The welfare cost of inflation in a cash-in-advance economy with costly credit

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The paper presents a modification of the Lucas-Stokey (1983) cash-in-advance economy in which the representative consumer decides, based on relative prices, which goods to buy with cash and which with costly credit. An explicit Baumol (1952) condition emerges that guides this consumer choice. Deriving and estimating a closed-form welfare cost function in an example economy, the paper shows that the welfare cost of inflation depends on the margins of substitution. The consumer avoids inflation through costly credit and faces higher welfare costs of inflation than in standard cash-in-advance economies.

1. Introduction

The cash-in-advance economies of Lucas (1980, 1984) and Lucas and Stokey (1983, 1987) serve monetary theory well by explicitly modeling the exchange function of cash. However, some criticism centers on the requirement that the consumer use cash: the exogenously imposed Clower (1967) constraint.1 Relatedly, the exogenous determination of goods as cash-purchased or credit-purchased according to preference specification arbitrarily impedes consumer choice; the consumer lacks the flexibility to use cash or credit in the purchase of

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1For example, Plosser (1984) comments on the limits of the exogenous Clower constraint.

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any particular good. This paper redresses these issues, within a cash-in-advance economy, by specifying an exchange function through which the consumer decides whether to use cash or costly credit to purchase a good.

Making credit costly in time creates the flexibility in exchange and facilitates empirical applications like the welfare costs of inflation. The theory of the welfare costs of inflation [Bailey (1956)] describes how consumers spend real resources in alternative means of exchange to avoid the inflation tax. Yet estimates of the welfare cost of inflation, such as in Cooley and Hansen (1989), follow from the Lucas (1980) cash-only economy that lacks any alternative means of exchange to cash. Such estimates capture only the inefficiency of inflation-induced substitution from goods to leisure rather than any real resource cost. Estimates such as in Cooley and Hansen (1991) follow from the Lucas–Stokey (1983) economy that has a costless alternative means of exchange to cash. These estimates capture only the inflation-induced inefficiency of substitution towards leisure and credit goods, and again exclude any Bailey-type real resource cost of avoiding inflation.

In this paper, the consumer chooses between a foregone-interest cost of cash and a time cost of credit when purchasing any one good. Avoiding the inflation tax means switching from fiat that uses no resources to exchange credit that uses up societal resources. Inflation acts through cash as a public tax with real proceeds returned in a lump sum fashion, while it acts through credit as a private societal tax with real proceeds destroyed.

Having the ability to switch to costly credit during a stable inflation, the consumer faces higher welfare costs in comparison to standard cash-in-advance economies. This may seem counter-intuitive. However, it results because of the unrealistic assumption in standard cash-in-advance economies that exchange credit is either absent or costless, while here the consumer dissipates real resources when avoiding inflation.

Driving the result of comparatively higher welfare costs, the consumer substitutes away from cash until the marginal costs of avoiding inflation, through credit use, equal the marginal inflation rate tax on cash use. This balances the marginal costs of the means of exchange through a tax avoidance margin analogous to Baumol’s (1952) exchange margin. The Baumol-type function of balancing exchange costs extends the Lucas–Stokey (1983) economy, and

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3 I owe the societal tax point to the referee.
distinguishes it from Townsend (1989) and Den Haan (1990), both of which also endogenize the cash-credit mix but lack an explicit Baumol-type condition. Romer (1986, 1987) also generalizes the Baumol condition, but does not employ the cash-in-advance framework.

The Baumol condition emerges from the exchange technology that specifies the use of Beckerian time [Becker (1965)] for exchange credit. In Den Haan (1990), the exchange technology also induces tradeoffs between the means of exchange. His novel model differs from that presented here by requiring time for cash exchange as well as for credit-type exchange, and by making the cash-in-advance structure apply only in the special case where the consumer uses only cash. The related exchange technologies make the tradeoffs in Den Haan similar to this paper, while the explicit Baumol margin here makes the study of the cash-credit tradeoff simpler.

This paper indicates that the margins of substitution significantly affect the welfare costs of inflation. Computing a closed-form welfare cost function for an example economy, the paper finds a higher welfare cost of inflation and a more negative interest elasticity of money demand in the costly credit economy than in the cash-only and the costless credit economies. The higher welfare cost and more negative interest elasticity align with Bailey's (1956) logic. Lastly, the paper estimates the consumer's welfare cost of inflation, compares it with the literature, and finds support for the analysis.

2. The deterministic costly credit economy

2.1. Exchange structure

The consumer as banker 'self-produces' exchange credit in an implicit banking sector that requires labor. Proportional to the size of the purchase per store and varying continuously by store, the consumer allocates time for exchange credit across a store continuum. Analogous to the color spectrum of Lucas (1980), each store on the continuum sells a different necessity produced with the same technology. The continuum here is similar to Prescott's (1987) continuum in its infinite number of stores and in its division into two segments of purchases.

4Lucas (1980, p. 144) suggests: 'With the introduction of some real cost associated with dealing in a credit market (say the time involved for one's credit worthiness to be established), one can imagine a model in which currency demand is governed by a mechanism such as that studied above coexisting with a credit mechanism . . . ' While lacking such a mechanism, McCallum (1983) introduces similar time as 'shopping time'. Hicks (1935) states: ' . . . my suggestion can be expressed by saying that we ought to regard every individual in the community as being, on a small scale, a bank. Monetary theory becomes a sort of generalization of banking theory.' Baltensperger (1980) emphasizes the potential from modeling real banking costs in developing the theory of the banking firm.
by cash or by exchange credit, but differs in that the consumer chooses the point of division on the continuum rather than taking it as given.

