of goods output allows for a nesting of this modified simple accumulation equation.[5] Table 6 provides the specification details.

Comparing the two definitions, of $\widetilde{i}_{Ht}$ from the last section 4 and i_{Ht} in this section, it could be stated that $\widetilde{i}_{Ht} = s_{Ht}k_t$, just as physical capital instead of goods are used in the section 3.1 model that compares to Ireland (1994). The human capital accumulation process now becomes $h_{t+1} = h_t(1 - \delta_H) + s_{Ht}k_t$ and the accounting of the shares of human and physical capital are now $1 = s_{Gt} + s_{Ht}$ and $1 = x_t + l_{Gt}$. The resource constraint is the same as in section 2, in (4) and (14).

With $A_F = a_2 = 0$ and $a_t = A_H = \varepsilon = 1$, the consumer maximizes the preference-discounted stream of utility in (1) subject to (5), (6) and (10), with respect to c_t, x_t, M_{t+1}, k_{t+1}, h_{t+1} and s_{Gt}. The growth rate is given by

$$1 + g = \frac{1 + r - \delta_K}{1 + \rho} = \frac{1 + \frac{w}{r}(1 - x) - \delta_H}{1 + \rho}, \qquad (22)$$

whereby balanced growth implies an equivalence of the marginal products of physical capital and human capital.

Solving the economy numerically, the baseline calibration of the change in the growth rate is given in Table 6. The growth rate decreases are significant and within the range of empirical estimates. A problem, however, is that with human capital so productive relative to goods production, in that $A_H = 1$ and $A_G = 0.0877$, most of the capital is directed to human capital. The capital-to-effective-labour ratio at the optimum of $R = 0$ is only $(s_G k / l_G H) = 0.0184$. This is not a very plausible ratio and represents an indication of the problem with the general specification. However, qualitatively, the calibration shows the positive Tobin (1965) effect of a rising capital-to-effective-labour ratio as the nominal interest rate rises, while the growth rate falls.

Figure 3 graphs the inflation–growth profile over a range of inflation rates. The line representing the model is the dot-dash one. It shows a marginal degree of non-linearity that tends to be much less than found empirically. Although there is no exact empirical measure for the degree of non-linearity, evidence indicates that the growth rate never becomes negative, while in the model here, it does become negative.

The model with the alternate assumption of using goods in the simple human capital accumulation equation was also calibrated, but is not shown here as it is not nested. The results for the growth rate are quite similar, and hence, in this respect, the models compare closely. In both, the model of this section and the alternate, the effect of inflation on growth is about half the magnitude of that effect when the human capital function also includes time, as in the next section.[6]

