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Inflation, Investment and Growth: a Money and Banking Approach

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Output growth, investment and the real interest rate in long-run evidence tend to be negatively affected by inflation. Theoretically, inflation acts as a human capital tax that decreases output growth and the real interest rate, but increases the investment rate, opposing evidence. This paper resolves this puzzle by requiring exchange for investment as well as consumption. Inflation then decreases the investment rate, and still decreases both output growth and real interest up to some moderately high rate of inflation, above which increasingly low investment finally causes capital to fall relative to labour, and the real interest rate to rise.

INTRODUCTION

A host of recent long-run evidence indicates that inflation causes a negative long-run effect on economic growth, using both international panel data (Gillman *et al*. 2004) and international G7 time series (Fountas *et al*. 2006).[1] Starting as far back as Feldstein (1982) and including Barro (1995), inflation is also found empirically to cause a decrease in investment; recent long-run evidence supports this (Madsen 2003; Byrne and Davis 2004).[2] A theoretically negative long-run investment effect is found as well (Stockman 1981; Smith and Egteren 2005; Mansoorian and Mohsin 2006), and this result is viewed by Stockman (1981) as an 'inverse', or negative, Tobin (1965) effect. The conundrum comes about in that there is also significant long-run evidence that inflation causes a lower real interest rate (Rapach 2003; Rapach and Wohar 2005; Ahmed and Rogers 2000), which is viewed as a positive Tobin effect, and which is possible theoretically (see, for example, Lioui and Poncet 2008). This appears to be a puzzling contradiction: evidence indicating both long-run negative and long-run positive Tobin effects. Resolving this puzzle theoretically, in a way consistent with the empirical long-run inflation effects on growth, real interest rates and investment, has not been done within standard general equilibrium analysis.

In Tobin (1965), the Solow (1956) model is extended by adding on a money demand function in which money and physical capital are substitutes. Then, in the long run, an increase in inflation induces substitution away from real money towards capital. The consequent long-run equilibrium increase in the capital to labour ratio, or 'capital intensity', is the focus in Tobin, and so can be thought of as a *positive* Tobin effect. It results in a lower marginal product of capital and a lower real interest rate. There is a temporary increase in output growth along the transition path until the new steady state is reached, within the exogenous growth Solow world (see also Ireland (1994) and the Walsh (1998) treatment). Thus the long-run real interest rate falls, the long-run investment rate rises, and output growth only temporarily rises.[3]

Fairly robust evidence supports only the first element of the original Tobin (1965) theory: that the long-run real interest rate falls. And in contrast to Tobin, evidence supports that the investment rate falls and that the long-run growth rate of output falls. Thus the answer to this dilemma of seemingly opposite Tobin effects, along with a negative long-run growth effect, cannot be found in Tobin's extension of Solow. Instead, for the puzzle's resolution we show that it is sufficient to view the inflation mechanism more broadly. And it is necessary to carefully define what is meant by the Tobin effect in this broader framework: it is defined as in Tobin as 'capital intensity', but in particular in terms of the effect of inflation in causing higher capital to effective labour ratios across sectors (as in Gillman and Nakov 2003). Our definition is almost identical to what underlies the Tobin effect in his original model, except that our capital intensity is the stationary capital to effective labour ratio, which includes the Lucas (1988) indexing of labour by endogenous human capital instead of the Solow indexing of labour by exogenous technological change.

Our approach is therefore the inflation tax effect along the balanced growth path equilibrium with Lucas (1988) endogenous growth (Section I). Previously, it has been shown in this setting how inflation acts as a tax on goods and productive time, causing the real interest rate and the output growth rate to fall (Gillman and Kejak 2005b), as appears to be consistent with evidence. But the problem is that the investment to output ratio rises in such models (Gomme 1993), rather than falling as in recent empirical evidence and as in the theory of Stockman (1981). Stockman's approach is to require money for the exchange of not just goods consumption, but also all output, including investment. This reasonable assumption, in that all output does in fact have to be exchanged for, means extending the cash-in-advance constraint beyond its typical specification for only consumption to additionally include investment.[4] But a simple approach of just using the actual Stockman model is not sufficient: there inflation causes the real interest to rise as the capital stock is decreased, which is contrary to the evidence showing that the real interest rate decreases.

The key to resolving this puzzle is to consider that the real interest rate effect need not be positive, as in Stockman (1981), when the Stockman exchange constraint is included in a more general model. An increase in the inflation rate can still decrease the real interest rate (unlike Stockman), while at the same time the investment rate decreases (as in Stockman). Consider that there can be two opposing effects on the real interest rate, when it is determined exclusively by the capital to effective labour ratio in the goods sector in an economy such as Gomme (1993). If the Stockman constraint covers all of the consumer's expenditures, then an increase in the inflation tax discourages the consumer's supply of physical capital (or savings), causes the savings schedule to 'shift backwards' and pressures the real interest rate upwards as the equilibrium investment decreases. But with the increase in the inflation tax also falling on consumption, the consumer substitutes away from (exchanged for) goods, towards (non-exchange) leisure and away from labour, which pressures the real wage to rise relative to the real interest rate; meanwhile, this substitution also is from current to future consumption, towards more savings and pressuring the real interest rate downwards. As long as the labour decrease is large enough relative to the decrease of the physical capital available, then the real wage to real interest rate ratio will rise, the capital to effective labour ratios in the all sectors will rise, and the real interest rate will fall (Section II). This means that the investment to output ratio can continue to fall even while the capital to effective labour ratios rise across sectors and the real interest rate falls. This scenario, if it occurs, solves the puzzle.

This paper shows that applying the cash-in-advance constraint to both consumption and investment within the endogenous growth framework indeed does fit the described inflation evidence, within a realistically calibrated model of the US economy, for inflation rates rising up towards moderately high levels (Section III). Besides the Stockman (1981) constraint, the growth part of the model is also a key ingredient. Inflation reduces the return to human capital and the economic growth rate. It does this because the inflation-induced goods to leisure substitution causes a lower 'capacity utilization rate' of human capital when leisure increases; this directly lowers the return on human capital and the growth rate. But since the *after-inflation-tax* return on physical capital must equal the now-lowered return on human capital along the balanced growth equilibrium path, the savings–investment rate falls throughout the whole inflation range under consideration. This means that the inflation-induced fall in the investment rate is robust within a full range of the inflation rate, while the fall in the real interest rate becomes less and reverses to become an increase in the real interest rate once inflation continues to rise past a moderately high level. At this point the positive pressure on the real interest rate from the savings decline dominates the negative pressure from the labour decrease, which in turn has become increasingly smaller in magnitude because of the critical role played by exchange credit: it is used increasing more to avoid the inflation tax, allowing leisure to be used increasingly less as an avoidance device.

The results rest on the human capital endogenous growth feature, which is a widely used paradigm,[5] and on leisure use, which is ubiquitous in dynamic macroeconomic models and strongly emphasized (for example, by Chari *et al.* 2008) as a key channel. The paper's economy is the same as the nesting model of Gillman and Kejak (2005a), except that here it is extended by decentralizing the banking sector that produces exchange credit. This explicit banking production approach, which is known as the financial intermediation approach in the banking literature (Matthews and Thompson 2008), is based on a well-established industry-production function for financial intermediation services.

The credit production function still yields the same empirically plausible generalized (Cagan 1956) money demand (Mark and Sul 2003; Gillman and Otto 2007), as is found in Gillman and Kejak (2005b), which is essential for a realistic simulation of the negative inflation effect on growth. And the money to credit substitution implicit in the money demand determines how much leisure increases when inflation goes up, determining in part the effect of inflation on the real interest rate and therefore the plausibility of the model's Tobin (1965) effect on interest rates. Decentralizing the banking sector is important in that it makes more exacting the calibration of the money demand, since this now depends explicitly on parameters of a micro-founded credit production technology. Comparative statics of these technology parameters show how they affect money velocity and the balanced growth rate, which in turn affects the investment rate and real interest rate. These results are also shown through full model simulations (Section III).

Therefore the paper contributes a theoretical explanation of seemingly conflicting Tobin (1965) evidence on investment and real interest rates, within an economy that is calibrated realistically to US postwar data. At the same time, this model is theoretically consistent with other long-run inflation-related evidence—on money demand, the output growth rate and the employment rate—as well as with the effect of financial sector productivity increases on output growth and with the assumed structure of financial intermediation services production (Section IV). The consistency of the economy with these other empirical effects helps create greater confidence in the model's robustness for its resolution of the Tobin evidence.

