

A COMPARISON OF EXCHANGE ECONOMIES WITHIN A MONETARY BUSINESS CYCLE

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The paper sets out a monetary business cycle model with three alternative exchange technologies: the cash-only, shopping time and credit production models. The goods productivity and money shocks affect all three models, while the credit model has in addition a credit productivity shock. The paper compares the performance of the models in explaining the puzzles of the monetary business cycle theory. The credit model improves the ability to explain the procyclic movement of monetary aggregates, inflation and the nominal interest rate.

1 INTRODUCTION

The contribution of monetary factors to business cycle movements has been studied using the general equilibrium approach in the cash-in-advance economies of Cooley and Hansen (1989, 1995, 1998) and the shopping time model of Gavin and Kydland (1999) and Dittmar *et al.* (2005). While money supply shocks have been found to have little effect on business cycles, supported also in Ireland (2004) and Benk *et al.* (2005), there are still many nominal features that present a challenge for general equilibrium monetary modeling. For example, inflation persistence results in the model of Dittmar *et al.* (2005) through the use of Taylor rules of money rather than simple growth rate rules. Liquidity features have not been well explained in the ‘inflation tax’ models although recent work has brought a rudimentary liquidity effect into otherwise standard exchange-based economies without imposing nominal rigidities; this is through the use of a credit production sector in Li (2000). Explaining procyclic monetary aggregates and inflation rate move-

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ments has been even more elusive. A procyclic inflation movement is found only in Dittmar *et al.* (2005) when there is negative or near-zero feedback from output in the Taylor rule, whereas this feedback parameter is typically estimated at higher positive levels.

Extending the exchange economy by allowing for the production of credit as an alternative to cash, while maintaining a simple money supply growth rule, has found success in other related areas besides the liquidity effect. These include the modeling of the income velocity of base, M1 and M2 monetary aggregates (Gillman and Kejak, 2004), the explanation of the effect of inflation on growth (Gillman and Nakov, 2004; Gillman and Kejak, 2005a, 2005b) and the specification of a role for financial development within the inflation–growth nexus (Gillman and Harris, 2004; Gillman *et al.*, 2004). Using the credit production technology also has shown promise in explaining output movements during financial deregulatory periods at business cycle frequencies (Benk *et al.*, 2005).

Here the paper applies the credit production approach to the business cycle in order to compare this exchange technology extension to more standard approaches, the cash-in-advance and shopping time models. A simple money supply rule is maintained.¹ Velocity is endogenous and the results suggest that the credit production approach improves the ability of the inflation tax models to explain business cycle movements. In particular the paper demonstrates that the credit production model can explain procyclic movements in monetary aggregates, inflation and nominal interest rates while the standard models cannot.

Such potential improvements make sense intuitively in that they result from exploitation of an additional margin, relative to the standard cash-in-advance economy. A similar margin exists in the shopping time model but it is rarely exploited there; and shocking the shopping time is awkward in its rationale. The margin included by the credit approach is the ability of the agent to trade-off between using cash and using credit in exchange, depending on relative costs. Cash-only models do not have this freedom and shopping time approaches specify a general transactions cost that induces a margin between using money and using time for exchange. This money–time trade-off can be described as a broad-brush approach that the credit approach refines by specifying labor time that is used in a diminishing returns production function for credit services as an alternative to money in exchange. A distinct advantage of the credit approach relative to shopping time is that the credit production function can be shocked, and calibrated using time-series data from the bank sector. For example, the credit shock in a credit production approach has been identified robustly in Benk *et al.* (2005).

¹Both Alvarez *et al.* (2001) and Schabert (2003) show conditions under which Taylor interest rate rules can be equivalent to simple money supply growth rules.

Exploitation of the additional margin allows for additional income and substitution effects that improve the monetary business cycle model's performance during certain periods. The income effect is important when for example there is a positive credit shock that also contributes significantly to GDP. Benk *et al.* (2005) demonstrate that several of these appear to exist in the USA during the 1980s and 1990s, and for example that these contributed to even bigger increases in GDP during the upswings starting in 1982 and 1991. The income from the positive credit shock causes an additional upward increase in consumption and money demand not present in the other models. And this is the interpretation given for the model's ability to explain procyclic monetary aggregate (M1) movement.

The substitution effect is important in terms of the use of money versus credit in the purchase of the consumption basket. Consider that a positive shock to the productivity of the credit sector causes credit use to become less expensive, and induces more credit to be used relative to cash in exchange. This acts to decrease money demand in the face of an unchanged money supply growth rate. The level effect on money demand causes the price level to jump and the inflation rate to pulse upwards. Continuing with the example of the financial deregulation of the 1980s in the USA, the inflation rate would have been pulsed upwards from the deregulatory acts even while the money supply growth rate began to fall; the result would be an inflation rate that did not fall as quickly as expected (by the money supply growth rates) and a tendency for a procyclic inflation rate when the credit shock contributes significantly to output changes. This significant effect on output would only occur with relatively large, occasional, credit shocks such as major deregulations. This type of substitution likewise carries over to explain how the credit model better explains observed procyclic nominal interest rate movements not explained with the shopping time or standard cash-in-advance models. And so the credit model improves upon the ability to explain an observed procyclic nature of monetary aggregates, the inflation rate and the nominal interest rates, but does this most plausibly during subperiods containing strong credit shocks.

2 EXCHANGE-BASED BUSINESS CYCLE MODELS

Three representative agent models are examined: the standard cash-in-advance, a shopping time economy and the credit production economy. Here a nested model of the three economies is presented. With utility over consumption c_t and leisure x_t given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t (\log c_t + \Psi \log x_t) \quad 0 < \beta < 1 \quad (1)$$

the consumer faces a minimum of two shocks in all three models: an aggregate output productivity shock and a money supply growth rate shock. The

third shock introduced in the credit economy is to the productivity of credit production.