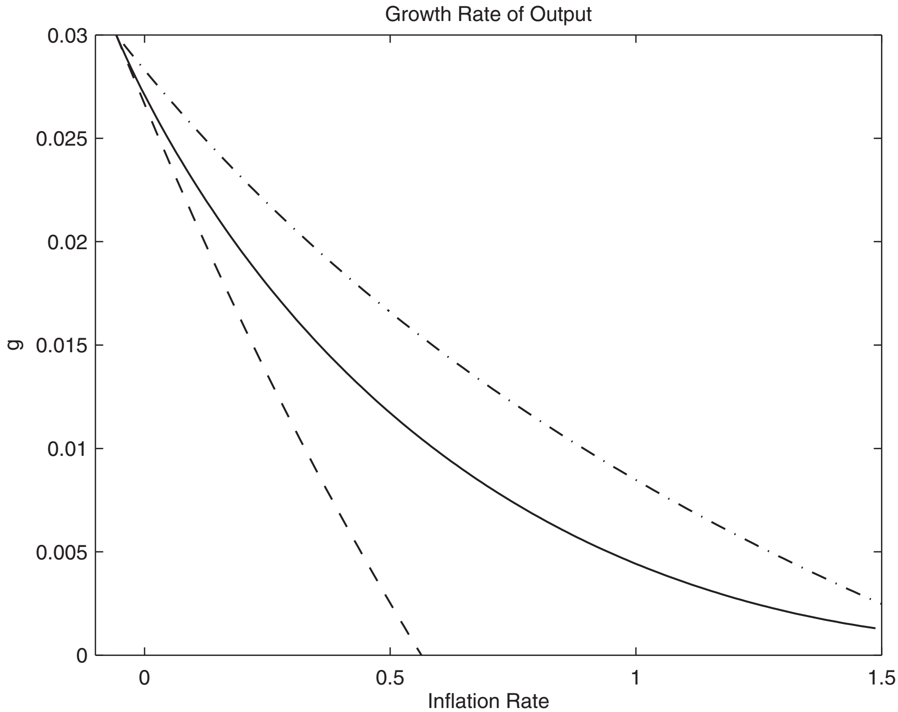

Figure 3. Calibration of Inflation and Growth: Section 5 Models.

5.2 *Becker–Lucas Model*

Using a more general Becker (1975) function as in the section 2 model relaxes the constraint on A_H and ε, as in Gillman and Kejak (2002). The human capital investment function uses both effective labour and capital. Table 7 summarizes the specification.

With $A_F = a_2 = 0$ and $a_t = 1$, the consumer maximizes the preference-discounted stream of utility in (1) subject to (5), (6) and (10), with respect to c_t, x_t, M_{t+1}, k_{t+1}, h_{t+1}, s_{Gt} and l_{Gt}. The balanced-path growth rate can be expressed as $1 + g = (1+\beta A_G(l_G h/s_G k)^{1-\beta} - \delta_K)/(1+\rho) = (1+(1-x)(1-\varepsilon)A_H(s_H k/l_H h)^{\varepsilon} - \delta_H)/(1+\rho)$.

Solving for this system numerically, the baseline calibration is given in Table 7. The general nature of the human capital investment function makes the magnitude about double of the previous model in section 5.1. The only significant problem here in matching the evidence is the marginal degree of non-linearity in the inflation–growth effect. The inflation–growth profile is graphed in Figure 3 as the dashed line. It is nearly linear and indicates a negative growth rate as inflation increases contrary to evidence.

5.3 *Becker–Lucas Model with a Credit Sector*

Finally, consider adding a credit sector to the model of section 5.2. This adds one more margin to the consumer and yields considerable flexibility to avoid the inflation

Table 7. Calibration for Section 5.2 Economy.

Assumptions			Production Functions	
$A_F = a_2 = 0$ $a_t = 1$			$y_t = A_G(s_{Gt}k_t)^{\beta}(l_{Gt}h_t)^{1-\beta}$ $i_{Ht} = A_H(s_{Ht}k_t)^{\varepsilon}(l_{Ht}h_t)^{1-\varepsilon}$	
Parameters			Variables	
		I. Baseline calibration		
$\rho = \delta_H = \delta_K = 0.03, \beta = 0.4, \varepsilon = 0.3, \alpha = 3,$ $A_G = 0.3110, A_H = 0.2609$			$R = 0, g = 0.03, r = 0.0909$	
		II. Calibration		
$\varepsilon = 0.4, A_G = 0.3110, A_H = 0.3318$			$R = 0, g = 0.03, r = 0.0909$	
		III. Calibration		
$\alpha = 2, A_G = 0.3110, A_H = 0.2609$			$R = 0, g = 0.03, r = 0.0909$	
		I	II. $\varepsilon = 0.4$	III. $\alpha = 2$
Δ Nominal interest rate	Δ Growth rate	Δ Real interest rate	Δ Growth rate	Δ Growth rate
0.00→0.10	−0.00572	−0.00607	−0.00492	−0.00439
0.00→0.20	−0.00538	−0.00570	−0.00459	−0.00420

tax. This gives the model the ability to explain not only a significant inflation–growth effect, and Tobin (1965) effects, but also the non-linearity of the inflation–growth effect. In addition, while not detailed here (Gillman *et al.*, 2003), it can show differences in the inflation–growth effect across regions as based on financial development. Here, a credit sector is added using only effective labour, in contrast to the capital-only model of section 3.1. The assumptions are given in Table 8.

Table 8. Calibration for Section 5.3 Economy.