I. Representative Agent Model

(a) Consumer problem

The representative agent's discounted utility stream depends on the consumption of goods c_t and leisure x_t in a constant elasticity fashion:

$$(1) \qquad \sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t(\log c_t + \alpha \log x_t).$$

Exchange is required for both consumption and investment goods, denoted by i_t, whereby the consumer uses either nominal money, M_t, or credit from a credit card. Let q_t denote the real quantity of credit, and let P_t denote the nominal goods price. This makes the exchange constraint

$$(2) \qquad M_t + P_t q_t \geqslant P_t c_t + P_t i_t.$$

It is assumed that all expenditures are sourced from deposits, denoted in real units by d_t, held at the financial intermediary. The consumer buys shares in the intermediary by making a deposit, whereby the price per share is given by the intermediary at a fixed price of 1, so there is no possibility of a capital gain. However, the share—or unit deposit—yields a dividend that is paid by the intermediary to the consumer, so that the intermediary has no remaining profits after the dividend distribution; the intermediary is a 'mutual bank' owned by the consumer, as is consistent with a representative agent model. The per unit dividend is in essence the payment of a nominal interest rate on deposited funds. Denote the per unit nominal dividend as R_{qt}; total nominal dividends are then $P_t R_{qt} d_t$ (see subsection (b) for the intermediary problem).

Since all expenditures come out of the deposits, this means that

$$(3) \qquad P_t d_t = P_t(c_t + i_t).$$

The fractions of capital allocated across the three sectors, of goods (G), human capital (H) and credit (Q), add up to 1:

$$(4) \qquad 1 = s_{Gt} + s_{Ht} + s_{Qt}.$$

The fractions of labour add up to the total productively utilized time, or $1 - x_t$:

$$(5) \qquad 1 - x_t = l_{Gt} + l_{Ht} + l_{Qt}.$$

Physical capital, k_t, changes according to

$$(6) \qquad k_{t+1} = i_t + (1 - \delta_K)k_t.$$

Human capital, h_t, is accumulated through a constant returns to scale (CRS) production function using effective labour and capital; with $A_H > 0$, $\varepsilon \in [0, 1]$,

$$(7) \qquad h_{t+1} = A_H(l_{Ht}h_t)^{\varepsilon}(s_{Ht}k_t)^{1-\varepsilon} + (1 - \delta_H)h_t.$$

The change in the nominal money stock, $M_{t+1} - M_t$, is equal to income minus expenditure. The nominal income received from capital and labour, with P_t denoting the price of goods, and r_t and w_t denoting the real rental and wage rates, is $P_t r_t (s_{Gt} + s_{Qt}) k_t + P_t w_t (l_{Gt} + l_{Qt}) h_t$. Also, there is a lump sum government transfer V_t and the dividend distribution from the intermediary $R_{qt} d_t$. Expenditures are on consumption and investment, $P_t (c_t + i_t)$, and for the payment of the fee for credit services; with P_{qt} denoting the nominal price per unit of credit, this fee is $P_{qt} q_t$. Together, these items make the income constraint

$$M_{t+1} = M_t + P_t r_t (s_{Gt} + s_{Qt})k_t + P_t w_t (l_{Gt} + l_{Qt})h_t + V_t + P_t R_{qt} d_t - P_t c_t - P_t i_t - P_{qt} q_t. \tag{8}$$

(b) Financial intermediary problem

There are two approaches to positing the production function for financial intermediary services: the 'production' approach and the 'financial intermediation' approach. In the first, only labour and capital are used to produce the financial service, typically in CRS fashion. In the second, a third input is added—the deposits into the bank—and again a CRS function is used, but now of the three inputs instead of just labour and capital. The distinction between the two approaches, when nested as part of a general equilibrium, is crucial. As King and Plosser (1984) insightfully point out, if the CRS assumption is made using just labour and capital as inputs, then there is a flat marginal cost curve of credit supply, where the intratemporal credit is used for exchange. And with an alternative of money for making exchanges, with a marginal shadow cost that is also 'flat' at the nominal interest rate of R, there is no unique equilibrium between money and credit use.

In this section it is demonstrated that the financial intermediation approach, of including deposits as an input, solves this problem of the definition of equilibrium, by giving an upward sloping marginal cost, *per unit of deposits*. Then a unique equilibrium between money and credit results. This is impossible following the production approach without deposits, as is proved in the section on the full equilibrium analysis (subsection (a) of Section II).[6] And the financial intermediation approach is supported empirically (see Section IV).

The intermediary is assumed to operate competitively. It sets the price of deposits, and then the consumer determines the quantity of deposits it wants to hold, d_t, as with a mutual bank. The production function for credit services is CRS in effective labour, since the human capital indexes the raw labour in all production sectors of the endogenous growth model, capital and the deposited funds d_t. With $A_Q \in (0, \infty)$, $\gamma_1 \in [0, 1)$, $\gamma_2 \in [0, 1)$ and assuming that $\gamma_1 + \gamma_2 < 1$, the production function is given by[7]

$$q_t = A_Q (l_{Qt} h_t)^{\gamma_1} (s_{Qt} k_t)^{\gamma_2} d_t^{1-\gamma_1-\gamma_2}. \tag{9}$$

Dividing equation (9) by d_t and defining normalized variables as

$$l_{qt} \equiv \frac{l_{Qt} h_t}{d_t}, \quad s_{qt} \equiv \frac{s_{Qt} k_t}{d_t}, \quad q_t^* \equiv \frac{q_t}{d_t},$$

the production function can be written as

$$q_t^* = A_Q l_{qt}^{\gamma_1} s_{qt}^{\gamma_2}. \tag{10}$$

The solvency restriction that assets equal liabilities is given by

$$P_t q_t + M_t = P_t d_t. \tag{11}$$

The liquidity constraint is that money withdrawn by the consumer is covered by deposits:

$$P_t d_t \geqslant M_t. \tag{12}$$

When no credit is used, the liquidity constraint holds with equality and is equal to the solvency constraint.

Defining the residual return per unit of deposit as R_{qt}, which results after profit maximization, the total nominal profit is then $R_{qt}d_tP_t$, and it is returned to the consumer as owner of the bank, and its deposits. The competitive profit maximization problem can then be written as maximizing profit, denoted by Π_{Qt}, with respect to the three inputs of capital, labour and deposits, subject to the production function in equation (9)—profit here is the revenue $P_{qt}q_t$ minus the costs $w_tl_{Qt}h_tP_t + r_ts_{Qt}k_tP_t$, and the dividend payout $R_{qt}d_tP_t$:

$$(13) \qquad {}_{l_{Qt}, s_{Qt}, d_t}\mathrm{Max}\, \Pi_{Qt} = P_{qt}q_t - w_tl_{Qt}h_tP_t - r_ts_{Qt}k_tP_t - R_{qt}d_tP_t,$$

subject to equation (9). More simply, with normalized variables $P_{qt}/P_t \equiv p_{qt}$ and $\Pi^*_{Qt} \equiv \Pi_{Qt}/(d_tP_t)$, and using equation (10), the firm's problem is

$$(14) \qquad {}_{l_{qt}, s_{qt}}\mathrm{Max}\, \Pi^*_{Qt} = p_{qt}A_Ql_{qt}^{\gamma_1}s_{qt}^{\gamma_2} - w_tl_{qt} - r_ts_{qt} - R_{qt}.$$

The solvency and liquidity constraints in equations (11) and (12) are always satisfied in this simple problem. Zero profit, or $\Pi^*_{Qt} = 0$, results through the distribution of the dividends according to the number of shares of bank ownership as given by the real quantity of deposits d_t, at the dividend rate R_{qt}. Therefore $R_{qt} = p_{qt}q_t^*(1 - \gamma_1 - \gamma_2)$, as follows directly from the CRS properties of credit production. This residual dividend rate in equilibrium is equal to the per unit of credit revenue R_t minus the per unit cost $(\gamma_1 + \gamma_2)R_t$, as shown below (in Proposition 4), by using in addition the equilibrium price of credit (equation (22) below).