Current investment i_t plus the depreciated capital from the last period comprise the current capital stock k_t :

$$k_t = (1 - \delta)k_{t-1} + i_t \tag{2}$$

Output y_t is produced by the agent with the previous period capital stock k_{t-1} and current labor n_t via a Cobb–Douglas constant-returns-to-scale production function with the productivity shock z_t :

$$y_t = e^{\varepsilon_t} k_{t-1}^\alpha n_t^{1-\alpha} \tag{3}$$

$$z_t = \varphi_z z_{t-1} + \varepsilon_{zt} \quad \varepsilon_{zt} \sim N(0, \sigma_{\varepsilon z}^2) \quad 0 < \varphi_z < 1 \tag{4}$$

Firms maximize their profits $y_t - r_t k_{t-1} - w_t n_t + (1 - \delta)k_{t-1}$, implying the equilibrium real wage rate w_t and the real gross capital rate of return net of depreciation δ , or r_t :

$$w_t = (1 - \alpha)e^{\varepsilon_t} k_{t-1}^\alpha n_t^{-\alpha} \tag{5}$$

$$r_t = \alpha e^{\varepsilon_t} k_{t-1}^{\alpha-1} n_t^{1-\alpha} + 1 - \delta \tag{6}$$

Current income from labor, capital and lump-sum transfers of new money T_t are spent on consumption c_t and capital, yielding the change in money stock $M_t - M_{t-1}$. With P_t the nominal price of the consumption good, this gives the period t budget constraint as

$$w_t P_t (1 - x_t - l_{Ft}) + P_t r_t k_{t-1} + T_t - P_t c_t - P_t k_t \geq M_t - M_{t-1} \tag{7}$$

The money supply is subject to a sequence of random nominal transfers that satisfy

$$T_t = \Theta_t M_{t-1} = (\Theta^* + e^{u_t} - 1)M_{t-1} \tag{8}$$

where Θ_t is the random growth rate of money, Θ^* is the stationary growth rate of money and u_t is a random autoregressive process given by

$$u_t = \varphi_u u_{t-1} + \varepsilon_{ut} \quad \varepsilon_{ut} \sim N(0, \sigma_{\varepsilon u}^2) \quad 0 < \varphi_u < 1 \tag{9}$$

The other resource constraint allocates the total time endowment among leisure, labor hours in producing the aggregate output and time spent in exchange activity, denoted by l_{Ft} :

$$n_t + x_t + l_{Ft} = 1 \tag{10}$$

2.1 Exchange

An extended cash-in-advance constraint is specified so that it encompasses three alternative exchange technologies. The general form is

$$M_{t-1} + T_t \geq P_t c_t (B_1 - B_2 c_t^{b_1} \tilde{A}_{Ft} l_{Ft}^{b_2}) \tag{11}$$

where B_1 , B_2 , b_1 and b_2 are parameters, and \tilde{A}_{Ft} is a variable, specified in the following special cases.

2.1.1 Cash-only. For the standard cash-in-advance economy that uses only cash, let $B_1 = 1$ and $B_2 = 0$.

2.1.2 Shopping Time. The shopping time case assumes that \tilde{A}_{Ft} is a positive parameter A_F , $B_1 = 0$, $B_2 = -1$, $b_1 = 0$ and $b_2 = -1$; or

$$M_{t-1} + T_t \geq P_t c_t A_F / l_{Ft} \quad (12)$$

This implies a proportionality of the time spent in ‘shopping’ to the consumption velocity of money; or that $l_{Ft} = A_F [c_t / (M_t / P_t)]$. While the more general form of the shopping time function is $l_{Ft} = f(c_t, M_t / P_t)$, $f_c > 0$, $f_{M/P} < 0$, the particular specification with proportionality to velocity is found in Gavin and Kydland (1999) and Lucas (2000), justified because it yields a constant interest elasticity of money demand equal to -0.5 as in Baumol (1952).

Given that time in exchange activity is proportional to velocity, this implies a unitary elasticity of exchange time with respect to velocity; $(\partial l_{Ft} / \partial V_t)(V_t / l_{Ft}) = 1$ where $V_t \equiv c_t / (M_t / P_t)$. Or if the elasticity is defined in terms of the ratio of exchange time to consumption, where $\eta \equiv [\partial (l_{Ft} / c_t) / \partial V_t] [V_t / (l_{Ft} / c_t)]$, then again $\eta = 1$.

2.1.3 Credit Production. Here $\tilde{A}_{Ft} = A_F e^{v_t}$, $B_1 = 1$, $B_2 = 1$, $b_1 = -\gamma$ and $b_2 = \gamma$, or

$$M_{t-1} + T_t \geq P_t c_t (1 - c_t^{-\gamma} A_F e^{v_t} l_{Ft}^{\gamma}) \quad (13)$$

It is assumed that $\gamma \in (0, 1)$, $A_F > 0$ and that the shock v_t follows an autoregressive process:

$$v_t = \varphi_v v_{t-1} + \varepsilon_{vt} \quad \varepsilon_{vt} \sim N(0, \sigma_{\varepsilon_v}^2) \quad 0 < \varphi_v < 1 \quad (14)$$

Note that the credit sector specification, supplying only a means of exchange and not intertemporal credit, is parallel to the aggregate output sector specification in several ways. First the credit shock is similar to the productivity shock above, except that the credit shock is a sectoral productivity shock rather than an aggregate shock across all sectors. But it is still a shock to the shift parameter of the production function both in the credit sector case and in the aggregate production case. To see this, consider letting $a_t \in (0, 1]$ denote the fraction of consumption goods that are purchased with money. Then $c_t a_t$ is the total amount purchased with money and $c_t(1 - a_t)$ is the remainder: the total amount of goods purchased with credit. Now consider producing this quantity of credit used for exchange with the following production function involving labor time: $c_t(1 - a_t) = A_F e^{v_t} (l_{Ft} / c_t)^{\gamma} c_t$, where l_{Ft} is the labor time. This can be rewritten as $1 - a_t = A_F e^{v_t} (l_{Ft} / c_t)^{\gamma}$ which says that

the share of credit production is produced with the labor per unit of consumption, with a diminishing marginal product of normalized labor. Solving for $a_t = 1 - A_F e^{\nu} (I_F/c_t)^{\gamma}$, writing the exchange constraint as $M_t = a_t P_t c_t$ and substituting for a_t gives the exchange constraint (13). This clarifies that the assumption behind the exchange constraint is simply that the credit share is produced in a diminishing returns fashion. And it shows that the shock affects the productivity factor of this production function.

The credit production function is also similar to the Cobb–Douglas form of the aggregate production function. Writing it as $c_t(1 - a_t) = A_F e^{\nu} I_F^{\gamma} c_t^{1-\gamma}$, it is of the Cobb–Douglas form in I_F and c_t . However, just as American Express offers credit for exchange (no intertemporal loans) with its standard card, and just as American Express takes the total economic activity as a given in its production of the exchange credit for the economy, so also does our credit production take the total output as a given in its production of the exchange credit.