Assumptions		Production Functions	
$a_2 = \gamma_1 = 0;$		$y_t = A_G(s_{Gt}k_t)^{\beta}(l_{Gt}h_t)^{1-\beta}$ $i_{Ht} = A_H(s_{Ht}k_t)^{\varepsilon}(l_{Ht}h_t)^{1-\varepsilon}$ $d_t = (1-a_t) = A_F(l_{Ft}h_t/c_t)^{\gamma_2}c_t$	
	Baseline calibration		
Parameters $\rho = \delta_H = \delta_K = 0.03, \beta = 0.4$ $\varepsilon = 0.3, \alpha = 3, \gamma_2 = 0.3, A_F = 0.5184$ $A_G = 0.3110, A_H = 0.3279,$		Variables $R = 0, g = 0.03, r = 0.0909$ $s_Gk/l_Gh = 1.6871$	
Δ Nominal interest rate	Δ Growth rate	Δ Real interest rate	Δ Capital–labour ratio
0.00→0.10	−0.00472	−0.00500	0.16696
0.00→0.20	−0.00381	−0.00401	0.15398

With $a_2 = \gamma_1 = 0$, the consumer maximizes the preference-discounted stream of utility in (1) subject to (5), (6) and (10), with respect to c_t, x_t, M_{t+1}, k_{t+1}, h_{t+1}, s_{Gt}, l_{Gt} and l_{Ft}. The added first-order condition is the Baumol (1952) equation $R = w/\left[\gamma_2 A_F(l_F h_t/c_t)^{\gamma_2 - 1}\right]$.

The calibration results are reported in Table 8, and details of similar calibrations can be found in Gillman and Kejak (2002, 2005). Figure 3 graphs the inflation–growth profile in the solid line. This conforms roughly to evidence on the shape of the non-linearity (Gillman *et al.*, 2003), in which the growth rate does not become negative as the inflation rate increases.

6. Comparison of Models

Table 9 summarizes the findings across the different models. The table summarizes, in its second column, that all models but the first have inflation–growth effects close to the Chari *et al.* (1996) range of −0.2 to −0.7% for a change in the inflation rate from 10 to 20% (although here we report the results for similar changes in R). While some of the models have calibrations that are a bit high, none are too low. This establishes clearly a robust significant negative inflation–growth effect across a range of models. Distinguishing further among the models requires use of the third and fourth columns, on non-linearity and Tobin-type (1965) effects. With growing evidence of a strong non-linearity, whereby the inflation–growth effect is marginally weaker as higher levels of the inflation rate, and on positive Tobin-type (1965) effects, only two models, 5.1 and 5.3, meet all criterion. The model of section 5.1, however, does not provide a sense of plausibility, in that the parameter assumption of $A_H = 1$ leads to a nearly insignificant capital-to-effective-labour ratio. Also, its non-linearity is only marginal and generates negative levels of the growth rate. The model of section 5.3 has no such plausibility problems and has a strong non-linearity without negative levels of the growth rate.

Only the last model also is jointly consistent with the Aiyagari *et al.* (1998) money and banking findings that the banking sector expands in size in conjunction with the level of the inflation rate. Furthermore, Gillman and Kejak (2002, 2005) show that this section 5.3 model yields a money demand closely comparable

Table 9. Summary of Growth and Tobin Effects.

Models of Section	BGP Inflation–Growth Decrease R: 0→0.10	BGP Inflation–Growth Decrease R: 0.10→0.20	Non-Linearity	Tobin Effect
3.1	0	0	NA	Positive
3.2	−0.0080	−0.0067	Marginal	Inverse
4.1	−0.0082	−0.0081	Near linear	None
5.1	−0.0028	−0.0026	Marginal	Positive
5.2	−0.0057	−0.0054	Near linear	Positive
5.3	−0.0047	−0.0038	High	Positive

to a Cagan-type (1956) constant semi-interest elasticity model for which Mark and Sul (2002) find recent broad-based cointegration support.

7. Conclusions

The paper presents a general monetary endogenous growth model with both human and physical capital, and then categorizes a set of models as being nested within this model. The first subset of models considered are *Ak* models in which inflation acts as a tax on physical capital with a negative long-run Tobin-type (1965) effect. Next presented are *Ah* models in which inflation acts as a tax on human capital and there is a positive Tobin (1965) effect. Then come the more general models with human and physical capital, in which inflation acts more as a tax on human capital and there is a positive Tobin (1965) effect.