The first-order conditions of the simplified problem in equation (14) can be written in terms of average and marginal products: with

$$AP_{l_{qt}} \equiv \frac{q_t^*}{l_{qt}}, \quad AP_{s_{qt}} \equiv \frac{q_t^*}{s_{qt}}, \quad MP_{l_{qt}} \equiv \gamma_1 AP_{l_{qt}}, \quad MP_{s_{qt}} \equiv \gamma_2 AP_{s_{qt}},$$

and the marginal cost per unit of credit, denoted by MC_t,

$$(15) \qquad p_{qt} = \frac{w_t}{\gamma_1(q_t^*/l_{qt})} \equiv \frac{w_t}{\gamma_1 AP_{l_{qt}}} = \frac{w_t}{MP_{l_{qt}}} = MC_t,$$

$$(16) \qquad p_{qt} = \frac{r_t}{\gamma_2(q_t^*/s_{qt})} \equiv \frac{r_t}{\gamma_2 AP_{s_{qt}}} = \frac{r_t}{MP_{s_{qt}}} = MC_t.$$

These Baumol (1952) conditions equate the marginal cost of credit funds to the value of the marginal products of effective labour and capital in producing the credit, the standard price-theoretic conditions for factor markets; the marginal products are fractions, γ_1 and γ_2, of the average products. And from these conditions, the marginal cost schedule can be derived traditionally in terms of input prices, parameters and the output level q_t^*.

From equation (15),

$$MC_t = \frac{w_t}{\gamma_1}\frac{l_{qt}}{q_t^*}.$$

Substituting for

$$l_{qt} = A_Q^{-\frac{1}{\gamma_1}} s_{qt}^{\frac{-\gamma_2}{\gamma_1}} (q_t^*)^{\frac{1}{\gamma_1}}$$

from the production function in equation (10), gives that

$$MC_t = \frac{w_t}{\gamma_1} A_Q^{\frac{-1}{\gamma_1}} s_{qt}^{\frac{-\gamma_2}{\gamma_1}} (q_t^*)^{\frac{1-\gamma_1}{\gamma_1}}.$$

Finally, substituting for s_{qt} from the bank's first-order condition in equation (16), in which $s_{qt} = (\gamma_2 MC_t/r_t)q_t^*$, and simplifying, gives

$$(17) \qquad MC_t = \left(\frac{w_t}{\gamma_1}\right)^{\frac{\gamma_1}{\gamma_1+\gamma_2}} \left(\frac{r_t}{\gamma_2}\right)^{\frac{\gamma_2}{\gamma_1+\gamma_2}} A_Q^{\frac{-1}{\gamma_1+\gamma_2}} (q_t^*)^{\frac{1-\gamma_1-\gamma_2}{\gamma_1+\gamma_2}}.$$

For simplification, define $\gamma \equiv \gamma_1 + \gamma_2$ and rewrite the marginal cost as

$$MC_t = B_t (q_t^*)^{\frac{1-\gamma}{\gamma}}, \quad \text{where } B_t \equiv \left(\frac{w_t}{\gamma_1}\right)^{\frac{\gamma_1}{\gamma}} \left(\frac{r_t}{\gamma_2}\right)^{\frac{\gamma_2}{\gamma}} A_Q^{\frac{-1}{\gamma}}.$$

Consider the following proposition; the proofs for this and Propositions 2–7 are given in the Appendix.

Proposition 1. The marginal cost curve is upward sloping for $\gamma \in (0, 1)$, convex for $\gamma \in (0, 0.5)$, and concave for $\gamma \in (0.5, 1)$, when plotted against output q_t^*.

Figure 1 illustrates the convex case of the marginal cost curve (curved line), with $\gamma = 0.3$, $B = 1.3541$, and with the nominal interest rate $R = 0.15$ also drawn in as a horizontal line.

(c) Goods producer problem

The goods producer competitively hires labour and capital for use in its Cobb–Douglas production function. Given $A_G \in (0, \infty)$, $\beta \in [0, 1]$,

$$(18) \qquad y_t = A_G (l_{Gt} h_t)^{\beta} (s_{Gt} k_t)^{1-\beta},$$

with the first-order conditions

$$(19) \qquad w_t = \beta A_G (l_{Gt} h_t)^{\beta-1} (s_{Gt} k_t)^{1-\beta},$$

$$(20) \qquad r_t = (1-\beta) A_G (l_{Gt} h_t)^{\beta} (s_{Gt} k_t)^{-\beta}.$$

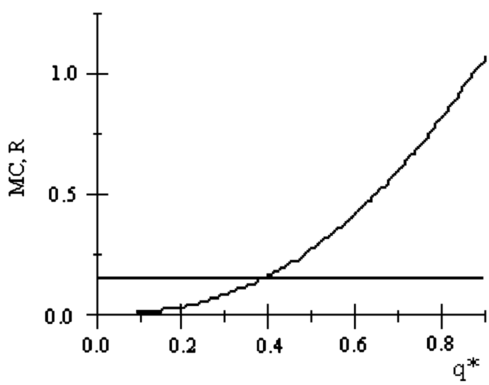

FIGURE 1. Marginal cost of credit per unit of q_t^*.

(d) Government financing problem

The government money supply changes according to a lump sum transfer of cash, V_t, given to the consumer each period:

$$M_{t+1} = M_t + V_t. \tag{21}$$

Assuming that this supply is such that there is a constant rate of money supply growth, defined by $\sigma \equiv V_t/M_t$, this money supply is

$$M_{t+1} = M_t(1+\sigma).$$

(e) Balanced growth path equilibrium

Given prices r_t, w_t, P_t, P_{qt} and R_{qt}, the consumer maximizes utility in equation (1) subject to the constraints in equations (2) to (8), with respect to c_t, x_t, l_{Gt}, l_{Ht}, l_{Qt}, s_{Gt}, s_{Ht}, s_{Qt}, q_t, d_t, i_t, k_{t+1}, h_{t+1} and M_{t+1}. Given prices r_t, w_t, P_t, P_{qt}, and the technology of equation (10), the financial intermediary maximizes profit (equation (14)) with respect to normalized inputs, yielding equilibrium equations (15) and (16). The goods producer maximizes profit subject to the CRS production function constraint (18), giving conditions (19) and (20). And the government's budget constraint (21) provides the market clearing condition for the money market; the deposit condition (3) provides market clearing for the intermediary's deposit market; and goods market clearing of income equal to expenditure is given by equation (8).

Along the balanced growth path (BGP), all growing real variables (c_t, y_t, q_t, d_t, $m_t \equiv M_t/P_t$, i_t, k_{t+1}, h_{t+1}) grow at the same rate, with this balanced growth rate denoted by g. Other stationary variables on the BGP also are denoted without the time index in the following BGP equilibrium conditions (with $\gamma \equiv \gamma_1 + \gamma_2$); these are then used in the next section to describe the effect of inflation.

$$p_q = R, \tag{22}$$

$$R = \sigma + \rho + \sigma\rho, \tag{23}$$

$$\frac{m}{y} = 1 - \left[R^{\frac{\gamma}{1-\gamma}} A_Q^{\frac{1}{1-\gamma}} \left(\frac{\gamma_1}{w}\right)^{\frac{\gamma_1}{1-\gamma}} \left(\frac{\gamma_2}{r}\right)^{\frac{\gamma_2}{1-\gamma}} \right], \tag{24}$$

$$\frac{x}{\alpha c_t} = \frac{1+\tilde{R}}{wh_t}, \tag{25}$$

$$\tilde{R} = (1-q^*)R + \gamma R q^*, \tag{26}$$

$$\frac{w}{r} = \frac{\beta}{1-\beta}\frac{s_G k_t}{l_G h_t} = \frac{\varepsilon}{1-\varepsilon}\frac{s_H k_t}{l_H h_t} = \frac{\gamma_1}{\gamma_2}\frac{s_Q k_t}{l_Q h_t}, \tag{27}$$

$$r_H = \varepsilon A_H \left(\frac{s_H k_t}{l_H h_t}\right)^{1-\varepsilon}(1-x), \tag{28}$$

$$1+g = \frac{1+r_H-\delta_H}{1+\rho} = \frac{1+r/(1+\tilde{R})-\delta_K}{1+\rho}, \tag{29}$$

$$(30)\qquad \frac{i_t}{k_t} = \frac{k_{t+1} - k_t(1-\delta_K)}{k_t} = g + \delta_K,$$

$$(31)\qquad \frac{i_t/k_t}{y_t/k_t} = \frac{g+\delta_K}{A_G\left((l_G h_t)/(s_G k_t)\right)^{\beta} \cdot s_G} = \frac{g+\delta_K}{r \cdot s_G} = \frac{r/(1+\tilde{R}) - \rho(1-\delta_K)}{r \cdot s_G(1+\rho)}.$$

II. Analysis of the Effect of Inflation

The price of credit per unit is simply the nominal interest rate, by equation (22), giving the perfectly elastic demand for credit at the price R; thus the marginal cost of money (R) equals the marginal cost of credit, in a generalization of the margin found in Baumol (1952). At the Friedman optimum, the nominal interest R equals zero (equation (23)), no credit is used (equation (17)), and normalized money demand (inverse money velocity) is equal to 1 (equation (24)), which gives the special case of a cash-only economy.