The degree of diminishing returns depends on the parameter γ . Gillman and Kejak (2005b) illustrate that a value of γ between 0 and 0.5 results in a marginal cost of credit production that is upward sloping and convex, as in the right-hand side of a standard U-shaped marginal cost curve, while values between 0.5 and 1 give an upward sloping but concave marginal cost curve. The values used in the robustness (Section 5 below) range between 0 and 1 but values above 0.5 are suspect in that they yield a marginal cost that rises at a diminishing rate, unusual if found in the industrial organization literature. The baseline value in the simulations is $\gamma = 0.21$, as estimated in Gillman and Otto (2003) from the time-series estimation of US money demand that is derived from a similar credit technology.

2.1.4 Comparison. In comparison to the shopping time case, one key difference is the ability to shock the productivity of the credit production in a standard way, in that it is similar to the shock to any sector or to the aggregate output. The other key difference concerns the elasticities of these models to nominal type changes. Consider that the exchange time in the credit model is not proportional to the consumption velocity of money as it is in the common shopping time specification. Rather the exchange time to velocity ratio rises with the inflation rate. This implies a significant difference in the underlying money demand function. And a similar difference exists between the cash-only and the credit production economies.

Consider the elasticity of exchange time relative to velocity ($1/a_t$). While zero in the cash-only case, and one in the shopping time case, the elasticity of exchange time with respect to velocity is larger than one in the credit production case. For the credit case, let $V \equiv c/(M/P)$ and $\eta \equiv [\partial(I_F/c_t)/\partial V_t][V_t/(I_F/c_t)]$; then $\eta = (1/\gamma)[1/(V - 1)]$. If, for example, $a_t = 0.5$ and $\gamma = 0.21$, then $V = 2$ and $\eta \approx 5$. This means that the exchange time rises much more than proportionally with increases in the velocity. And this is just a

standard feature of a production function with a diminishing marginal product in each of its factors. To see this, consider a standard Cobb–Douglas production function of output, say Y , that depends on a labor quantity L and capital K , as in $Y = L^\eta K^{1-\eta}$. Then the elasticity of the ratio of labor to capital with respect to the ratio of capital to output, denoted by $\tilde{\eta}$, compares directly to η , the labor elasticity of velocity as defined above; this Cobb–Douglas elasticity can be found to be equal to $\tilde{\eta} = -1/\eta$. With $\eta = 0.21$, $\tilde{\eta} \approx -5$, similar to $\eta \approx 5$ when $V = 2$ (the difference in signs results because the credit output is $1 - a_t$ and not a_t). These elasticity results in the production functions reflect the same thing: that the marginal cost curve is positively sloped and rising at an increasing rate. Increasingly more labor time is used because of increasing marginal costs of production. So the elasticity result in the credit production function is a natural consequence of using a standard microeconomic relation and is not found in the standard shopping time and cash-only models.

The consequence of the credit specification can be put in terms of income and substitution effects. There can be significant income effects from using an increasing amount of time in banking, as the inflation rate increases. Cash-only has no such real resource use in avoiding inflation and shopping time has what might be called a unitary elastic cost. During the business cycle, a significant positive credit productivity shock can free up a measurable amount of time and have a significant income effect on the credit model.

The substitution effect can be stated in terms of the interest elasticity of money demand. The cash-only model has a very sluggish interest elasticity of money that rises slightly in magnitude as the inflation rate goes up; it does not allow for exchange time to be used as an alternative to money; and therefore the consumer has no alternative by which to buy goods and only slightly substitutes away from money as inflation rises. The shopping time model has a constant interest elasticity similar to the Baumol (1952) model that results from its assumption of a unitary time elasticity with respect to velocity. And the credit, or banking time, model produces an interest elasticity that rises in magnitude with the inflation rate in a way very similar to the Cagan (1956) model;² this is a result of using a more standard production function. These differing substitution effects can influence business cycle results if there is a large shock that significantly affects the use of money versus its credit alternative in the credit model. The only exchange alternative in the cash-only model is leisure, not typically subject to shocks; in the shopping time model, the exchange alternatives are leisure or shopping time, also not typically shocked. And note that, at high rates of inflation, the elasticity tends to be higher in the credit model than in both the cash-only and shopping time (depending on calibrations) and the substitution effect would then be sig-

²See Gillman and Kejak (2002).

nificantly greater and the effect of a shock larger, such as one that possibly may have occurred during the moderately high US inflation of the early 1980s when deregulation began.

2.2 Equilibrium

The consumer's exchange constraint can alternatively be written in the nested model as

$$M_{t-1} + T_t \geq a_t P_t c_t \tag{15}$$

where

$$\begin{aligned} a_t &= 1 \quad \text{cash-only} \\ &= A_F / l_{Ft} \quad \text{shopping time} \\ &= 1 - A_F e^{v_t} \left(\frac{l_{Ft}}{c_t} \right)^\gamma \quad \text{credit production} \end{aligned} \tag{16}$$

or, expressed in terms of l_{Ft} , in each of these cases, gives that

$$\begin{aligned} l_{Ft} &= 0 \quad \text{cash-only} \\ &= A_F / a_t \quad \text{shopping time} \\ &= [(1 - a_t) / (A_F e^{v_t})]^{1/\gamma} c_t \quad \text{credit production} \end{aligned} \tag{17}$$

This formulation summarizes the nested model developed above and is convenient for defining the equilibrium and for calibration.

The consumer chooses consumption, leisure, capital stock, the fraction of goods bought with money, time spent in exchange activity and the money balances over time, $\{c_t, x_t, k_t, a_t, l_{Ft}, M_t\}_{t=0}^\infty$, to maximize lifetime utility (1) subject to the budget constraint (7), the cash-in-advance constraint (15) and the exchange technology given in equation (17) for the three cases:

$$\begin{aligned} L = E \sum_{t=0}^\infty \beta^t & \left\{ (\log c_t + \Psi \log x_t) + \lambda_t \left(\frac{M_{t-1} + T_t}{P_t} - a_t c_t \right) \right. \\ & \left. + \mu_t \left[w_t (1 - x_t - l_{Ft}) + r_t k_{t-1} + \frac{M_{t-1} + T_t}{P_t} - c_t - k_t - \frac{M_t}{P_t} \right] \right\} \end{aligned} \tag{18}$$

A competitive equilibrium for this economy consists of a set of allocations $\{c_t, x_t, l_t, n_t, l_{Ft}, k_t, a_t, M_t\}_{t=0}^\infty$, a set of prices $\{w_t, r_t\}_{t=0}^\infty$, exogenous shock processes $\{z_t, v_t, u_t\}_{t=0}^\infty$, a money supply process and initial conditions k_{-1} and M_{-1} such that, given the prices, shocks and government transfers, the allocations solve the consumer's utility maximization problem, solve the firm's profit maximization problem and make the goods, labor and money markets clear.