Assume the index \( s \in [0, 1] \) marks a store's place along the continuum of stores. Let \( \tau(s, t) \) be the proportional time per good that the consumer employs when buying a good with credit at store \( s \). Assume \( \tau(s, t) \geq 0 \) and \( \partial \tau / \partial s < 0 \). Since \( \tau(s, t) \) is decreasing in \( s \), at low \( s \) stores the consumer requires more time for credit use and at high \( s \) stores less time for credit use. Since \( \tau(s, t) \) is strictly monotonic, there exists a store, say \( \bar{s}(t) \in [0, 1] \), which divides the continuum between cash and credit use.

The consumer chooses \( \bar{s} \) in deciding where to use credit and where to use cash, buying goods with cash from stores with high time costs of credit, indexed from 0 to \( \bar{s} \), and buying goods with credit from stores with low time costs of credit, indexed from \( \bar{s} \) to 1. Let \( c(s, t) \) be the amount of good purchased at store \( s \) at time \( t \); then the consumer makes the good a cash good for \( 0 \leq s < \bar{s} \) and a credit good for \( \bar{s} \leq s \leq 1 \). That the exchange technology induces the consumer to choose low \( s \) goods as cash goods and high \( s \) goods as credit goods makes them analogous to the Lucas–Stokey (1983) cash good \( c_1 \) and credit good \( c_2 \). Alternatively, the choice of \( \bar{s} \) can be thought of as determining the color composition of each of a single cash good and a single credit good.

Assume perfect competition and identical production technology in the market of each store's good. Then the consumer pays the same positive price at time \( t \), denoted as \( P(t) \), for any good across all of the stores. This price holds whether using cash or credit. The consumer either uses cash held in advance of trading or uses credit and pays off the debt at the beginning of the next period. Either way, the storekeeper finds the receipts from the period's trading available to him for further trading only at the beginning of the next period.

### 2.2. Cash constraint

To buy goods with cash across the low \( s \) stores, the consumer receives a lump sum transfer of cash, \( H(t) \), at the end of each period \( t \). Given an initial cash stock of \( M(0) \), the cash stock at the beginning of period \( t + 1 \) is

\[
M(t + 1) = M(t) + H(t). \tag{1}
\]

The consumer cash expenditures are constrained by the cash stock:

\[
P(t) \int_0^s c(s, t) \, ds \leq M(t). \tag{2}
\]
2.3. Credit time constraint

When buying goods with credit across the high s stores, the amount of time spent at each store equals $\tau(s, t)c(s, t)$. With a given time endowment of one, the total time spent buying goods with credit falls between zero and one:

$$0 \leq \int_{s(t)}^{1} \tau(s, t)c(s, t)\, ds \leq 1. \tag{3}$$

2.4. Production

The consumer as producer uses the same production function for all goods, assumed linear in the labor input. Labor input equals the time endowment, 1, minus leisure time, $x(t)$, and minus total time in exchange credit activity, as given in eq. (3). Total goods production equals $w(t)$ multiplied by the labor input:

$$\int_{0}^{1} c(s, t)\, ds = w(t)\left[ 1 - x(t) - \int_{s(t)}^{1} \tau(s, t)c(s, t)\, ds \right]. \tag{4}$$

In a decentralized economy with profit-maximizing firms, $w(t)$ would be equal to the positive real wage.

2.5. Wealth constraint

The consumer’s end-of-period receipts equal the nominal wages from labor, $P(t)w(t)[1 - x(t) - \int_{s(t)}^{1} \tau(s, t)c(s, t)\, ds]$, plus the lump sum cash transfers, $H(t)$, and the nominal goods endowment, $P(t)a(t)$. End-of-period expenditures equal the cash set aside for next period’s cash purchases, $M(t + 1)$, and the payment of debt from exchange credit, $P(t)\int_{0}^{1} c(s, t)\, ds$. Defining $i(t)$ as the nominal (discrete time) interest rate, with $q^t = 1/[1 + i(1)][1 + i(2)] \cdots [1 + i(t)]$, the consumer discounts the stream of nominal income minus expenditures to get net wealth:

$$\sum_{t=0}^{\infty} q^t \left( P(t)w(t)\left[ 1 - x(t) - \int_{s(t)}^{1} \tau(s, t)c(s, t)\, ds \right] \right. + H(t) + P(t)a(t) - M(t + 1) - P(t)\int_{0}^{1} c(s, t)\, ds = 0. \tag{5}$$

2.6. Preferences

The consumer’s utility at time $t$, $U(t)$, with $\alpha \geq 0$, is defined as

$$U(t) \equiv \int_{0}^{1} [\ln c(s, t) + \alpha \ln x(t)]\, ds. \tag{6}$$
2.7. Equilibrium

The representative consumer defines the equilibrium as the quantity and price sequence \( \{c(s, t), x(t), s(t), M(t), P(t)\} \), for \( t = 0, \ldots, \infty \), which maximizes utility in eq. (6), subject to the cash constraint in eq. (2), discounted by time preference over the infinite horizon, and subject to the wealth constraint in eq. (5). The equilibrium sequence also satisfies nonnegativity constraints on \( c(s, t) \) and \( x(t) \), the credit time constraint of eq. (3), and the cash market clearing condition of eq. (1). To keep the economy monetary with defined prices, assume henceforth that \( s \neq 0 \), so that \( s \in (0, 1) \). This excludes the case of a pure credit economy, which can occur only during extreme hyperinflation with no cash use and which apparently has never been experienced.\(^5\) For a study of a nonbinding cash-in-advance constraint, see Svensson (1985).