While there is no unemployment *per se* in any of these three classes of models, the employment rate moves in the opposite direction of the inflation rate in the models with human capital. This direction and causality of the employment effect is not inconsistent with evidence in Shadman-Mehta (2001). They find cointegration of inflation and unemployment for historical UK data, including Phillips original sample period, and that inflation Granger causes unemployment in the long run.

The reviewed models show a strong linkage between the magnitude of the inflation–growth and the Tobin-type (1965) effects, and between the non-linearity of both of these effects. The growth rate decrease, when the inflation rate rises from 0 to 10% and then to 20%, is proportional in its strength of magnitude and its degree of non-linearity directly to the real interest rate decrease and the capital-to-effective-labour rate increase. This linkage is a general characteristic across models that act as a key distinguishing feature. If the non-linearity is in fact significant, as evidence suggests, then models without this overstate significantly the inflation effects for rates of inflation above the baseline level. The explanation of this non-linearity comes back to the money demand elasticity that underlies the model. A rising interest elasticity, with inflation rising, leads to easier substitution away from inflation and causes the non-linearity. A near-constant interest elasticity money demand, as in the standard cash-in-advance model, leads to a near-linear response.

Debate on the monetary growth models and on the existence within these models of Tobin-type (1965) effects on the real interest rate and the Great Ratios, c/y and i/y, goes back to when monetary growth models used the Solow model as modified by Tobin (1965) to include money as the basis of debates (Johnson, 1969; Niehans, 1969). The advent of endogenous growth theory as ushered in by Lucas (1988) marked a substantial leap in progress that reframed this debate once money was included within these models using the Lucas (1980) approach (Gomme, 1993). The resulting endogeneity of the growth rate relative to changes in the inflation rate, as working through the labour–leisure channel, allowed for calibration of the inflation–growth effect within the estimated empirical range. However, the *Ak* models also have been able to accomplish the same feat, making unclear what approach is more advantageous. Updating the traditional focus on the Great Ratios has allowed for a re-focusing on how these models can be differentiated. The Gomme-type (1993) models capture general equilibrium decreases in the real interest rate and the consumption-to-output ratio, and increases in the investment-to-output ratios, all as a result of inflation and as consistent with evidence. This makes the simpler *Ak* models more dated, in that they cannot so easily, if at all, achieve similar results. A further distinguishing factor, going beyond the magnitude of the inflation–growth effect, and beyond the direction of the Tobin-type (1965) effects, is how these effects behave over the range of inflation rate levels. Evidence shows a strong non-linearity in the inflation–growth effect. And the Lucas (1988)–Gomme (1993) model that is extended to include credit production as a substitute to cash, in a Baumol-type (1952) fashion, can account for this non-linearity by producing an implied interest elasticity of money demand that rises in magnitude with the inflation rate, as in the successful Cagan (1956) model. There are currently few such models that link the evidence in favour of near-constant semi-interest elasticities of money demand, with the non-linearity of the inflation–growth effect, along with a significant negative magnitude of this effect, while also capturing the Tobin-type (1965) effects.

Acknowledgements

We thank Tony Braun, Bee Jeong, Jan Kmenta and seminar participants at CERGE-EI. Max Gillman is grateful for research grants from Central European University. Michael Kejak is grateful for research grants from CERGE-EI (Centre of Economic Research and Graduate Education – Economics Institute) in Prague.

Notes

1. See also Haslag (1998) for *Ak* economies in which money is required as bank reserves; these models have negative inflation–growth effects in a way similar to Stockman's (1981) model.
2. As an exception, Paal and Smith (2000) present an overlapping generations model in which a threshold inflation rate exists.
3. Denote nominal discount bonds that are purchased at time t by B_{t+1}, and their price by q_{t+1}. Then the receipts B_t and costs $-q_{t+1}B_{t+1}$ are added to the income constraint in equation (6), and the derivative with respect to B_{t+1} gives that $(1+R_{t+1}) \equiv (1/q_{t+1}) = (1+g_{t+1})(1+\pi_{t+1})(1+\rho)$. This combined with equation (16) gives the Fisher equation (17).
4. See Otto and Crosby (2000) for some related empirical work.
5. For an alternative view, see Barro and Sala-i-Martin (1995), footnote 13, page 181.
6. Chari *et al.* (1996) use a Lucas and Stokey (1983) cash-good, credit-good, preference function, which cannot be nested in the section 2 model of this paper, and report an insignificant inflation–growth effect. It is possible that if the Lucas and Stokey (1983)