In a stationary deterministic steady state we use the transformation $p_t = P_t / M_t$ (and also denote real money balances by $m_t = M_t / P_t$). There is no uncer-

tainty and time indices can be dropped, denoting by an asterisk the steady-state values and by $R^* = r^*(\Theta^* + 1)$ the steady-state interest rate factor.

2.3 Log-linearization and Calibration

The first-order conditions and log-linearization of the model, following Uhlig (1995), are presented in Appendix A1. This uses the first-order Taylor approximation of the log variables around the steady state and replaces all equations by approximations which are linear functions in the log-deviations of the variables. For example, the variable x_t is replaced with $x_t = x^*(1 + \hat{x}_t)$, where \hat{x}_t is the percentage deviation (log-deviation) from the steady state, or $\hat{x}_t \approx d \log x_t$, and x^* is the steady-state value of the variable x_t .

The baseline calibration uses standard values that are found in the literature. For the more novel credit sector parameter A_F , its value is set to 1.422 which follows from setting $\gamma = 0.21$ (as estimated in Gillman and Otto (2003)). The table in Appendix A2 presents the values used in all three models.

3 IMPULSE RESPONSES

Figures 1–3 show the impulse responses for the credit model to goods productivity shocks, money shocks and the additional credit productivity shock. The impulse responses of the cash-only and shopping time models to goods productivity and money shocks are similar to those of the credit model, with the exceptions mentioned below.

3.1 Goods Productivity Shock

Across the three models, a positive goods productivity shock (Fig. 1) causes more output, consumption, capital, labor, real wages, real interest rate and real money, and lower leisure and prices. Shopping time falls slightly while banking time falls a lot, as labor time is more valuable.

3.2 Money Shock

Across the three models, a positive shock to the nominal money supply growth rate (Fig. 2) causes an increase in capital, real wages and prices, and a decrease in output, consumption, labor, the real interest rate and real money. Leisure falls in the shopping time model but increases in the cash-only and credit models. At the same time, the exchange time in the credit model rises by some 10-fold more than the shopping time. Also consumption falls strongly in the cash-only model, less so in the credit model, and hardly at all in the shopping time model. The cash-only and credit models show the typical goods to leisure substitution, but the shopping time model does not. This can be interpreted as the shopping time model having ‘too much’

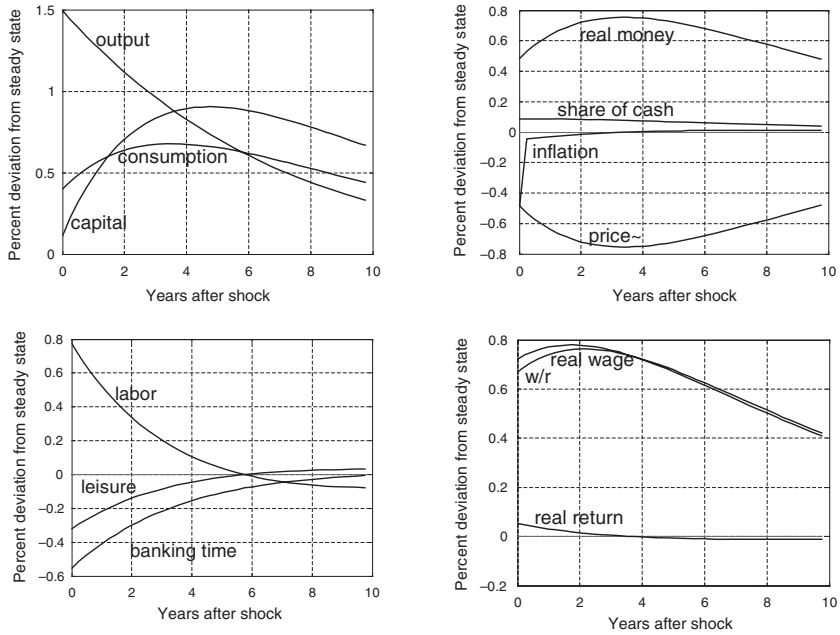


FIG. 1 Impulse Responses to 1 Per Cent Productivity Shock; Credit Model

substitution towards exchange time at low inflation rates, because of the constant -0.5 interest elasticity of money; the credit model in contrast has a near-zero interest elasticity of money at very low inflation rates. The credit model's inelastic money demand at low inflation rates causes more substitution from goods to leisure.³

3.3 Credit Productivity Shock

The third shock (Fig. 3) appears only in the credit model, giving it potentially more explanatory power through this additional dimension. Here the key difference, with a positive credit productivity shock, is that while consumption and output rise, so do prices. In comparison, for a money shock, consumption and output fall as prices rise, in all three models. This is the reason why the additional shock allows for a better explanation of procyclic inflation. And this feature makes sense: an increase in credit productivity during say financial deregulation causes more banking and less money use, with the same money supply growth rate; thus more inflation. If the credit

³See also how Lucas (2000) contrasts the constant interest elasticity function versus the constant semi-interest elasticity function at low inflation rates.