2.8. The consumer maximization problem

Discounting utility by \( \beta \in (0, 1) \), the consumer's Lagrangian is

\[
\max_{\{c(s, t), x(t), s(t), M(t)\}} \mathcal{L}^t = \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{t} \int_0^t \left[ \ln c(s, t) + \alpha \ln x(t) \right] ds + \right.
\]

\[
+ \lambda(t) \left[ M(t) - P(t) \int_0^t c(s, t) ds \right]
\]

\[
+ \mu \left\{ \sum_{t=0}^{\infty} q^t \left( P(t) w(t) \left[ 1 - x(t) - \frac{1}{t} \int_0^t \tau(s, t) c(s, t) ds \right] - \right. M(t + 1) - P(t) \int_0^t c(s, t) ds + H(t) + P(t) a(t) \right) \right\},
\]

\[\text{(7)}\]

\(^5\)More generally, uniqueness and existence of the general equilibrium follows if utility, defined \( U(c(s, t), x(t)) : [0, 1] \times N^2 \rightarrow \mathbb{R}^+ \), is monotonic, pairwise continuous, and twice differentiable in goods and leisure, if for all \( s \in (0, 1) \), \( \lim_{c(t), n \rightarrow -\infty} U(c, t, n) = 0 \), \( \lim_{c(t), n \rightarrow 0} U(c, t, n) = \infty \), \( \lim_{x(t) \rightarrow \infty} U(x(t)) = 0 \), \( \lim_{x(t) \rightarrow 0} U(x(t)) = \infty \), and if \( U_{cc} < 0 \), \( U_{xx} < 0 \), and \( U_{cc} U_{xx} - U_{cx}^2 \geq 0 \), for the satisfaction of second-order conditions, and \( U_{xx} > 0 \) for uniqueness. Uniqueness also requires continuity of various functions, for the implicit function theorem and a simple fixed point theorem. Proof of the existence of the general equilibrium follows with modification from Lucas and Stokey (1987).
with first-order conditions

\[ \beta t \lambda'(s) - \mu q - 1 = 0, \quad (8) \]

\[ \beta t \frac{1}{c(s, t)} - \beta t \lambda'(t) P(t) = 0 \quad \text{for} \quad 0 < s < \bar{s}, \quad (9) \]

\[ \beta t \frac{1}{c(s, t)} - \mu qP(t)[w(t)\tau(s, t) + 1] = 0 \quad \text{for} \quad \bar{s} \leq s \leq 1, \quad (10) \]

\[ \beta t \frac{1}{c(s, t)} - \mu qP(t)[w(t)\tau(s, t) + 1] = 0. \quad (11) \]

\[ - \beta t \lambda'(r) P(t)c(\bar{s}, t) + \mu qP(t)[w(t)\tau(s, t)c(\bar{s}, t) + c(\bar{s}, t)] = 0. \quad (12) \]

3. Discussion of first-order conditions

The relative price of cash to wealth, from eq. (8) and with \( \tau \) the inflation rate, equals \( \lambda'(t)/\mu = (1 + i)/(1 + \pi) \) and shows the relative discounting of the value of money due to inflation. Substituting on the basis of eq. (8), write eq. (9) as

\[ \beta t \lambda'(s) - \mu qP(t)[w(t)\tau(s, t) + 1] = 0, \quad (13) \]

This states that the discounted marginal utility of cash goods equals the product of the discounted marginal utility of nominal wealth and the shadow price of the cash good. The shadow price of the cash good consists of a real goods cost of 1 and a real exchange cost of \( i(t) \). Eq. (10) shows that the marginal utility of credit goods equals a similar product \( \mu qP(t)[w(t)\tau(s, t)] \), \( \bar{s} \leq s \leq 1 \). The shadow price of the credit good consists of a real goods price of 1 and a real exchange cost of \( w(t)\tau(s, t) \). Similarly, in eq. (11), the real shadow price of leisure equals \( w(t) \).

Combining the first-order condition for money, in eq. (8), with the first-order condition with respect to the marginal store \( \bar{s} \) at which to use exchange credit, in eq. (12), shows the Baumol-type condition that balances marginal exchange costs:

\[ i(t) = w(t)\tau(\bar{s}, t). \quad (14) \]

The consumer sets the time cost of cash equal to the time cost of credit at the marginal store \( \bar{s} \) (see fig. 1). In Baumol (1952), this equating of the marginal costs
of exchange appears by algebraically rearranging the first-order condition to show that the interest rate equals the marginal (and average) costs of a dollar from the bank. This condition also relates to the legal restrictions description of equilibrium, such as in Eichenbaum and Wallace (1985), in which equilibrium among different types of money occurs at the point of equality among the marginal transactions costs for each of the different types of money. Equating marginal costs here provides the solution to $s$ from eq. (14), $s = \frac{\tau}{\tau'}(i(t)/w(t))$, shows how the consumer's decision depends on relative prices, and provides the additional margin that extends the standard cash-in-advance economies.