preference parameters are specified so that cash and credit goods are near-perfect substitutes, while at the same time, there is no real resource cost to using the credit, which is true in the Lucas and Stokey (1983) model, then inflation can cause near-perfect substitution to the credit good, with close to zero increase in leisure, resulting in an insignificant growth effect.

References

Ahmed, S. and Rogers, J. H. (2000). Inflation and the great ratios: Long term evidence from the US. *Journal of Monetary Economics* 45(1): 3–36.

Aiyagari, S., Braun, R. and Eckstein, Z. (1998). Transaction services, inflation, and welfare. *Journal of Political Economy* 106(6): 1274–1301.

Barro, R. (2001). Human capital and growth. *American Economic Review* 91(2): 12–17.

Barro, R. and Sala-I-Martin, X. (1995). *Economic Growth*. New York: McGraw-Hill, Inc.

Baumol, W. (1952). The transactions demand for cash: An inventory – theoretic approach. *Quarterly Journal of Economics* 66: 545–566.

Becker, G. S. (1965). *Human Capital*. Chicago: University of Chicago Press.

Becker, G. S. (1975). *Human Capital*, 2nd edn. Chicago: University of Chicago Press.

Cagan, P. (1956). The monetary dynamics of hyperinflation. In M. Friedman (ed.), *Studies in the Quantity Theory of Money*. Chicago: University of Chicago Press, pp. 25–120.

Chari, V., Jones, L. E. and Manuelli, R. E. (1996). Inflation, growth, and financial intermediation. *Federal Reserve Bank of St. Louis Review* 78(3): 41–58.

Clower, R. (1967). A reconsideration of the microfoundations of monetary theory. *Western Economic Journal* 6(1): 1–9.

Crowder, W. J. (1998). The long-run link between money growth and inflation. *Economic Inquiry* 36(2): 229–243.

Cziraky, D. and Gillman, M. (2003). Stable money demand and nominal money causality of output growth: A multivariate cointegration analysis of Croatia (manuscript).

Dotsey, M. and Sarte, P. D. (2000). Inflation uncertainty and growth in a cash-in-advance economy. *Journal of Monetary Economics* 45(3): 631–655.

Erosa, M. and Ventura, M. (2000). On inflation as a regressive consumption tax. University of Western Ontario Department of Economics Working Papers 2000/1.

Ghosh, A. and Phillips, S. (1998). Inflation, disinflation and growth. IMF Working Paper WP/98/68.

Gillman, M. (1993). Welfare cost of inflation in a cash-in-advance economy with costly credit. *Journal of Monetary Economics* 31: 22–42.

Gillman, M. (2000). On the optimality of restricting credit: Inflation-avoidance and productivity. *Japanese Economic Review* 51(3): 375–390.

Gillman, M., Harris, M. and Matyas, L. (2003). Inflation and growth: Explaining a negative effect. *Empirical Economics* 29(1): 149–167.

Gillman, M. and Kejak (2002). Modeling the effect of inflation: Growth, levels, and Tobin. In D. Levine (ed.), *Proceedings of the 2002 North American Summer Meetings of the Econometric Society: Money*. (http://www.dklevine.com/proceedings/money.htm).

Gillman, M. and Kejak, M. (2004). The demand for bank reserves and other monetary aggregates. *Economic Inquiry* 42(3): 518–533.

Gillman, M. and Kejak, M. (2005). Inflation and balanced-path growth with alternative payment mechanisms. *Economic Journal* 115: 1–25.

Gillman, M., Kejak, M. and Valentinyi, A. (1999). Inflation and growth: Non-linearities and financial development. *Transition Economics Series 13*. Vienna: Institute of Advanced Studies.