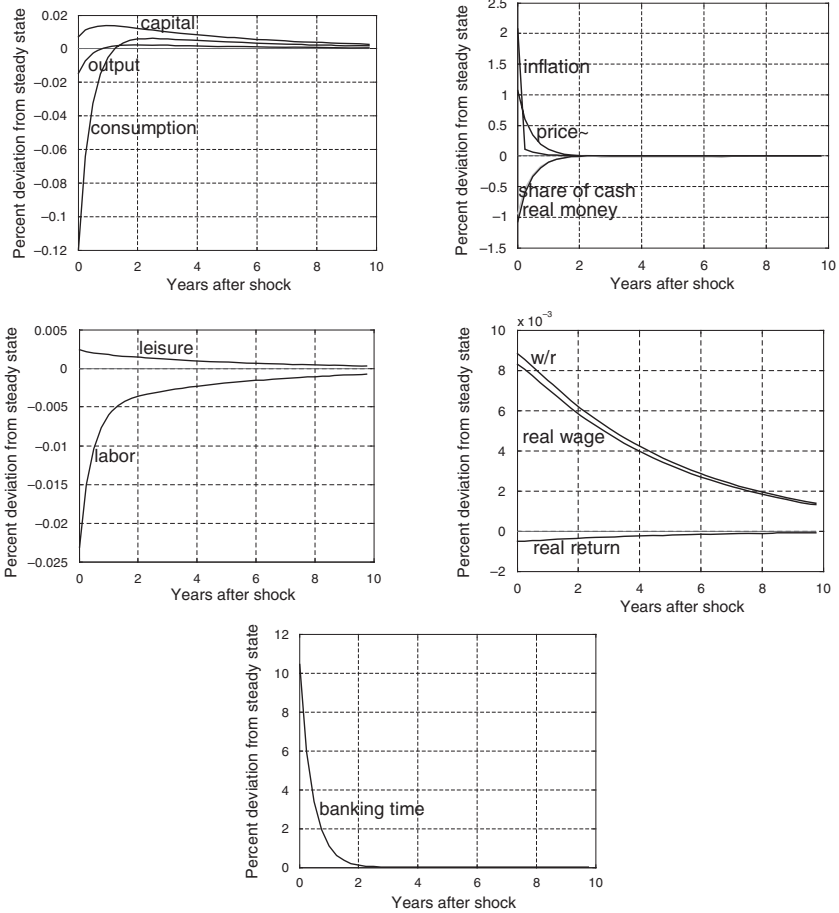


FIG. 2 Impulse Responses to 1 Per Cent Money Supply Shock; Credit Model

shock also leads to a positive GDP impulse, then inflation moves up at the same time as GDP. This is a feature found in US post-war data, and as elaborated upon next, the impulse responses show that neither the goods productivity or the money shock yields such procyclic inflation.

4 PUZZLES

Table 1 first sets out the actual cyclical behavior of the post-war US economy over the 1959:I–2000:IV period. This updates the facts presented in Cooley and Hansen (1995). It shows the standard deviations and the cross-correlations with real GDP and with M1 growth for real and nominal variables.

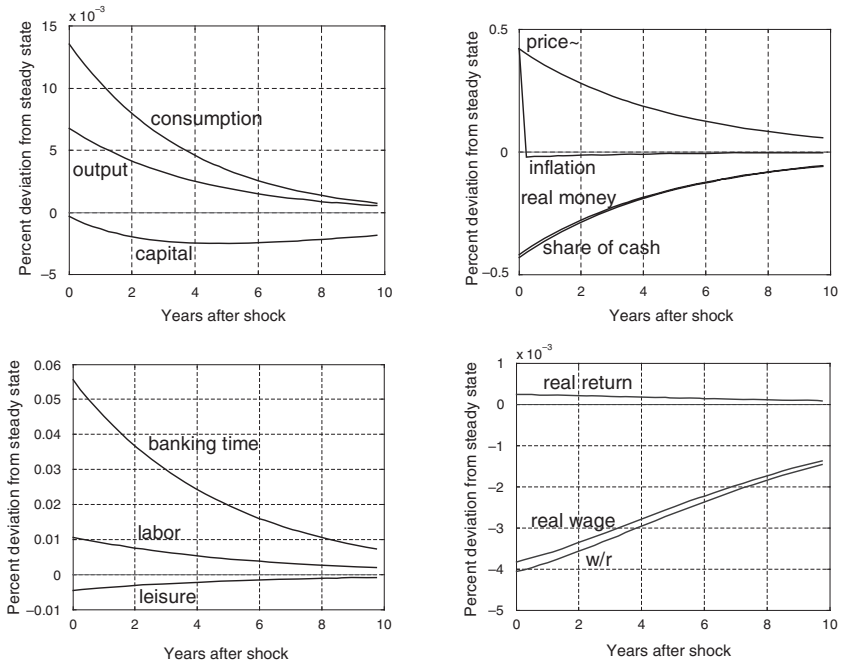


FIG. 3 Impulse Responses to 1 Per Cent Credit Productivity Shock; Credit Model

4.1 Simulations

Simulations were conducted for all three models, in order to see how they perform compared with the puzzles in the literature; only the credit model simulations are presented in Table 2. This table presents the results of simulating the credit model economy 50 times, each simulation being 168 periods long, to match the number of observations underlying the US statistics reported in Table 1. Each simulated time series is filtered with the Hodrick–Prescott filter; the standard deviations of the key variables are reported as well as their cross-correlation with output.

A comparison with the actual cross-correlations in Table 1 shows noteworthy features. While the credit model does not capture the actual output correlation with banking hours, it does do rather well with the inflation rate and the nominal interest rate. The actual data show a positive correlation of future output with inflation and nominal interest rates, and a negative correlation with lagged output with inflation and nominal interest rates. The credit model simulation shows a similar pattern although it is not exactly in phase with actual data. For example, the actual data show a positive current output correlation, and in the simulation the correlation turns positive only with the one-period-ahead output.

TABLE I
CYCLICAL BEHAVIOR OF THE US ECONOMY: 1959:I–2000:IV

Variable	SD %	Corr. with <i>M</i> growth	Cross-correlation of output with										
			<i>x</i> (-5)	<i>x</i> (-4)	<i>x</i> (-3)	<i>x</i> (-2)	<i>x</i> (-1)	<i>x</i>	<i>x</i> (+1)	<i>x</i> (+2)	<i>x</i> (+3)	<i>x</i> (+4)	<i>x</i> (+5)
Output	1.43	-0.13	0.06	0.26	0.46	0.67	0.86	1.00	0.86	0.67	0.46	0.26	0.06
Consumption	1.47	0.01	0.42	0.57	0.69	0.80	0.84	0.79	0.62	0.43	0.23	0.04	-0.13
Investment	4.77	-0.11	0.20	0.37	0.52	0.70	0.83	0.89	0.80	0.63	0.43	0.20	-0.01
Banking hours	1.23	-0.01	-0.53	-0.50	-0.42	-0.32	-0.15	0.04	0.21	0.34	0.45	0.47	0.50
Real wage	1.16	0.18	0.43	0.53	0.57	0.60	0.58	0.45	0.29	0.14	-0.02	-0.18	-0.30
Prices (CPI)	1.22	-0.15	-0.61	-0.68	-0.71	-0.70	-0.64	-0.51	-0.37	-0.22	-0.09	0.04	0.16
Inflation	0.42	-0.32	-0.33	-0.25	-0.10	0.02	0.20	0.38	0.47	0.47	0.49	0.49	0.41
Money (M1)	3.98	0.11	0.14	0.15	0.15	0.14	0.11	0.07	0.00	-0.06	-0.09	-0.12	-0.15
Money growth	1.00	1.00	0.08	0.09	0.05	0.03	-0.09	-0.13	-0.25	-0.19	-0.14	-0.08	-0.07
Real money	3.32	0.20	0.35	0.40	0.43	0.44	0.39	0.30	0.17	0.06	-0.04	-0.13	-0.21
Interest rate (TBill)	1.15	-0.51	-0.61	-0.50	-0.33	-0.14	0.13	0.36	0.48	0.51	0.50	0.47	0.43
Cons. velocity	2.68	-0.25	-0.24	-0.21	-0.17	-0.11	-0.02	0.07	0.14	0.18	0.18	0.18	0.17
Income velocity	3.27	-0.28	-0.38	-0.34	-0.26	-0.16	-0.01	0.15	0.23	0.27	0.27	0.26	0.23