4. Substitution rates in the cash-in-advance economies

The consumer's marginal rate of substitution between cash and credit goods, from eqs. (13) and (10), equals $(1 + i(t))/(1 + w\tau(s, t))$, with $0 < s < 1$. In cash-only economies like Lucas (1980), $1 + i(t)$ gives the only relevant shadow price for goods consumption. In Lucas and Stokey (1983), the rate between cash goods and credit goods equals $(1 + i(t))/1$. To see how this paper's economy includes the deterministic cash-only and costless credit cases, first note the cash-only case. If $\tau(s, t) = \infty$ for all $s$, so that $w\tau(s, t) > i(t)$ for all $s$, then the Baumol-type eq. (14) becomes nonbinding, $\bar{s} = 1$, and prohibitive credit costs

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6In Baumol's notation, $i = h[(T/C)/(C/2)]$. 

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Fig. 1. Determination of the marginal credit store. Balancing exchange costs through a Baumol-type condition. Example: $w\tau(s, t) = Aw(1 - s)$.

$Aw = 0.54, i = 0.133.$
reduce the economy to the cash-only case; 1 + i(t) would be the only shadow price of consumption goods.

Secondly, again relax the assumption of strict monotonicity for τ(s, t) to monotonicity, so that τ ≤ 0 instead of τ < 0. Exogenously divide the goods into cash and costless credit goods by setting τ(s, t) = 0 for s < q and τ(s, t) = 0 for s ≥ q, and by assuming that q = τ^{-1} i(t)/w(t); the consumer takes q as given instead of as a choice variable. The marginal rate of substitution between cash and credit goods collapses to (1 + i(t))/1 as in Lucas and Stokey (1983). This gives a cash good with a goods’ cost of 1 and an exchange cost of i(t), and a credit good with a goods’ cost of 1 and an exchange cost of 0. Through a dichotomized relative price specification instead of their preference specification, these assumptions create the same marginal rates of substitution as in the Lucas-Stokey economy.

A more general case exists in the model here: the color spectrum division between cash and credit changes in response to changes in the relative costs of exchange. The resulting marginal rate of substitution of (1 + i(t))/(1 + w(t) τ(s, t)) is generally lower than (1 + i(t))/1, since credit is costly here instead of costless as in Lucas and Stokey (1983). A rate of (1 + i(t))/1 still holds in the economy here for the least credit-costly store (s = 1) if those costs equal 0 [if τ(1, t) = 0]. Otherwise the rate lies below (1 + i(t))/1, falls as s goes from 1 to q, and equals 1 at the maximal store q where the shadow exchange costs of cash and credit are equal [i = wτ(q, t)].

The rate of substitution also equals 1 at the optimum, both here and in Lucas and Stokey (1983). However, the Lucas-Stokey consumer at the optimum faces zero exchange costs both for cash and for credit and so uses both. The consumer at the optimum here faces zero exchange costs for cash, but positive costs for credit, and so uses only cash. The latter result naturally follows in economies where, independent of preferences, costly credit serves only an exchange function.7

Altogether, the consumer finds an added first-order condition and a generally lower marginal rate of substitution between cash and credit goods as compared to Lucas and Stokey (1983). These differences show the sense in which the economy is an extension of the costless credit economy, now including costly credit use alongside cash use and additionally having the consumer decide on the composition of the cash and credit goods. Without preferences tying down cash usage, the consumer can substitute between cash and costly credit to buy any good: doing so alters the welfare costs of inflation.

5. The welfare costs of stable inflation

Through derivation of a closed-form welfare cost function, this section compares the costs of inflation among the costly credit, cash-only, and costless credit

7Pointed out to me by Gary S. Becker.
The welfare cost of inflation

The log-utility example economy yields a relatively simple closed-form function for the welfare costs of inflation and facilitates comparison to Cooley and Hansen (1989, 1991) who also use log-utility. For the technology of exchange credit, assume a linear function: \( \tau(s, t) = A(1 - s) \), for all \( t \), with \( A > 0 \). The consumer as storekeeper can be thought of as providing credit 'paper work' time for a variable fee, dependent on the store index, so that the time cost of credit is replaced by an explicit cost \( A(1 - s) \).

Define the welfare cost of inflation as the real goods endowment, \( a(t) \), needed to make the consumer indifferent between the optimum with zero goods endowment and a nonoptimal inflation rate combined with the real goods endowment. Assume the inflation rate, the real goods endowment, and the marginal product of labor constant over time. Appendix A presents the implied closed-form stationary equilibrium. Using the equilibrium solution, the welfare cost function follows by first forming indirect utility.

Define \( V \) as indirect utility,

\[
V = U[c^*(i, a, \cdot), x^*(i, a, \cdot)],
\]

where \( U \) equals the log-utility of eq. (6) and \( c^*(i, a, \cdot) \) and \( x^*(i, a, \cdot) \) equal the equilibrium goods and leisure consumption from eqs. (26)–(28) of appendix A. Note that \( i = 0 \) is the Friedman (1969) optimum of deflation at the rate of time preference, which equates the competitive equilibrium to the Pareto optimum.