Consider what happens when inflation increases. As inflation rises, R rises and the shadow cost of exchange $\tilde{R}$ (equation (26)) rises; the agent then substitutes from money to credit as in equation (24) and from goods towards leisure according to the marginal rate of substitution given in equation (25). This $\tilde{R}$ is the average exchange cost per unit of output and is equal to a weighted average of the cost R when using cash, with the weight m/y, and the average cost when using credit, $(\gamma_1+\gamma_2)R$, as weighted by $1-m/y$.[8] Substitution towards leisure x reduces the employed time $(1-x)$; the capital to effective labour ratio also rises across all sectors as the real wage w rises and the real interest rate r falls (equations (19), (20) and (27)). But in equation (28), the rise in $(s_H k_t)/(l_H h_t)$ is dominated by the increase in leisure so as to reduce r_H. The growth rate, in equation (29), therefore falls as R rises, because r_H falls and because the after-inflation tax return on physical capital, which can be defined as $r_K \equiv r/(1+\tilde{R})$ (see equation (29)), also falls. And so the returns to capital remain the same but lower, in that $r_H = r_K$, but now at a lower level, and the growth rate falls accordingly.

The negative inflation effect on the investment to capital ratio of equation (30) follows directly from the growth rate effect. The effect of inflation on the investment to output ratio, i/y, as given in equation (31), similarly depends on the growth rate effect, but also on the changes in the interest rate and in the capital share of the goods sector, s_G. In the simulations below (Section III), it is clear that the changes in r and s_G go in opposite directions and are therefore offsetting to some extent, leaving the growth effect to dominate and cause i/y to fall when inflation increases.

The role of the Tobin effect here is actually rather secondary, as it affects the growth rate and the investment rate. The reallocation away from expensive labour and towards cheaper capital acts to better realign factor inputs given the inflation tax. This ameliorates the negative growth and investment effects, but does not reverse them. However, this positive Tobin effect, in terms of the increase in the capital to effective labour ratio, uniquely determines that there is a decrease in the real interest rate as R rises up from zero.

(a) Credit supply and money demand

As the money demand is residually determined by the credit supply, the fundamentals of the credit supply also underlie those of the money demand and ultimately impact upon the sensitivity of the Tobin effect. The comparative statics of the money demand with respect to the credit production parameters are qualitatively the same as for the comparative statics for the marginal cost curve. And a focus on marginal cost (MC) allows for simple graphical illustration, with respect to changes in the three structural parameters of the credit technology: A_Q, γ_1 and γ_2. While an increase in inflation causes more use of exchange credit, with a movement along the marginal cost curve up to a new higher MC, a change in the structural parameters causes the MC to shift graphically.

Proposition 2. Given q^*, an increase in A_Q decreases the MC.

Figure 2 graphs how an increase in the credit productivity parameter A_Q pivots down the marginal cost (dotted line) from its baseline (solid line). This also causes more credit supply and a lower money demand at a given nominal interest rate. And it increases the balanced path growth rate (see subsection (b) below, and Section III).

The scale parameters γ_1 and γ_2 have different effects on marginal cost and on growth. These scale parameters are important for the calibration of the growth, investment and interest rate effects. First, consider that their sum must be less than 1 in order for the economy's equilibrium to be well defined.

Proposition 3. Assume that $\gamma 1 + \gamma_2 = 1$ and that both credit and goods sectors are equally labour-intensive ($\gamma_1 = \beta$). Then there exists no equilibrium.

If $\gamma_1 + \gamma_2 = 1$, then there is no third factor, deposited funds, entering into the credit production function, and there is no equilibrium, so that the proposition shows the importance of deposited funds as a nontrivial factor. With $\gamma_1 + \gamma_2 < 1$, the marginal cost per unit of funds is upwards sloping as in Figure 1, and there is a unique equilibrium of credit supplied and money demanded, at a given nominal interest rate.

A second important characterizing feature is that the sum of the scale parameters is in fact equal to a measure of the per unit interest cost of the credit. Here we define $R^*_{qt} \equiv R_{qt}d_t/q_t$ as the per unit of credit dividend.

Proposition 4. The proportional per unit cost of credit is equal to the degree of the economies of scale, in that $(R_t - R^*_{qt})/R_t = \gamma_1 + \gamma_2$.

Consider that the total financial intermediary dividends returned to the consumer are $R_{qt}d_t$, or $R_{qt}d_t/q_t$ per unit of credit. The differential between the price of credit per unit of credit output, R_t, and the dividend rate of return per unit of credit, R^*_{qt}, gives the average cost of the resource use per unit of credit, $(\gamma_1 + \gamma_2)R_t$. This makes the degree of the

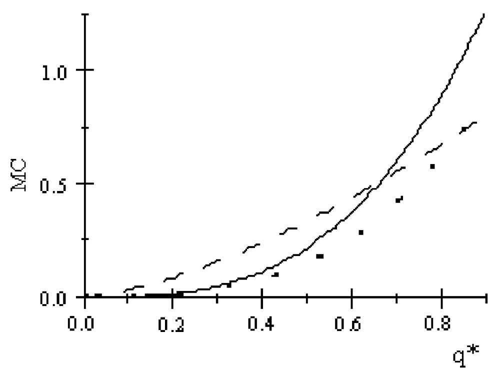

FIGURE 2. Marginal cost with changes in A_Q and γ.

'returns to scale', $\gamma_1 + \gamma_2$, equal to *the fraction of the nominal interest rate that is used up by the production costs per unit of credit*, which is the basis for calibration in Section II.

Given the per unit cost interpretation of $\gamma_1 + \gamma_2$, consider how changes in these parameters affect the marginal cost of credit function:

Proposition 5. Defining curvature as

$$\eta \equiv \left(\frac{\partial MC_t}{\partial q_t^*}\right) / \left(\frac{MC_t}{q_t^*}\right),$$

for a given w and r, an increase in γ_1 causes a decrease in the curvature of the MC curve and an increase in the level of MC for a given level of credit output, given a sufficiently low quantity of credit output.

Figure 2 also illustrates Proposition 5. For $MC = B(q^*)^{(1-\gamma)/\gamma}$, where B is given by equation (17), it graphs an increase in γ from $\gamma = 0.25$ (solid line) to $\gamma = 0.40$ (dashed line), while B actually depends on γ and falls in turn in this example from 1.73 to 0.94. The increase causes less curvature and a higher marginal cost for a given, sufficiently low, q^*. Therefore increasing γ causes greater 'scale', which leads to lower marginal costs at high output levels but higher marginal costs at low output levels.

The effects on money demand of changes in A_Q and γ can be understood in terms of shifting the marginal cost curve. If the MC shifts down, credit is cheaper, less money is used and velocity is higher; the reverse holds if the MC shifts up. A higher money velocity means that the inflation tax falls on less real money and so the tax reduces the growth rate by less. Subsection (b) shows these growth effects analytically for a human-capital-only economy.

(b) Growth

Analytically, for the case of no physical capital, the comparative statics of the balanced path growth rate are qualitatively the same as in the full model simulations. Consider for the next two propositions that $\beta = \varepsilon = 1$ and $\gamma_2 = 0$; then the technology is $y_t = c_t = A_G l_{Gt} h_t$, $h_{t+1} = (1 + A_H l_{Ht} - \delta_H) h_t$ and $q_t^* = A_Q l_{qt}^{\gamma_1}$.

Proposition 6. An increase in the credit sector productivity level, A_Q, causes an unambiguous decrease in the BGP leisure use and growth rate.

This reflects the intuition that greater productivity in producing credit results in a lower marginal cost of credit production (Proposition 2), a higher money velocity, a lower effective inflation tax ($\tilde{R}$) in equation (26), less leisure use and a higher growth rate. Increasing the scale γ gives the opposite results for sufficiently low nominal interest rates, since it causes marginal cost MC to rise (Proposition 5):

Proposition 7. Given that $R' < R \equiv (2/A_Q)\mathrm{e}^{-3/4}$, an increase in γ_1 causes on the BGP an increase in leisure use and a decrease in the growth rate.

(c) The real interest rate

Whether the capital intensities are rising across the sectors depends on whether w/r is rising. And when sectoral capital intensities are rising, the real interest rate is falling. A way to think intuitively of the overall forces determining r is to think in terms of what is happening to capital intensities when inflation increases.