TABLE 2
STANDARD DEVIATIONS IN PER CENT AND CORRELATIONS WITH OUTPUT OF THE SIMULATED ECONOMY (Hodrick–Prescott FILTERED SERIES)

Variable	SD %	Corr. with <i>M</i> growth	Cross-correlation of output with										
			<i>x</i> (-5)	<i>x</i> (-4)	<i>x</i> (-3)	<i>x</i> (-2)	<i>x</i> (-1)	<i>x</i>	<i>x</i> (+1)	<i>x</i> (+2)	<i>x</i> (+3)	<i>x</i> (+4)	<i>x</i> (+5)
Output	1.44	-0.01	-0.03	0.09	0.25	0.45	0.70	1.00	0.70	0.45	0.25	0.09	-0.03
Consumption	0.47	-0.25	-0.23	-0.12	0.03	0.24	0.51	0.86	0.73	0.60	0.48	0.37	0.26
Investment	4.51	0.09	0.04	0.15	0.30	0.49	0.71	0.99	0.65	0.38	0.17	0.00	-0.11
Capital	0.40	0.02	-0.45	-0.39	-0.29	-0.14	0.07	0.36	0.54	0.63	0.66	0.64	0.59
Banking hours	11.02	1.00	0.02	0.01	0.00	-0.01	-0.04	-0.06	-0.05	-0.03	-0.03	-0.02	-0.01
Share of cash	1.09	-0.92	-0.02	0.00	0.00	0.02	0.05	0.08	0.06	0.04	0.04	0.03	0.02
Real wage	0.72	-0.01	-0.12	0.00	0.16	0.38	0.65	0.98	0.74	0.54	0.37	0.22	0.10
Leisure	0.31	0.04	-0.06	-0.18	-0.32	-0.50	-0.72	-0.98	-0.62	-0.34	-0.13	0.04	0.15
Labor	0.75	-0.06	0.06	0.18	0.32	0.50	0.72	0.98	0.62	0.34	0.13	-0.04	-0.15
Prices	2.79	0.59	0.05	0.03	0.01	-0.03	-0.08	-0.16	-0.13	-0.11	-0.09	-0.08	-0.07
Inflation	2.00	0.84	0.00	-0.03	-0.03	-0.05	-0.08	-0.09	0.03	0.03	0.02	0.02	0.02
Real return	0.05	-0.04	0.10	0.21	0.35	0.52	0.72	0.96	0.59	0.29	0.07	-0.09	-0.20
Money	2.33	0.23	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.00	-0.01	-0.02
Money growth	1.06	1.00	0.02	0.01	0.02	0.01	0.00	-0.01	-0.02	-0.01	-0.02	-0.02	-0.02
Real money	1.31	-0.87	-0.10	-0.04	0.01	0.10	0.22	0.37	0.31	0.25	0.21	0.16	0.11
Interest rate	2.00	0.84	0.00	-0.02	-0.02	-0.03	-0.06	-0.07	0.05	0.04	0.02	0.02	0.02
Wage rate	0.68	-0.01	-0.14	-0.02	0.15	0.36	0.63	0.97	0.75	0.55	0.39	0.24	0.12
Cons. velocity	1.09	0.92	0.02	0.00	0.00	-0.02	-0.05	-0.08	-0.06	-0.04	-0.04	-0.03	-0.02
Income velocity	1.54	0.73	0.06	0.13	0.22	0.34	0.46	0.60	0.38	0.21	0.06	-0.05	-0.12

4.2 *Explanation of Puzzles with Simulations across Models*

The various puzzles from Cooley and Hansen (1989, 1995, 1998) and Gavin and Kydland (1999) are enumerated in Table 3 and organized into credit effects and inflation tax effects categories (Table 3). Columns 2–4 summarize the extent to which the three models, credit, cash-only and shopping time respectively, are able to explain puzzles when faced with joint productivity and money shocks. Columns 5–8 show when the credit shock is also active, applying only to the credit model.

First note that, when subject to joint productivity and money shocks, the credit model generates the procyclic monetary aggregates and the money–output phase shift, as found in the actual data. These facts are not replicated by the two alternative models with the joint shocks. This shows an advantage of the credit model using standard shocks.

Credit shocks alone (column 5) generate procyclic monetary aggregates and income velocity as well as the phase shift between money and output, as seen in the data. This simulation also replicates the procyclic inflation and nominal interest rate, with values very close to the data. The other models cannot match the data here. Column 8 presents results of the credit model with all three shocks, as in the simulations presented in Table 2. Here the inflation procyclic movement with current output is lost, but as noted above the simulation still matches the correlation of inflation with one-period-ahead output.

What emerges primarily from this comparison with the puzzles is that the credit shock can be important in explaining inflation movements. Put differently, when the economy is in a period during which the credit shock is important, such as banking deregulation, the procyclic inflation movement can be explained in this way.

5 SENSITIVITY AND ROBUSTNESS

It is important that the simulations prove robust to variations in key parameters, in particular the degree of diminishing returns in credit production, γ , the productivity shift parameter in credit production, A_F , and the inflation rate level.