Then the welfare cost of inflation is the \( a = f(i, \cdot) \) that solves

\[
V(i, a, \cdot) - V(0, 0, \cdot) = 0.
\]

Presenting welfare costs as a percent of full income, where full income equals the time endowment 1 multiplied by the marginal product of labor \( w \), percentage welfare costs equal

\[
a \frac{w}{w} = \frac{f(i)}{w} = e^* \cdot (1 + i)^y \cdot Z - 1,
\]

\[
X = \frac{-i}{(1 + \alpha)Aw}, \quad Y = \left( \frac{1}{1 + \alpha} \right) \left( 1 + \frac{1}{Aw} \right),
\]

\[
Z = 1 - \frac{i \left( 1 - \frac{i}{Aw} \right)}{(1 + i)(1 + \alpha)}.
\]

*Martin J. Bailey suggested to me the analogy of paper work time.

The Friedman (1969) optimum is verified by evaluating the derivative of indirect utility with respect to \( i \) at \( i = 0 \).
This assumes that $i < Aw$ so that the economy remains monetary with 
$s = [1 - i/(Aw)] > 0$.

As the interest rate rises from 0, percentage welfare costs rise monotonically
from 0, cash use falls, and the economy approaches all credit. Increases in $\alpha$ and
$Aw$ cause percentage welfare costs to decrease. Increasing $Aw$ to infinity, and
making $s = 1 - i/(Aw) = 1$, collapses the cost function in (17) and (18) to the
function for the cash-only economy, with an ‘$N$’ subscript:

$$\frac{a_N}{w} = \frac{f_N(i, \cdot)}{w} = (1 + i)^{\hat{Y}}Z - 1,$$

$$\hat{Y} = \frac{1}{1 + \alpha}, \quad \hat{Z} = 1 - \frac{i}{(1 + i)(1 + \alpha)}.$$  \hspace{1cm} (19)

The $X$-term of eqs. (17) and (18), which contains the shadow exchange cost of
cash goods relative to that of credit goods, drops out in this cost function as do
the other components with $Aw$. Without these credit costs and the flexibility
between cash and credit, the welfare costs of inflation in the cash-only economy
are less than welfare costs in the costly credit economy: $f_N \leq f$. Except in the
optimum when welfare costs are the same in both economies, the consumer is
better off without the option to spend resources avoiding the inflation tax.

The consumer faces the same welfare costs in the cash-only economy of eqs.
(19) and (20) as found in Cooley and Hansen (1989) in the special case of
certainty, zero capital, and linear production technology. Cash-in-advance
economies with costless credit, as in Lucas and Stokey (1983) and Cooley and
Hansen (1991), face a different cost function than in either the costly credit or the
cash-only economy above. To derive the welfare cost of inflation for the costless
credit economy, as before set up the indirect utility eq. (16) from which to form
the cost function; then set $\hat{s}$ equal to some arbitrary number, say $\hat{s}$, and make
credit costs equal 0, $Aw = 0$. This yields the welfare cost function, similar to the
cash-only case, for the economy with a predetermined division between cash and
costless credit:

$$\frac{a}{w} = \frac{f(i, \hat{s})}{w} = (1 + i)^{\hat{Y}}\hat{Z} - 1,$$

$$\hat{Y} = \frac{\hat{s}}{1 + \alpha}, \quad \hat{Z} = 1 - \frac{i\hat{s}}{(1 + i)(1 + \alpha)}.$$  \hspace{1cm} (21)

For $\alpha \geq 1$ and $i > 0$, the welfare cost of inflation in this costless credit
economy is less than the cost in the cash-only economy and the difference
increases as $\hat{s}$ decreases; $\partial f/\partial \hat{s} \geq 0$, for $\alpha \geq 1$, and so $f(i, \hat{s}, \cdot) \leq f_N(i, \cdot)$. The
estimate for $\alpha$ in the paper is $\alpha = 2.27$ (see appendix B), and a similar estimate in Den Haan (1990) is 2.571. With $\alpha \geq 1$ satisfied, this means that the consumer faces a simple hierarchy of welfare costs across the costless credit, no-credit, and costly credit economies: $f(i, \tilde{s}, \cdot) \leq f_N(i, \cdot) \leq f(i, \cdot)$.

6. Comparison of the interest elasticities of money demand

The interest elasticity of money demand is highest in absolute value in the costly credit economy, given that $\alpha \geq 1$, $Aw \leq 1$, just as the welfare cost of inflation is highest in the costly credit economy for $\alpha \geq 1$. The smallest elasticity in absolute value is found in the cash-only economy, whereas the smallest welfare cost of inflation occurs in the costless credit economy. From eqs. (29) and (30) of appendix A, the costly credit elasticity, denoted by $\sigma$, equals

$$\sigma = -\frac{1 - \bar{s}}{\bar{s}} - \frac{i}{1 + i} + \frac{i[\bar{s} - (1 - \bar{s})(1 + i)]}{(1 + i)^{2}\left(1 + \alpha - \bar{s}\left(\frac{i}{1 + i}\right)\right)}.$$  \hspace{1cm} (23)

The three terms of eq. (23) compare to the $X$, $Y$, and $Z$ terms of the welfare cost function in eqs. (17) and (18). The first term captures the substitution between cash and credit in the face of inflation. At a 10% inflation rate, and with parameter values as given in appendix B, this term accounts for the main part of the elasticity.