Gillman, M. and Nakov, A. (2003). A revised Tobin effect from inflation: Relative input price and capital ratio realignments, US and UK, 1959–99. *Economica* 70(279): 439–450.
Gillman, M. and Nakov, A. (2004). Granger causality of the inflation-growth mirror in accession countries. *Economics of Transition* (forthcoming).
Gillman, M. and Wallace, M. (2003). Growth and inflation in a Baltic Sea transition economy: The case of Latvia (manuscript).
Gomme, P. (1993). Money and growth: Revisited. *Journal of Monetary Economics* 32: 51–77.
Gylfason, T. and Herbertsson, T. (2001). Does inflation matter for growth? *Japan and the World Economy* 13(4): 405–428.
Haslag, J. H. (1998). Monetary policy, banking, and growth. *Economic Inquiry* 36: 489–500.
Ireland, P. (1994). Money and growth: An alternative approach. *American Economic Review* 55: 1–14.
Johnson, H. G. (1969). Inside money, outside money, income, wealth, and welfare in monetary theory. *Journal of Money, Credit and Banking* 1(1): 30–45.
Judson, R. and Orphanides, A. (1999). Inflation, volatility and growth. *International Finance* 2(1): 117–138.
Khan, S. and Senhadji, A. (2001). Threshold effects in the relationship between inflation and growth. *IMF Staff Papers* 48(1): 1–21.
King, R. G. and Rebelo, S. (1990). Public policy and economic growth: Deriving neoclassical implications. *Journal of Political Economy* 98(2): S126–S150.
Lucas, R. E. Jr (1967). Adjustment costs and the theory of supply. *Journal of Political Economy* 75(4): 321–334.
Lucas, R. E. Jr (1980). Equilibrium in a pure currency economy. *Economic Inquiry* 43: 203–220.
Lucas, R. E. Jr (1988). On the mechanics of economic development. *Journal of Monetary Economics* 22: 3–42.
Lucas, R. E. Jr (2000). Inflation and welfare. *Econometrica* 68(2): 247–275.
Lucas, R. E. and Stokey, N. L. Jr (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12: 55–93.
Mark, N. and Sul, D. (2002). Cointegration vector estimation by panel DOLS and long run money demand. Technical Working Paper 287, NBER, Cambridge, MA.
Niehans, J. (1969). Efficient monetary and fiscal policies in balanced growth. *Journal of Money, Credit and Banking* 1(2): 228–251.
Otto, G. and Crosby, M. (2000). Inflation and the capital stock. *Journal of Money, Credit and Banking* 32(2): 236–253.
Paal, B. and Smith, B. (2000). The sub-optimality of the Friedman rule and the optimum quantity of money (manuscript, Stanford University).
Rapach, D. (2003). International evidence on the long-run impact of inflation. *Journal of Money, Credit and Banking* 35(1): 23–48.
Schultz, T. W. (1964). *Transforming Traditional Agriculture*. New Haven: Yale University Press.
Shadman-Mehta, F. (2001). A re-evaluation between wages and unemployment in the United Kingdom. *Economica* 68(272): 567–606.
Sidrauski, M. (1967). Inflation and economic growth. *Journal of Political Economy* 75: 796–810.
Stockman, A. (1981). Anticipated inflation and the capital stock in a cash-in-advance economy. *Journal of Monetary Economics* 8(3): 387–393.
Stokey, N. and Lucas, Jr., R. (1989). *Recursive Methods in Economic Dynamics*. Cambridge: Harvard University Press.
Tobin, J. (1965). Money and economic growth. *Econometrica* 33(4): 671–684.