For the γ values of 0.21 (the baseline calibration), 0.3, 0.5, 0.6 and 0.8, two of the most important cases are examined: the credit-shock-only case and the case when the economy is faced with all three shocks. When faced with credit shocks only, the procyclicality of monetary aggregates remains unchanged under all γ values except for the largest value 0.8. The procyclic natures of income velocity, inflation and nominal interest rate are extremely robust; the correlation coefficients remain approximately constant under all values of γ . The same robustness is found in the phase shift between output and money. When subject to all three shocks, the economy demonstrates the

TABLE 3
THE EXTENT TO WHICH PRODUCTIVITY, MONEY OR CREDIT SHOCKS CAN EXPLAIN THE MONETARY PUZZLES

(1)	(2) <i>Credit model</i>		(3) <i>CIA model</i>		(4) <i>SHT model</i>		(5)		(6) <i>Credit model with credit shocks</i>		(7) <i>CR + M shocks</i>		(8) <i>CR + PR + M shocks</i>	
	<i>PR + M shocks</i>	<i>PR + M shocks</i>	<i>PR + M shocks</i>	<i>PR + M shocks</i>	<i>PR + M shocks</i>	<i>PR + M shocks</i>	<i>CR shock</i>	<i>PR + CR shocks</i>	<i>CR + M shocks</i>	<i>CR + PR + M shocks</i>				
<i>Facts and puzzles</i>														
A. Credit														
1. Monetary aggregates are procyclical (+0.07; +0.33 in CH, 1995)	Yes	No	No	No	Yes	Yes	YES	YES	No	Yes	No	Yes	Yes	Yes
2. Phase shift in the correlation between output and money = lagged money is correlated with present output	0.03	-0.04	0	0	0.26	0.05	0.05	0.05	-0.05	0.01	-0.05	0.01	0.01	0.01
3. Positive correlation between output and inflation (+0.38)	Yes	No	No	No	YES	No	YES	No	Yes	Yes	Yes	Yes	Yes	Yes
4. Positive correlation between output and nominal interest rate (+0.35)	No	No	No	No	YES	No	YES	No	No	No	No	No	No	No
	-0.11	-0.12	-0.08	-0.08	0.39	0.39	0.39	0.39	-0.79	-0.09	-0.79	-0.09	-0.09	-0.09
	No	No	No	No	YES	No	YES	No	No	No	No	No	No	No
	-0.08	-0.09	-0.05	-0.05	0.39	-0.30	-0.30	-0.30	-0.79	-0.07	-0.79	-0.07	-0.07	-0.07
B. Inflation tax														
1. Income velocity is procyclical (+0.15)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	YES	No	YES	YES	YES
	0.63	0.88	0.61	0.61	1	0.90	0.90	0.90	-0.73	0.60	-0.73	0.60	0.60	0.60
2. Negative correlation between money growth and output (-0.13)	YES	YES	Yes	Yes	N/A	N/A	N/A	N/A	Yes	Yes	Yes	Yes	Yes	Yes
	-0.03	-0.05	-0.02	-0.02					-0.91	-0.04	-0.91	-0.04	-0.04	-0.04
3. Negative correlation between money growth and hours (-0.15 in CH, 1995)	YES	YES	Yes	Yes	N/A	N/A	N/A	N/A	N/A	Yes	N/A	Yes	Yes	Yes
	-0.05	-0.10	-0.03	-0.03					-0.91	-0.06	-0.91	-0.06	-0.06	-0.06
4. Negative correlation between money growth and consumption (-0.1 in CH (1998) but 0.02 in CH (1995); 0.01 here)	Yes	Yes	YES	YES	N/A	N/A	N/A	N/A	Yes	YES	Yes	YES	YES	YES
	-0.30	-0.79	-0.08	-0.08					-0.99	-0.25	-0.99	-0.25	-0.25	-0.25
5. Negative correlation between output and prices (-0.51)	Yes	Yes	Yes	Yes	No	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	-0.15	-0.20	-0.15	-0.15	1	0.72	0.72	0.72	-0.35	-0.16	-0.35	-0.16	-0.16	-0.16

CH, Cooley and Hansen; CIA, cash-in-advance; SHT, shopping time; PR, productivity; M, money; CR, credit; N/A, not available; 'Yes' indicates that the shock explains the puzzle; 'YES' indicates the best explanatory power of the different models for a given puzzle.

same robustness. Moreover, when γ increases, the correlation coefficients of the money growth with output and hours worked move closer towards their observed values. The only exception is the correlation of output with monetary aggregates, which, at higher γ values, becomes acyclical or slightly countercyclical.

For the productivity parameters (A_T) of 0.6, 1.0, 1.4, 1.7 and 2.0, when only credit shocks operate in the economy, the model remains robust under various productivity parameters with one exception: at low productivity the nominal money supply becomes slightly countercyclical. Under joint productivity, money and credit shocks the system proves to be robust; however, just as with varying γ values, monetary aggregates display a rather acyclical pattern, although the shift in the correlation coefficient is almost negligible.

Under various inflation rates (-4, -2, 0, 2, 5, 10, 20, 100 per cent), the results are robust with all of the shock processes. The exception is the behavior of nominal money supply under credit shocks, which turns out to be procyclic only at moderate inflation rates but countercyclical at deflationary or hyperinflation rates.

6 DISCUSSION

The impulse responses show that the shopping time model has differences such as its leisure decrease when the money supply growth rate is shocked upwards. This feature is not found in the other two models and it appears to be related to the assumption of its exchange time moving proportionally with velocity. This may create a lesser performance of the shopping time model to explain the inflation tax puzzles. For example, the credit model with goods productivity and money shocks seems better at explaining procyclic monetary aggregates.

However, the performance differences among the three models are somewhat marginal in comparison with the advantage of having the additional credit shock in the credit model. This gives the procyclic aggregate movements found in the data and can generate procyclic inflation rate movements. A related type of shopping time shock can be added to the shopping time framework, as Dittmar *et al.* (2005) show, but this has less intuition in that the specification of the shopping time function is not linked to any microfoundations other than a fixed interest elasticity of money demand. The advantage of the credit model is that the additional credit productivity shock helps to capture substitution away from money use during important financial sector innovation periods, and to generate income effects in terms of saved time in banking.

The inflation movements are not persistent in the credit model, however, when using the simple money supply growth rule, and this makes the overall model's performance with all three shocks still inconsistent with observed inflation–output contemporaneous correlation. But since the credit-shock-

only model gives the right magnitude and positive sign for the inflation correlation, an increase in inflation persistence such as from a Taylor feedback rule as in Dittmar *et al.* (2005) may lead to overall improvement. Another area for improvement in the model is liquidity effects. Cooley and Hansen (1995, 1998) modify cash-in-advance economies with nominal rigidities and the non-neutralities so introduced cause larger velocity and interest rate volatility that are closer to the facts. However, the inflation tax models of Section 2 better fit for example the negative correlation between current output and the price level. And the nominal rigidity models poorly explain real variable movements, and do not capture money growth, inflation and interest rate correlations. A credit approach may still be useful for the liquidity problem if cash transfers can be injected first into the credit sector with a subsequent increase in the supply of credit before the inflation rate increases.