To illustrate these effects, consider the Becker (1965) concept of 'full income', y^F, that includes the shadow income from non-market output (human capital investment) as well as the explicit income from market output, from all sectors of the economy. Looking at his full income in terms of the total cost (TC_t) of all output, where $TC_t \equiv y_F$, we have $(s_{Gt} + s_{Ht} + s_{Qt})r_t k_t + (l_{Gt} + l_{Ht} + l_{Qt})w_t h_t = TC_t$. Note here that the banking sector cost does not include the interest cost of the deposits (an input to production), since this is just the residual profit that is redistributed back to the consumer; then only the labour and capital costs remain. Substituting into the TC_t using the goods and time constraints of equations (4) and (5), the TC_t can be written as the 'isocost line'

$$\frac{k_t}{h_t} = A_t - \frac{w_t}{r_t}(1 - x_t), \tag{32}$$

where $A_t \equiv TC_t/(r_t h_t)$, with a vertical axis of normalized capital k_t/h_t, a horizontal axis of raw labour $1 - x_t$, and a slope of $-w_t/r_t$. The capital to effective labour ratios in all sectors have a slope that is proportional to w_t/r_t (see equation (27)). Therefore when w_t/r_t increases, the ratios

$$\frac{s_{Gt}k_t}{l_{Gt}h_t}, \quad \frac{s_{Ht}k_t}{l_{Ht}h_t} \quad \text{and} \quad \frac{s_{Qt}k_t}{l_{Qt}h_t}$$

increase.

Figure 3 indicates the capital intensity ratios of the two sectors, goods and human capital investment, by the slopes of the positively-sloped rays from the origin (the goods sector is more capital-intensive), and isocost lines of the form in equation (32) by the negatively-sloped lines. When the inflation rate rises, the labour time $1 - x$ falls and so does k/h, so that the initial, outermost (from the origin), isocost line shifts inwards until the middle isocost line is reached, with new higher sectoral capital intensities (dashed rays from origin). The input price ratio w/r is higher since the slope of the isocost is steeper and capital intensities are higher (equation (27)) so the real interest rate has fallen. When the inflation rate continues to rise, the k/h falls again but by less, and the labour time falls

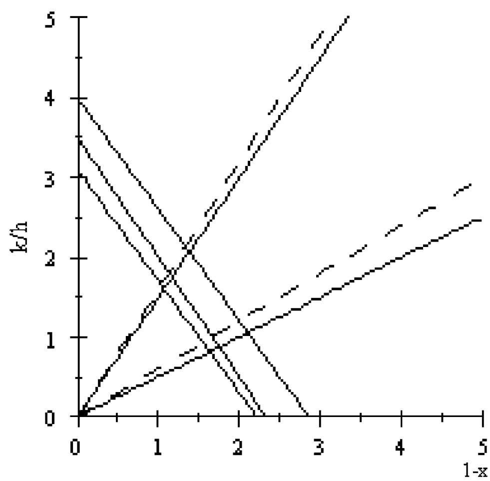

FIGURE 3. Changes in the isocost lines and the sectoral capital intensities.

by much less, resulting in the innermost isocost line. Here w/r (the slope of the isocost line) now has fallen back to what it was in the outermost isocost line, and the capital to effective labour ratios have fallen back to the original ray from the origin. When w/r falls, the real interest rate rises. The falling k/h and $1 - x$ that underlie Figure 3, along with the i/y and r effects, are shown in simulations of the calibrated model in Section III.

III. Calibration and Simulation

The baseline calibration sets parameters and BGP target values of variables as based on postwar US annual data for 1954–2000; these are given in Table 1. Based on the postwar US quarterly calibrations of Gomme *et al.* (2006) and Gomme and Rupert (2007), the shares of effective labour in the goods sector and human capital investment sectors are 0.64 and 0.70, respectively; the annual investment–capital ratio, $i/k = (i/y)/(k/y)$, is 0.088; the implied annual rate of physical capital depreciation, $\delta_K = i/k - g$, is 0.071; the depreciation rate of human capital is the same as for physical capital, $\delta_H = 0.071$; and the intertemporal elasticity of substitution θ is 1. The average annual rate of growth of real GDP, g, and the average inflation rate, π, are 1.68% and 5%, respectively, as in the data. This implies a BGP money supply growth rate of $\sigma = 6.68\%$. Given a time preference rate at the standard value of $\rho = 4\%$, the nominal interest rate is equal to $R = \sigma + \rho + \rho\sigma = 10.68\%$, and the gross real return on capital is $r_K = r_H = g + \delta_K + \rho + \rho_g = 12.8\%$.

Table 1
Baseline Calibration

Parameters		
Preferences		
θ	1	Relative risk-aversion parameter
α	1.935	Leisure weight
ρ	0.04	Discount rate
Goods production		
β	0.64	Effective labour share in goods production
δ_K	0.071	Depreciation rate of goods sector
A_G	1	Goods productivity parameter
Human capital production		
ε	0.7	Effective labour share in human capital production
δ_H	0.071	Depreciation rate of human capital sector
A_H	0.253	Human capital productivity parameter
Banking sector		
γ_1, γ_2	0.172, 0.096	Labour and capital shares in credit production
A_Q	1.44	Banking productivity parameter
Government		
σ	0.067	Money growth rate
Target values		
g	0.0168	Average annual output growth rate
π	0.05	Average annual inflation rate
l_G	0.255	Labour used in goods sector
x	0.5	Leisure
i/k	0.088	Investment–capital ratio
m/y	0.584	Inverse money velocity

To also achieve the Gomme *et al.* (2006) target values for working time $l_G = 0.255$ and leisure $x = 0.5$, the utility parameter for leisure is set at $\alpha = 1.935$.

The basis for the calibration for $\gamma_1 + \gamma_2 = \gamma$ is the interest differential formula of Proposition 5, whereby $\gamma_1 + \gamma_2 = (R - R_q)/R$. It is calibrated using financial industry data at $\gamma = 0.268$, assuming the use of data from just one year, on the basis that this parameter does not change over time. To see how this was calibrated, first note that the Cobb–Douglas production function implies that $R_q d = R_q(1 - \gamma)$ is the total dividend returned to the consumer (interest dividend on deposits); this makes γRq the resource cost of the credit. Per unit of credit this is γR, so γ is the per unit cost of credit divided by R. To compute this, consider that $\gamma = (\gamma Rq)/(Rq)$ is the total credit cost divided by Rq. For the total credit cost estimate, we use as the basis the average annual fee for an American Express credit card as a measure of how much interest is paid on average; it is assumed to reflect the total interest costs of using the annual exchange credit through a 'charge card', rather than a roll-over intertemporal credit card. For an average person this is calculated as \$170, comprised of the basic \$125 Gold Card annual fee plus add-on charges of \$45 for late payment penalties. For R, the average three-month Treasury Bill interest rate, on an annual basis and as an average for the postwar data sample period, gives $R = 0.0606$. Finally, for q, it is true that $q = (q/d)d = [1 - (m/y)]d$ and that in the economy $y = d$; therefore $q = [1 - (m/y)]y$. Using real GDP per capita at 2006 prices, $y = \$25{,}127$, while the US M2 average annual income velocity for 1954–2000 is equal to $1/0.584$. Putting this together, $R_q = 0.0606(1 - 0.584)25{,}127 = 633.44$, and $\gamma = 170/633.44 \approx 0.268$. Dividing γ between capital and labour shares is done by assuming the same ratio of the labour and capital shares in the goods sector: $\gamma_1/\gamma_2 = \beta(1 - \beta) = 0.64/0.36$. This implies that $\gamma_1 = 0.172$ and $\gamma_2 = 0.096$, respectively. To then achieve the target value of $m/y = 0.584$, it requires that $A_Q = 1.44$.

(a) Credit production

Figure 4 simulates the baseline equilibrium credit $q^* \in [0, 1)$ (equation (10)) as graphed with respect to the l_q labour axis (curved line), including the tangency (circle) of the profit line (straight line) of equation (14) to the production function; its slope equals the marginal product of credit labour, or $w/R = 10.39$.

(b) Growth, investment rate and Tobin effects

Figure 5 simulates for the baseline calibration how the growth rate falls as the inflation rate goes up (solid lines), and the comparative statics (dashed lines) of a 5% rise in A_Q and a 20% rise in γ. As in Propositions 6 and 7, greater credit productivity increases the growth rate, and an increase in γ decreases the growth rate for a given inflation rate.