Setting $\bar{s}$ equal to 1, in the equilibrium solution given in appendix A, yields the interest elasticity for the cash-only economy, denoted by $\sigma_N$:

$$\sigma_N = -\frac{i}{1 + i} + \frac{i}{(1 + i)^{2}\left(1 + \alpha - \frac{i}{1 + i}\right)}.$$  \hspace{1cm} (24)

Compared to eq. (23), eq. (24) lacks the negative first term and has a more positive last term (a larger return of inflation proceeds). This makes the cash-only elasticity less negative than the costless credit elasticity: $|\sigma_N| \leq |\sigma|$. The main difference is the missing first term that reflects the flexibility between means of exchange.

Setting $\bar{s} = \tilde{s}$ and $Aw = 0$, in the equilibrium solution given in appendix A, yields the elasticity for the costless credit economy:

$$\sigma_{\tilde{s}} = -\frac{i}{1 + i} + \frac{i\bar{s}}{(1 + i)^{2}\left(1 + \alpha - \frac{i\bar{s}}{1 + i}\right)}.$$  \hspace{1cm} (25)
This elasticity is generally less than the elasticity in the costly credit economy; with $z \geq 1$ and $Aw \leq 1$, nonbinding constraints according to the parameter specifications of $z = 2.27$ and $Aw = 0.54$ in appendix B, $|\sigma_z| \leq |\sigma|$. Similar to the cash-only elasticity in eq. (24), the costless credit elasticity in eq. (25) also lacks the first term in eq. (23), and so reflects an inflexibility between means of exchange. In comparing the costless credit to the cash-only economy, $\partial|\sigma_z|/\partial z \leq 0$ implies that $|\sigma_z| \geq |\sigma_N|$; that is, the elasticity in the costly credit economy is higher in absolute value than in the cash-only economy. Intuitively, money demand is more interest-elastic in the costless credit economy than in the cash-only economy because the taxed good, cash, comprises a smaller percent of expenditures in the costless credit economy.

By Bailey's (1956) logic, for the costly credit and the cash-only economies, the ranking of the interest elasticities confirms the ranking of the welfare costs functions. The comparative rankings for the costless credit economy are different because the Bailey logic does not apply. The costless credit economy allows avoidance of the inflation tax without using real resources; it lacks the Bailey link between the interest elasticity, real resource use, and the welfare cost of inflation; and it provides the consumer a more negative higher elasticity than in the cash-only economy, but the lowest welfare costs of inflation.

7. Estimates of welfare costs and elasticities

Estimating the welfare costs of inflation allows a comparison to other estimates and further tests the paper's analysis. Details of the data specifications are in appendix B. Comparison to existing estimates in the literature requires distinguishing between welfare costs as a percent of full income, as in eqs. (17)-(22), and as a percent of current income, as is common. The full income measure captures the substitution effects of inflation, while the current income measure adds an income effect of less work because of inflation. Computing current income from either side of the resource constraint in eq. (4) (with $a = 0$), panel A of table 1 presents the estimates using both full and current income bases. The table shows that the 'sign' of the effect of $z$ on welfare costs, but not of the cost of credit $Aw$, depends on the income basis.

Using averaged annual U.S. data for the years 1948–1988, the measure of the welfare cost of a 10% inflation rate equals 2.19% of current income for the costly credit economy [eqs. (17) and (18)]. The comparable estimates for the cash-only and costless credit economies equal 0.582% and, assuming $\delta = 0.5$, 0.098% [eqs. (19)-(22)]. This assumes that the real rate of interest equals 3% [$i = (1.10)(1.03) - 1 = 0.133$], and that $Aw = 0.54$ and $z = 2.27$. The interest elasticity in the costly credit, cash-only, and costless credit economies, in eqs. (23)-(25), equal $-0.429$, $-0.085$, and $-0.101$, respectively.
Table 1
Sensitivity of welfare cost and velocity estimates to parameter changes.

<table>
<thead>
<tr>
<th>Parameter Changes</th>
<th>Welfare costs as percent of full income [from (17)–(18)]</th>
<th>Current income [from (4)]</th>
<th>Welfare costs as percent of current income</th>
</tr>
</thead>
<tbody>
<tr>
<td>[x = 2.27, Aw = 0.54]</td>
<td>[0.617]</td>
<td>[w \cdot (0.2818)]</td>
<td>[2.190]</td>
</tr>
<tr>
<td>[Aw = 0.594]</td>
<td>[0.576]</td>
<td>[w \cdot (0.2816)]</td>
<td>[2.046]</td>
</tr>
<tr>
<td>[Aw = 1.08]</td>
<td>[0.391]</td>
<td>[w \cdot (0.2532)]</td>
<td>[1.543]</td>
</tr>
<tr>
<td>[Aw = 0.54, x = 2.27]</td>
<td>[0.617]</td>
<td>[w \cdot (0.2818)]</td>
<td>[2.190]</td>
</tr>
<tr>
<td>[x = 2.497]</td>
<td>[0.579]</td>
<td>[w \cdot (0.2630)]</td>
<td>[2.202]</td>
</tr>
<tr>
<td>[x = 4.54]</td>
<td>[0.373]</td>
<td>[w \cdot (0.1644)]</td>
<td>[2.268]</td>
</tr>
</tbody>
</table>