APPENDIX

Section 2 First-Order Conditions

Define η_t, λ_t and μ_t as the Lagrangian multipliers for the human capital, income and money constraints, respectively, of equations. The first-order conditions of the section 2 model are:

$$c_t : \frac{1}{c_t} = \lambda_t P_t \left\{ 1 + \left(\frac{\mu_t}{\lambda_t}\right)\left[\gamma_1 + \gamma_2 + (1-\gamma_1-\gamma_2)\left(1 - A_F\left(\frac{s_{Ft}k_t}{c_t}\right)^{\gamma_1}\left(\frac{l_{Ft}h_t}{c_t}\right)^{\gamma_2}\right)\right]\right\};$$

$$x_t : \frac{\alpha}{x_t} = \eta_t A_H (1-\varepsilon) h_t \left(s_{Ht}k_t\right)^{\varepsilon}\left(l_{Ht}h_t\right)^{-\varepsilon} h_t;$$

$$M_{t+1} : -\lambda_t + \left(\frac{1}{1+\rho}\right)(\lambda_{t+1} + \mu_{t+1}) = 0;$$

$$\begin{aligned} k_{t+1} : & -\lambda_t P_t - \mu_t P_t a_2 + \left(\frac{1}{1+\rho}\right) P_{t+1} r_{t+1} s_{Gt+1} + \left(\frac{1}{1+\rho}\right) P_{t+1}(1-\delta_K) \\ & + \left(\frac{1}{1+\rho}\right)\mu_{t+1} a_2 (1-\delta_K) P_{t+1} + \left(\frac{1}{1+\rho}\right)\mu_{t+1}\gamma_1 \frac{s_{Ft+1}}{c_{t+1}} P_{t+1} A_F \left(\frac{s_{Ft+1}k_{t+1}}{c_{t+1}}\right)^{\gamma_1 - 1} \\ & \left(\frac{l_{Ft+1}h_{t+1}}{c_{t+1}}\right)^{\gamma_2} c_{t+1} + \left(\frac{1}{1+\rho}\right)\eta_{t+1}\varepsilon(s_{Ht+1}) A_H \left(s_{Ht+1}k_{t+1}\right)^{\varepsilon-1}\left(l_{Ht+1}h_{t+1}\right)^{1-\varepsilon} = 0; \end{aligned}$$

$$\begin{aligned} h_{t+1} : & -\eta_t + \left(\frac{1}{1+\rho}\right)\lambda_{t+1} P_{t+1} w_{t+1} l_{Gt+1} + \left(\frac{1}{1+\rho}\right)\eta_{t+1}(1-\delta_H) \\ & + \left(\frac{1}{1+\rho}\right)\mu_{t+1} P_{t+1}\gamma_2 l_{Ft+1} A_F \left(\frac{s_{Ft+1}k_{t+1}}{c_{t+1}}\right)^{\gamma_1}\left(\frac{l_{Ft+1}h_{t+1}}{c_{t+1}}\right)^{\gamma_2 - 1} \\ & + \left(\frac{1}{1+\rho}\right)\eta_{t+1}(1-\varepsilon) l_{Ht+1} A_H \left(s_{Ht+1}k_{t+1}\right)^{\varepsilon}\left(l_{Ht+1}h_{t+1}\right)^{-\varepsilon} = 0; \end{aligned}$$

$$s_{Gt} : \lambda_t P_t r_t - \eta_t \varepsilon A_H \left(\frac{l_{Ht}h_t}{s_{Ht}k_t}\right)^{1-\varepsilon} = 0;$$

$$l_{Gt} : \lambda_t P_t w_t - \eta_t (1-\varepsilon) A_H \left(\frac{l_{Ht}h_t}{s_{Ht}k_t}\right)^{-e} = 0;$$

$$s_{Ft} : \mu_t \gamma_1 P_t A_F \left(\frac{s_{Ft+1}k_{t+1}}{c_{t+1}}\right)^{\gamma_1 - 1}\left(\frac{l_{Ft+1}h_{t+1}}{c_{t+1}}\right)^{\gamma_2} - \eta_t \varepsilon A_H \left(\frac{l_{Ht}h_t}{s_{Ht}k_t}\right)^{1-e} = 0;$$

$$l_{Ft} : \mu_t \gamma_2 P_t A_F \left(\frac{s_{Ft+1}k_{t+1}}{c_{t+1}}\right)^{\gamma_1}\left(\frac{l_{Ft+1}h_{t+1}}{c_{t+1}}\right)^{\gamma_2 - 1} - \eta_t (1-\varepsilon) A_H \left(\frac{l_{Ht}h_t}{s_{Ht}k_t}\right)^{-e} = 0.$$