The left-hand panels of Figure 6 show the human capital return r_H (equation (29)) and i/y (equation (31)) falling as the inflation rate increases. The right-hand panels show that r falls and $(s_G k)/(l_G h)$ rises as inflation rises, up to a moderately high level of inflation, but then the graphs reverse at higher levels of inflation. This shows that the real interest rate r falls while i/y falls, but eventually r starts to rise, in concordance with the change in capital intensities. Note that here r does not begin rising until at a level of the inflation rate higher than those experienced in the postwar USA, thereby confirming a 'positive' Tobin effect, while having a negative i/y and growth effects, for the baseline calibration.

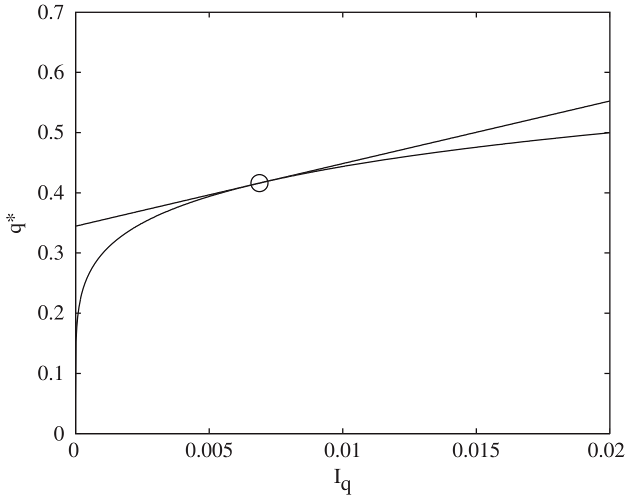

FIGURE 4. Baseline equilibrium credit production.

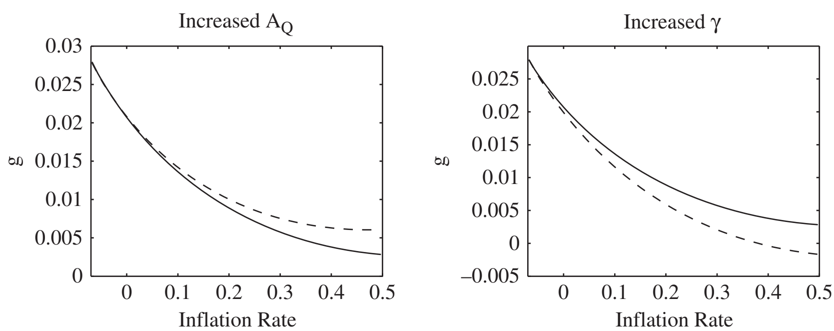

FIGURE 5. Inflation, growth, and changes in A_Q and γ.

Figure 7 shows related effects of inflation: on the rate of productively employed labour in all three sectors, $1-x$, exhibiting a similar nonlinearity as seen for other variables; on the physical capital to human capital ratio k/h, with it also falling in a similar nonlinear fashion; on i_H/y, which is the ratio of outputs in the human capital investment and goods sectors; and on s_G, the share of capital in goods production. The falling levels of both $1-x$ and k/h are consistent with the isocost line of Figure 3 shifting inwards towards the origin as inflation increases, while the decrease in i_H/y is consistent with the initial penalization of the labour-intensive sector, as w/r increases. These changes in i_H/y are reflected in the initial rise in s_G.

IV. DISCUSSION: CONSISTENCY WITH FACTS

This paper shows potential consistency with the negative effect of inflation on the balanced path growth rate of output, the investment rate and the real interest rate. It shows that the real interest rate goes down as inflation rises, for levels of inflation up to a rate that is above that found in the US postwar era. But the other long-run features of the model are also consistent with empirical experience.

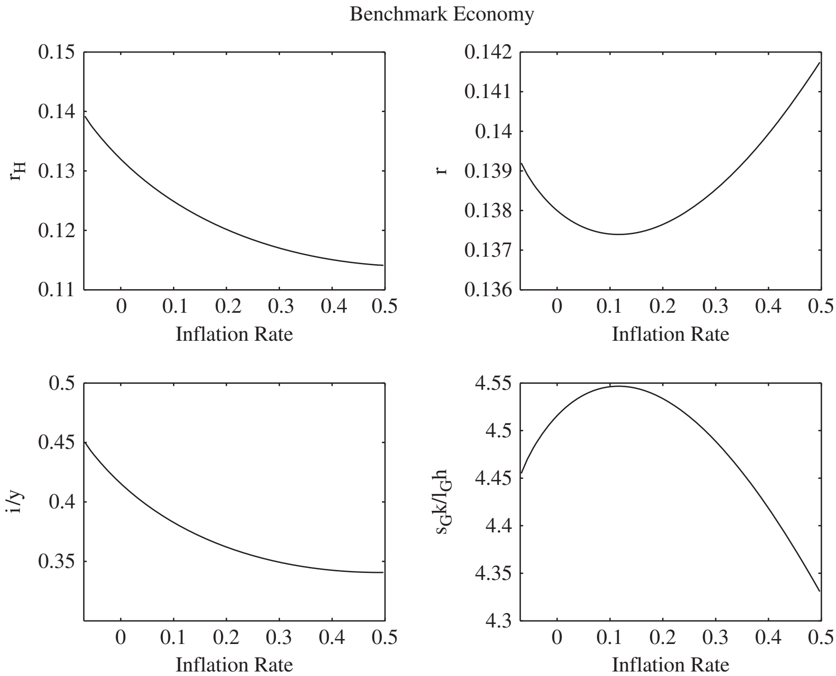

FIGURE 6. Inflation vs returns on capital, investment rate, capital/effective labour.

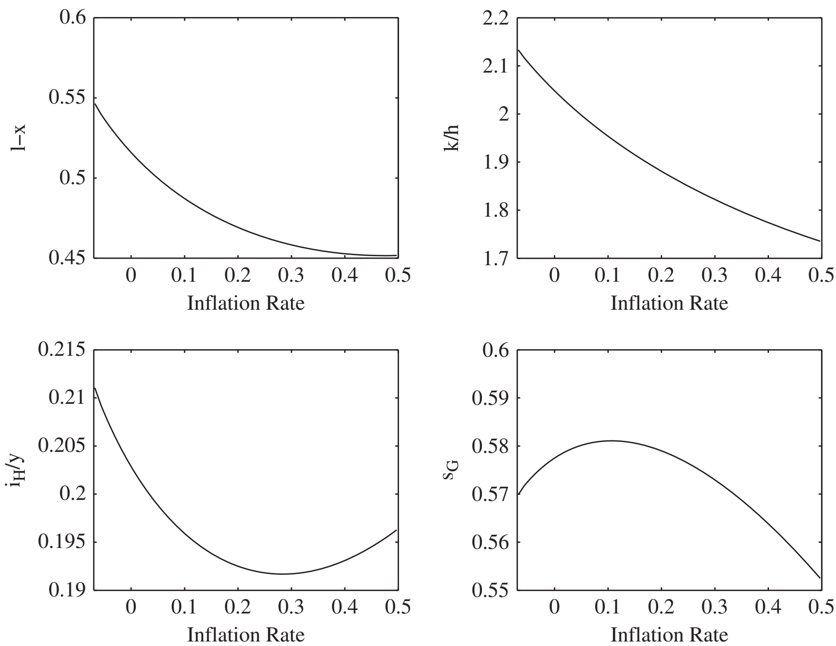

FIGURE 7. Effect of inflation on productive time $(1-x)$, k/h, i/y and s_G.

The money demand interest elasticity is a generalized version of Cagan's (1956) elasticity of $-bR$, where b is a positive parameter. Here the elasticity can be shown to be a function ε of bR^z, where

$$\varepsilon(bR^z) = -z\frac{-bR^z}{1-bR^z} \quad \text{with} \quad z = \frac{\gamma}{1-\gamma}$$

and with b a function of input prices and credit technology parameters. The result is that both elasticities rise in magnitude as the nominal interest rate rises. A Cagan function has been supported for international data (Mark and Sul 2002), and this particular generalized Cagan elasticity has been supported for US and Australian data (Gillman and Otto 2007). In stochastic form, this type of money demand is able to explain velocity at business cycle frequencies (Benk *et al.* 2008).

The money demand is residually determined by the credit supply, since these are perfect substitutes in exchange. So it is noteworthy that the credit production used here has found empirical support for its CRS specification in the financial intermediation/banking literature ever since this technology for financial intermediation services first emerged (Hancock 1985; Wheelock and Wilson 2006). This means that both parts of the money–credit solutions have empirical support.