B. Velocity: Approximation using \[1/\bar{s} = 1/(1 - i:Aw)\] with \[i = 0.133\] (10% inflation)

<table>
<thead>
<tr>
<th>Parameter Changes</th>
<th>[1/\bar{s} = ]</th>
<th>[\bar{s}]</th>
<th>[5.66]</th>
<th>[2.99]</th>
<th>[1.63]</th>
<th>[1.36]</th>
<th>[1.33]</th>
<th>[1.25]</th>
<th>[1.20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Aw = ]</td>
<td>[0.133]</td>
<td>[0.162]</td>
<td>[0.20]</td>
<td>[0.35]</td>
<td>[0.50]</td>
<td>[0.54]</td>
<td>[0.65]</td>
<td>[0.80]</td>
<td></td>
</tr>
<tr>
<td>[1/\bar{s} = ]</td>
<td>[x]</td>
<td>[5.66]</td>
<td>[2.99]</td>
<td>[1.63]</td>
<td>[1.36]</td>
<td>[1.33]</td>
<td>[1.25]</td>
<td>[1.20]</td>
<td></td>
</tr>
</tbody>
</table>

For a perspective on the economy's realism, the economy's income velocity of money can also be computed. Defining income equal to consumption, and approximating the consumption as \[c(s)\], for \(0 < s < \bar{s}\) [instead of \(|\int_{0}^{1} c(s)ds|\)], gives an income velocity of money equal to \(1/\bar{s}\). A 10% inflation sets this velocity equal to 1.33 (compared to 1 for the cash-only economy). As shown in panel B of table 1, to get a reasonable estimate of U.S. velocity at a 10% inflation, say equal to 5.66,\(^{10}\) would require a lower estimate of the cost of credit: \(Aw = 0.162\) instead of the actual \(Aw = 0.54\). \(Aw = 0.162\) would yield an interest elasticity of \(-4.73\) instead of \(-0.429\), and a welfare cost of 5.79% instead of 2.19%.

Still, the welfare cost estimates compare to the literature as expected. Cooley and Hansen’s (1989) estimate of the welfare cost of a 10% inflation is 0.521% of current income for quarterly data. Since their economy allows only cash use, 0.521% compares to the cash-only economy of this paper and its estimate of 0.582%. The closeness of the estimates (within 12%) and the small magnitude of 0.521% compared to the 2.19% estimate for the costly credit economy, does not contradict this paper’s finding that allowing only cash yields ‘low’ estimates of the welfare cost of inflation.

Cooley and Hansen’s (1991) estimate of the welfare costs of a 10% inflation is less than their (1989) estimate (0.357% compared to 0.521%). As in this paper,

\(^{10}\)The GNP velocity of M1 in the U.S. averaged 5.66 for the high inflation years of 1973–1981 when the annual percentage increase in the Consumer Price Index for all items averaged 9.24%; tables B-1, 62, 67 [Council of Economic Advisors (1991)].
Wright (1991) attributes the difference to the ability of the consumer to costlessly avoid the inflation tax through the credit goods in the Cooley–Hansen (1991) economy. These estimates support this paper's finding that the welfare costs of inflation in the costless credit economy are less than those in the cash-only economy.

Den Haan's (1990) estimate of the costs of going to a 5% inflation rate from 0% is comparatively high at 4.68%. While not directly comparable to the 2.19% estimate in the economy here, the principal difference is that the Den Haan exchange technology requires time use for all means of exchange; this would in itself give rise to expectations of a higher estimate than found here where only credit uses up time.

This paper's 2.19% estimate is significantly higher than the partial equilibrium estimates in the 0.3% range, such as in McCallum (1990) and Fischer (1981). The income decrease that occurs when inflation increases in the general equilibrium economy here, as opposed to holding income constant for the partial equilibrium estimates, explains a small part of the difference. The larger part of the difference seems to be that the partial equilibrium estimates assume 'small' interest (or semi-interest) elasticities. The economy here suggests assuming elasticities from the upper half of the estimated ranges, such as in Lucas (1988).

8. Qualifications

The cash-in-advance economy in this paper facilitates analyses that require flexibility between means of exchange, such as the examination of the income velocity of money, the specification of a stable money demand function, and the determination of a rule of money supply growth. Adding a feature of realism, the costly credit extension also leaves open the merging of the transactions and asset functions of money through the introduction of intertemporal credit. For the equity premium puzzle [Mehra and Prescott (1985)], the model adds one feature, that the marginal rate of substitution in consumption over time depends on the change in the relative price of exchange. Instead of depending on just the interest rate as in Bohn (1991), the marginal rate of substitution depends on changes in the interest-rate/real-wage-rate and the productivity in finance. In itself, this feature of the credit friction may make for better adjustment of prices to shocks, less movement in consumption, and even more difficult the generation of the change in consumption demanded by the Mehra and Prescott 'friction'.