7 CONCLUSION

The paper analyzes three different models of exchange technology within a business cycle framework. The first two are the standard cash-only and shopping time models and the third is a credit model that is a stochastic version of the Gillman and Kejak (2005b) economy. The credit model allows for an additional shock to the usual goods productivity and money shocks. It is found that this addition allows the co-movement of monetary aggregates, inflation and the nominal interest rate with output at different points in the phase of the business cycle to be captured better than other models. Impulse responses confirm this feature in the credit model that is not available in the cash-only and standard shopping time models. The paper thus is able to argue that the credit production approach is an extension that, based in a microfoundations-linked calibration, improves the performance of the monetary business cycle model. The contribution represents a step that allows the general equilibrium business cycle to account for important changes in banking and for the more standard inflation tax effects.

APPENDIX

A1 First-order Conditions and Log-linearization

The first-order conditions with respect to c_t , x_t , k_t , a_t , M_t are

$$\frac{1}{c_t} - \lambda_t a_t - \mu_t w_t \left(\frac{1-a_t}{A_F e^{v_t}} \right)^{1/\gamma} - \mu_t = 0 \tag{A1}$$

$$\frac{\Psi}{x_t} - \mu_t w_t = 0 \tag{A2}$$

$$-\mu_t + \beta E_t(\mu_{t+1} r_{t+1}) = 0 \tag{A3}$$

$$-\lambda_t c_t + \mu_t w_t c_t \frac{1}{\gamma A_F e^{v_t}} \left(\frac{1-a_t}{A_F e^{v_t}} \right)^{(1/\gamma)-1} = 0 \tag{A4}$$

$$\frac{-\mu_t}{P_t} + \beta E_t \left(\frac{\lambda_{t+1} + \mu_{t+1}}{P_{t+1}} \right) = 0 \tag{A5}$$

These can be simplified to

$$R^* - 1 = \frac{w^*}{\gamma A_F} \left(\frac{1-a^*}{A_F} \right)^{(1/\gamma)-1} \tag{A6}$$

$$\frac{x_t}{\Psi c_t} = \frac{1 + a^*(R^* - 1) + w^*(1 - a^*/A_F)^{1/\gamma}}{w^*} \tag{A7}$$

$$r^* = \frac{1}{\beta} \tag{A8}$$

The log-linearized system of equilibrium conditions includes the consumer's first-order conditions

$$(\lambda^* a^* c^* + \mu^* c^*) \hat{c}_t + \lambda^* a^* c^* \hat{a}_t + \mu^* w^* l_F^* \hat{w}_t + \mu^* w^* l_F^* \hat{l}_F + \lambda^* a^* c^* \hat{\lambda}_t + (\mu^* w^* l_F^* + \mu^* c^*) \hat{\mu}_t = 0 \tag{A9}$$

$$\hat{x}_t + \hat{\mu}_t + \hat{w}_t = 0 \tag{A10}$$

$$-\hat{\mu}_t + E_t \hat{\mu}_{t+1} + E_t \hat{r}_{t+1} = 0 \tag{A11}$$

$$-\hat{\lambda}_t + \hat{\mu}_t + \hat{w}_t + (1 - \gamma) \hat{l}_F - (1 - \gamma) \hat{c}_t - v_t = 0 \tag{A12}$$

$$-\hat{\mu}_t + \hat{p}_t + E_t \left(\frac{\lambda^*}{\lambda^* + \mu^*} \hat{\lambda}_{t+1} + \frac{\mu^*}{\lambda^* + \mu^*} \hat{\mu}_{t+1} - \hat{p}_{t+1} - u_{t+1} \right) = 0 \tag{A13}$$

the firm's equilibrium conditions

$$-\hat{w}_t + z_t + \alpha \hat{k}_{t-1} - \alpha \hat{n}_t = 0 \tag{A14}$$

$$-\hat{r}_t + [1 - \beta(1 - \delta)]z_t + (\alpha - 1)[1 - \beta(1 - \delta)]\hat{k}_{t-1} + (1 - \alpha)[1 - \beta(1 - \delta)]\hat{n}_t = 0 \tag{A15}$$

$$-\hat{y}_t + z_t + \alpha \hat{k}_{t-1} + (1 - \alpha)\hat{n}_t = 0 \tag{A16}$$

and the resource and money market constraints

$$-\hat{l}_F + \frac{a^*}{\gamma(a^* - 1)} \hat{a}_t + \hat{c}_t - \frac{1}{\gamma} v_t = 0 \tag{A17}$$

$$l_F^* \hat{l}_F + x^* \hat{x}_t + n^* \hat{n}_t = 0 \tag{A18}$$

$$\hat{p}_t + \hat{a}_t + \hat{c}_t = 0 \tag{A19}$$

$$-w^* n^* \hat{w}_t - w^* n^* \hat{n}_t - r^* k^* \hat{r}_t - r^* k^* \hat{k}_{t-1} + c^* \hat{c}_t + k^* \hat{k}_t = 0 \tag{A20}$$

$$\hat{p}_t - \hat{p}_{t-1} - \hat{\pi}_t + u_t = 0 \tag{A21}$$

Equations (A9)–(A21), together with the three shock processes for goods productivity, money supply and credit productivity, form the complete recursive system of linear stochastic difference equations in the endogenous state variable \hat{k}_t ,

exogenous state variables z_t, v_t, u_t , endogenous control variables $\hat{c}_t, \hat{x}_t, \hat{n}_t, \hat{l}_F, \hat{a}_t, \hat{w}_t, \hat{r}_t, \hat{y}_t, \hat{p}_t, \pi_t$ and shadow prices $\hat{\lambda}_t, \hat{\mu}_t$.

A2 Calibration

	Credit	Cash only	Shopping time
α	0.36	0.36	0.36
δ	0.05	0.05	0.05
β	0.99	0.99	0.99
A_F	1.422	N/A	0.0034
Ψ	2.03	2.03	1.876
Θ	0.0125	0.0125	0.0125
γ	0.21	N/A	N/A
φ_z	0.95	0.95	0.95
σ_z	0.0075	0.0075	0.0075
φ_v	0.95	N/A	N/A
σ_v	0.0075	N/A	N/A
φ_u	0.57	0.57	0.57
σ_u	0.01	0.01	0.01
c	0.8098	0.8072	0.8463
x	0.7055	0.7069	0.6847
n	0.2940	0.2930	0.3072
l_F	0.00049	0	0.0080
a	0.7002	1	0.425
w	2.3706	2.3706	2.3706
r	1.0101	1.0101	1.0101
π	1.0125	1.0125	1.0125
y	1.0891	1.0855	1.1381
k	11.1695	11.1333	11.6725
m	0.5670	0.8072	0.3598

N/A, not available.

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