The money demand determines the velocity effect and the subsequent goods to leisure substitution. The resulting decrease in employed time $(1-x)$ as a result of inflation (Figure 7), in the long run, is consistent with evidence finding cointegration of inflation and unemployment (Ireland 1999; Shadman-Mehta 2001), given that unemployment and the employment rate are found to move closely together. The fact that there is the nonlinear effect of inflation on the employment rate may not have been identified empirically but certainly is an area that might be further investigated.

The credit supply behind the money demand also has the feature that financial development from higher credit sector productivity leads to a higher balanced path growth rate. This result is consistent with the large literature on finance and growth, in which finance is found to positively affect growth.

The central feature for the Tobin effect of a co-movement between inflation and the capital to effective labour ratio is supported empirically in Gillman and Nakov (2003), for both US and UK data. Here cointegration is found between the two series, and Granger causality is found from inflation to the input ratio. This compliments the evidence on the negative effect of inflation on growth, investment and the real interest rate. So it appears that many related facets of the stationary equilibrium analysis are consistent with long-run evidence.

In qualification, the model's simulated decrease in the real interest rate (Figure 6) is small in magnitude, compared for example to Rapach (2003), who finds larger decreases in the real interest rate from inflation increases. However, there are no taxes in our model as in Feldstein (1982), and there may be other features not modelled that make the simulated effect relatively small. The model captures many features simultaneously, in terms of the signs of the changes of many variables, the profile of the changes across the range of inflation (for the inflation–growth effect) and the functional forms (for money demand and credit supply) that are also found in the empirical results. The restriction of calibrating the model carefully to US postwar data makes it challenging to get magnitudes of all such changes to correspond to empirical findings, especially given differently estimated models without precisely comparable results.

However, the model does capture, for example, the estimated magnitudes for the decrease in the output growth rate, which has been well investigated in empirical studies. For example, Barro (1995), using international panel data, finds a 0.24-percentage-point decrease in the growth rate from a 10-point increase in the inflation rate. Our Figure 5 shows that the growth rate falls by 0.4 percentage points when inflation rises from 10% to

20%, and this falls to a 0.2 decrease when inflation rises from 20% to 30%; others show that this magnitude does indeed decrease as the inflation rate goes up (Gillman *et al.* 2004).

Thus the paper has mostly restricted its theoretical description of the empirical findings to one of getting the direction of the changes correct, within a well-calibrated model, for ranges of the inflation rate as seen in the postwar US data. The profiles of the inflation–growth effect and the money demand functional form are exceptions, in that these are rather well studied over different inflation rates, and we can capture these accurately within the model. The nonlinearity in the inflation–growth effect has not been well studied in other dimensions. Our results in particular find this same profile for the investment–output ratio; further study of whether this profile exists empirically would be interesting.

V. Conclusion

This paper offers a solution to the puzzle of explaining conflicting Tobin-type evidence that is found in the literature. It focuses on a natural way to define the Tobin effect in terms of the effect of inflation on capital intensity as in Tobin. But the model's capital intensity is the capital to effective labour ratio, with effective labour indexed by Lucas (1988) human capital instead of by Solow (1956) technological change, as in Tobin. For inflation rates within the US postwar experience, the results within the calibrated economy are that inflation causes a rise in the ratio of the wage rate to the real interest, a rise in the capital to effective labour ratio across sectors, and a decrease in the real interest rate. This is consistent with Tobin's decrease in the real interest rate even though it includes a Stockman (1981) exchange constraint that causes an inflation tax on investment.

The ability to explain this evidence qualitatively, in a quantitatively precise calibration, along with related inflation effects, indicates some success with this approach. This suggests that it may be arbitrary to restrict the specification of cash-in-advance exchange constraints to cover only consumption goods, while leaving investment to be frictionlessly acquired. One way to test the appropriateness of the model's exchange constraint specification is to investigate stochastic extensions of this model, with shocks for example as in Benk *et al.* (2008). It might be possible to determine if the paper's Stockman (1981) approach, within endogenous growth and with money and banking, leads to a stronger explanation of the movements of real and nominal variables over time.

APPENDIX: PROOFS OF PROPOSITIONS

Proof of Proposition 1

Given $\gamma \in (0,1)$ and $MC_t = B_t(q_t^*)^{(1-\gamma)/\gamma}$, it is clear that $B_t > 0$, which implies that the 'slope' coefficient B_t is positive. For B_t held constant at $\overline{B}$,

$$\frac{\partial MC_t}{\partial q_t^*} = \frac{1-\gamma}{\gamma}\bar{B}(q_t^*)^{\frac{1-\gamma}{\gamma}-1} > 0 \quad \text{if } \gamma < 1,$$

establishing the MC upward slope. Then the exact value of γ determines the curvature:

$$\frac{\partial^2 MC_t}{\partial (q_t^*)^2}\begin{cases} = (1-\gamma\gamma-1)(1-\gamma\gamma)B_t(q_t^*)^{\frac{1-\gamma}{\gamma}-2} > 0 & \text{if } \gamma < 0.5, \\ > 0 & \text{if } \gamma > 0.5, \end{cases}$$

establishing convexity and concavity, respectively.

Proof of Proposition 2
From equation (17), for a given q^* and $(\gamma_1+\gamma_2)\in(0,1)$, it follows that $\partial(MC)/\partial A_Q<0$.

Proof of Proposition 3
From equation (10),

$$\frac{q_t}{d_t}=A_Q\left(\frac{l_Q h_t}{d_t}\right)^{\gamma_1}\left(\frac{s_Q k_t}{d_t}\right)^{\gamma_2},$$

and with $\gamma_1+\gamma_2=1$,

$$1=A_Q\left(\frac{l_Q h_t}{q_t}\right)^{\gamma_1}\left(\frac{s_Q k_t}{q_t}\right)^{\gamma_2}.$$

Using equations (10) and (24), it can be shown that $l_Q h_t/q_t=\gamma_1 R/w$ and $s_Q k_t/q_t=\gamma_2 R/r$; substituting these relations back into the previous equation yields

$$1=A_Q\left(\frac{\gamma_1 R}{w}\right)^{\gamma_1}\left(\frac{\gamma_2 R}{r}\right)^{\gamma_2},\quad \text{or}\quad R=A_Q^{-1}\left(\frac{\gamma_1}{w}\right)^{-\gamma_1}\left(\frac{\gamma_2}{r}\right)^{-\gamma_2}.$$

Substituting in for w and r from equations (19) and (20),

$$R=\left(\frac{\beta}{\gamma_1}\right)^{\gamma_1}\left(\frac{1-\beta}{\gamma_2}\right)^{\gamma_2}\left(\frac{A_G}{A_Q}\right)\left(\frac{l_G h_t}{s_G k_t}\right)^{\beta-\gamma_1}.$$

With $\gamma_1=\beta$, the last expression becomes $R=A_G/A_Q$. The nominal interest rate is a constant, independent of the growth rate: $R=\sigma+\rho+\sigma\rho$ (given the log-utility assumption), which in general is not equal to $R=A_G/A_Q$, giving a contradiction. In the case when $R=A_G/A_Q=\sigma+\rho+\sigma\rho$, there is no equilibrium since $A_G>0$ implies $R>0$. Then equation (24) implies that $q_t=\infty$, which violates that $q_t/d_t\in[0,1)$, derived by combining equations (2) and (3).