However, within a cash-in-advance economy that depends on the real-wage/interest-rate, it seems possible to envision incorporating both human and physical capital with adjustment costs, adding intertemporal credit, and moving towards resolution of the equity premium puzzle. If the real wage rises because
of rising labor productivity in a cycle, and its income effect combines with the income effect of rising real returns to physical capital in a cycle, then the income effect of more consumption may dominate the substitution effect of less consumption as the result of a cyclically rising interest rate. This may produce the needed covariance of consumption and the risk-free rate. Adjustment between human and physical capital would still require flexibility within the consumption–investment process, and perhaps even more flexibility within the cash–credit process, even while leaving room for an Eichenbaum and Christiano (1992) friction.

Appendix A: Stationary equilibrium solution

\[ c^*(s) = \frac{w + a}{(1 + i) \left[ 1 + x - \left( \frac{i}{1 + i} \right) \left( 1 - \frac{i}{Aw} \right) \right]} \quad \text{for } 0 < s < \bar{s}, \quad (26) \]

\[ c^*(s) = \frac{w + a}{[1 + Aw(1 - s)] \left[ 1 + x - \left( \frac{i}{1 + i} \right) \left( 1 - \frac{i}{Aw} \right) \right]} \quad \text{for } s \leq \bar{s} \leq 1, \quad (27) \]

\[ x^* = \frac{x(w + a)}{w \left[ 1 + x - \left( \frac{i}{1 + i} \right) \left( 1 - \frac{i}{Aw} \right) \right]^2}. \quad (28) \]

\[ \frac{M^*(t)}{P(t)} = \frac{\left( 1 - \frac{i}{Aw} \right) (w + a)}{(1 + i) \left[ 1 + x - \left( \frac{i}{1 + i} \right) \left( 1 - \frac{i}{Aw} \right) \right]}. \quad (29) \]

\[ \bar{s}^* = 1 - \frac{i}{Aw}. \quad (30) \]

\[ \lambda^*(t) P(t) = \frac{(1 + i) \left[ 1 + x - \left( \frac{i}{1 + i} \right) \left( 1 - \frac{i}{Aw} \right) \right]}{w + a}. \quad (31) \]
Appendix B: Parameter specification and data description

A, w, and z are the parameters requiring specification for the welfare cost and elasticity estimations. In these estimates, of eqs. (18), (20), (22), and (23), A and w appear everywhere as the product Aw and so are here computed as this product. Annual averages are computed from U.S. annual data for the 1948–1988 period.

The average time costs per good over the continuum of stores which offer exchange credit is

$$\int_{\hat{s}(t)}^{1} \tau(s, t) ds = \int_{\hat{s}}^{1} A(1 - s) ds = \frac{A}{2}.$$ 

These costs equal labor hours, L, per output, Y, in the credit sector, or L/Y, so that A/2 = L/Y. Multiply both sides of the latter equation by the real wage, w, to estimate Aw: Aw/2 = wL/Y. Aw equals twice the share of labor in output in the credit sector (abstracting from the model’s feature of no capital).

The Finance, Real Estate, and Insurance (FIR) sector is chosen to measure the model’s credit sector, picking a broader rather than narrower sector on the assumption that technological innovation would affect average costs equally across the entire sector. The broader sector also provides longer time series data on which to draw a stationary estimate. A gauge of the significance of labor in this sector is that annual hours worked by all employees in FIR, as a percent of total hours worked by all employees in all U.S. sectors, has risen steadily from 3% in 1948 to 6% in 1988.11

The labor share in output in FIR is measured by the ratio of the product of the annual wages and the number of full-time equivalent employees in FIR to the value of annual GNP in FIR; this yields an average of 0.27 for 1948–198812 and makes Aw = 0.54. Measurement errors in FIR make the Aw estimate uncertain, though the stability of the average may decrease this risk and is evidenced by the range of (0.22, 0.33) for the 1948–1988 period.

Derive z from eq. (26), which sets out the solution for equilibrium leisure in the log-utility economy. Assuming that a = 0 (zero goods endowment),

$$z = \left(\frac{x}{1 - x}\right)\left(1 - \frac{i\hat{s}}{1 + i}\right).$$

This implies that z is the share of leisure in hours worked at the economy’s optimum (i = 0). Then 1/(1 + z) and z/(1 + z) are the share of work and leisure.


in total allocated time. Excluding $1/3$ of time for sleep, measure the share of working time in total time to derive $z$. Divide annual hours worked, by all full- and part-time workers in the U.S., by the annual number of full- and part-time workers in the U.S. Then take the annual average, over 1948–1988, to get $\alpha = 2.27$.\(^{13}\) Heuristically, $1/(1 + \alpha)$ would be between about 1/4 for a 40-hour work week, or $40/168$ of awake time, and about 1/3 for a 60-hour work week, or $60/168$ of awake time; its measure of $1/(1 + 2.27)$ falls in between. Also $\alpha = 2.27$ is close to a similar measure (equal to 2.571) in Den Haan (1990).

References


Friedman, Milton, 1969, The optimum quantity of money and other essays (Aldine, Chicago, Ill.)

Hicks, J.R., 1935, A suggestion for simplifying the theory of money, Economica 2, 1–19.


\(^{13}\)Tables 6.6, 6.11, ibid.