Proof of Proposition 4
Since $R_t=p_{qt}$ by equation (22), by use of the CRS property of the production function of equation (9),

$$\frac{w_t l_{Qt} h_t}{R_t q_t}=\gamma_1\quad\text{and}\quad\frac{r_t s_{Qt} k_t}{R_t q_t}=\gamma_2.$$

From equation (14) and using the definitions above of

$$l_{qt}\equiv\frac{l_{Qt}h_t}{d_t},\quad s_{qt}\equiv\frac{s_{Qt}k_t}{d_t}\quad\text{and}\quad q_t^*\equiv\frac{q_t}{d_t},$$

it follows that

$$R_{qt}=R_t q_t^*-\gamma_1 R_t q_t^*-\gamma_2 R_t q_t^*=R_t q_t^*(1-\gamma_1-\gamma_2).$$

With the definition above that $R_q^*\equiv R_{qt}/q_t^*$,

$$R_q^*=R_t(1-\gamma_1-\gamma_2),\quad\text{or}\quad R_t=R_q^*+(\gamma_1+\gamma_2)R_t,$$

so

$$\frac{R_t-R_{qt}^*}{R_t}=\gamma_1+\gamma_2.$$

Proof of Proposition 5
With

$$\gamma\equiv\gamma_1+\gamma_2\quad\text{and}\quad\eta\equiv\left(\frac{\partial MC}{\partial q^*}\right)/\left(\frac{MC}{q^*}\right),$$

we have

$$\eta = \frac{1-\gamma}{\gamma} \quad \text{and} \quad \frac{\partial \eta}{\partial \gamma_1} < 0.$$

By equation (17),

$$\frac{\partial MC}{\partial \gamma_1} = \frac{\partial}{\partial \gamma_1}\left\{\exp\left[\frac{1}{\gamma}\left(\gamma_1 \log\frac{\mathrm{w}}{\gamma_1} + \gamma_2 \log\frac{\mathrm{r}}{\gamma_2} - \log \mathrm{A_Q} + (1-\gamma)\log \mathrm{q}\right)\right]\right\},$$

which can be written as

$$\frac{\partial MC}{\partial \gamma_1} = MC\left(\frac{-\gamma + \gamma_2 \log(w/r) - \gamma_2 \log(\gamma_1/\gamma_2) - \log(A_Q/q^*)}{\gamma^2}\right).$$

For ease of exposition, let $\gamma_1 = \gamma_2$. Then

$$\frac{\partial MC}{\partial \gamma_1} = MC\frac{-2\gamma_1 + \gamma_1 \log(w/r) - \log(A_Q/q^*)}{4\gamma_1^2} > 0, \quad \text{for } q^* < \mathrm{e}^{-2\gamma_1}\left(\frac{\mathrm{w}}{\mathrm{r}}\right)^{\gamma_1} \mathrm{A_Q^{-1}}.$$

Proof of Propositions 6 and 7
These propositions both use the following BGP equilibrium solution for the case with no physical capital:

$$q^* = \left(\frac{\gamma_1 R}{A_G}\right)^{\frac{\gamma_1}{1-\gamma_1}} A_Q^{\frac{1}{1-\gamma_1}},$$

$$\frac{c_t}{h_t} = \frac{A_G\rho[1 + A_H(1-x) - \delta_H]}{A_H(1+\gamma_1 A_G q^* R)(1+\rho)},$$

$$x = \frac{(\alpha\rho/(1+\rho))\Omega(R)(1+(1-\delta_H)/A_H)}{1+(\alpha\rho/(1+\rho))\Omega(R)},$$

where

$$\Omega(R) = \frac{1 + (1-q^*)R + (\gamma_1)q^* R}{1+\gamma_1 A_G q^* R}$$

(ratio of 'shadow price' of goods to 'social cost' of goods) and

$$1 + g = \frac{1 + A_H(1-x) - \delta_H}{1+\rho}.$$

Proof of Proposition 6 From the solution given above, it is clear that $\partial q^*/\partial A_Q > 0$, and since $\gamma_1 < 1$ that $\partial\Omega(R)/\partial q^* < 0$. With $\delta_H < 1$, it follows that $\partial x/\partial\Omega(R) > 0$. Consequently, $\partial x^*/\partial A_Q < 0$; with $\partial g/\partial x < 0$, then $\partial g^*/\partial A_Q > 0$.

Proof of Proposition 7 From the solution given above,

$$1 + g = \frac{1 + A_H(1-x) - \delta_H}{1+\rho}, \quad \text{so} \quad \frac{\partial g}{\partial \gamma_1} = -\frac{A_H}{1+\rho}\frac{\partial x}{\partial \gamma_1}.$$

Using sign(x) for the sign of x, it follows that sign($\partial g/\partial\gamma_1$) is a negative function of sign($\partial x/\partial\gamma_1$) = sign($\partial\Omega/\partial\gamma_1$). Given that $A_G = 1$ as in the baseline calibration, from the solution above,

$$\Omega = 1 + \frac{(1-q^*)R}{1+\gamma_1 q^* R}$$

and

$$\frac{d\Omega}{d\gamma_1} = \frac{\partial\Omega}{\partial q^*}\frac{\partial q^*}{\partial\gamma_1} + \frac{\partial\Omega}{\partial\gamma_1} = -\frac{R(1+\gamma_1 R)}{(1+\gamma_1 q^* R)^2}\frac{\partial q^*}{\partial\gamma_1} - \frac{(1-q^*)R}{(1+\gamma_1 q^* R)^2} q^* R,$$

where

$$\frac{\partial q^*}{\partial\gamma_1} = \frac{\partial}{\partial\gamma_1}\left\{\exp\left(\frac{1}{1-\gamma_1}\log[A_Q(\gamma_1 R)^{\gamma_1}]\right)\right\} = q^*\frac{1-\gamma_1+\log(\gamma_1 A_Q R)}{(1-\gamma_1)^2}.$$

Thus

$$\frac{d\Omega}{d\gamma_1} = -\frac{q^* R(1+\gamma_1 R)}{(1+\gamma_1 q^* R)^2}\left[\frac{1-\gamma_1+\log(\gamma_1 A_Q R)}{(1-\gamma_1)^2} + \frac{(1-q^*)R}{(1+\gamma_1 R)}\right],$$

and since $(1-q^*)R < 1+\gamma_1 R$ for $R \leqslant 1$,

$$\frac{1-\gamma_1+\log(\gamma_1 A_Q R)}{(1-\gamma_1)^2} + \frac{(1-q^*)R}{(1+\gamma_1 R)} < \frac{1-\gamma_1+\log(\gamma_1 A_Q R)}{(1-\gamma_1)^2} + 1 < 0$$

if

$$R < R' \equiv \frac{1}{\gamma_1 A_Q} e^{-(1-\gamma_1)(2-\gamma_1)}.$$

In the baseline calibration, $A_Q = 1.44$, $\gamma_1 = 0.268$ and so $R' = 0.73$, establishing that $\partial g/\partial\gamma_1 < 0$ for $R < 0.73$.

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NOTES

1. Fountas *et al.* (2006) find 'strong evidence' that inflation negatively Granger-causes output growth in the G7 countries (USA, UK, Germany, France, Japan, Canada and Italy) for data from 1957 to 2002 using a bivariate VAR-GARCH model, commenting that 'over the last 20 years, the contours of an inverse connection between inflation and growth across countries have begun to emerge from econometric studies' (p. 328). They also discuss more ambiguous results on how inflation uncertainty affects growth. See also Temple (2000) for a related review of this literature.
2. Some of this evidence is related to inflation uncertainty, as in Byrne and Davis (2004), although uncertainty is not introduced in our paper. Also, Ahmed and Rogers (2000) is an exception that shows evidence that the investment to output ratio rises in the USA when inflation goes up.
3. In the Solow–Tobin model, using standard notation, output y depends on capital k and labour n: $y_t = A_t k_t^{1-\beta} n_t^{\beta}$. Investment i with depreciation δ_k is $i_t = k_{t+1} - k_t(1-\delta_k)$, and the balanced path output growth rate $g = (k_{t+1} - k_t)/k_t$ is exogenous. It can be seen that $i_t/y_t = (g+\delta_k)(k_t/(A_t^{1/\beta} n_t))^{\beta}$; if k_t/n_t rises because of inflation, then so does i_t/y_t.
4. Note that the original Lucas (1980) cash-in-advance constraint on only consumption was applied to an economy in which there was no physical capital; investment is zero and not explicitly excluded from the exchange constraint.
5. In contrast, a positive Tobin (1965) effect will not result in an *Ak* endogenous growth model, even with the Stockman (1981) exchange constraint, in that the real interest rate is exogenously equal to *A*. This result and an overview of growth models is provided in Gillman and Kejak (2005a).
6. Assuming only labour and capital (the 'production' approach), King and Plosser (1984) note that: 'The constant returns to scale structure implies that at given factor prices the finance industry supply curve is horizontal'. Baltensperger (1980), in focusing on costly intermediation services, finds that the production function must be of decreasing returns to scale in capital and labour, or conversely that there needs to be a convex cost function, so that the constant marginal revenue per unit of funds equals the rising marginal cost per unit funds. Berk and Green (2004), in their study of mutual funds intermediation, specify a convex cost function, as do Wang *et al.* (2004) for a variety of value-added bank services. Using the 'production' approach, Aiyagari *et al.* (1998) also assume a money demand function, while Li (2000) sets

capital equal to one, both being ways to still get a unique equilibrium but requiring additional assumptions.

7. From Sealey and Lindley (1977) and Clark (1984), where this form of the function is first specified.
8. That $(\gamma_1 + \gamma_2)R$ is an average cost can be verified by dividing the total cost of credit production, net of deposit dividends, by the total output of credit production.

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