Advanced Modern Macroeconomics

International Trade

Max Gillman

Cardiff Business School

19 November 2010

- Endogenous growth with two agents, same output good;
  - different productivities in human capital investment,
  - all other parameters the same.
  - Two agents viewed as two different countries.
- Autarky: each agent in isolation
  - with different growth rates, interest rates, wages rates.
  - First agent: as in Example 12.2;
  - second agent 15% higher human capital investment productivity.
- Trade: world market clearing allowed;
  - agents adjust physical capital to human capital ratios
  - converge in growth rates, interest rates, wage rates.
  - Called: factor price equalization.
- Agents trade goods, labor.
  - First agent with lower human capital productivity
  - demands excess goods, supplies excess labor, consumes less.
  - Second agent supplies excess goods, demands excess labor.

Building on the Last Chapters

- Extends endogenous growth model by with two agents,
- Based on Example 12.2.
- Get wage equalization of Chapter 4,
  - interest rate equalization of Chapter 7,
  - plus growth rate convergence.
- Different from Chapters 4, 7 : capital state variables,
  - that vary as trade opens, factor prices equalize.
  - No simple computation of gains from trade,
  - since transition dynamics impact computation.
- Solution uses market clearing in both goods, labor markets,
  - Chapters 4, 7 can use either market clearing condition.
  - Methodology in Appendix $A12$.
- Trade Chapters 4, 7 mark end Parts 2, 3;
  - now this trade chapter marks end Part 5.

Learning Objective

- With one output good, world trade can equalize factor prices,
- only if human capital productivity factors are different.
- Endogenous growth is central to showing factor price equalization
  - interest rates, wage rates equalized across countries.
- Key: autarky to trade involves adjustment of capital ratios.
  - Drawing excess supply, demand for goods, labor,
  - requires capital ratios adjustment first.
- Standard macroeconomics within international trade.

Who Made It Happen

- Heckscher-Ohlin trade theory,
- Samuelson factor price equalization.
- Extended with dynamic endogenous growth setting.
- As in Maffezzoli 2000
  - "Human Capital and International Real Business Cycles"
  - with similar model except stochastic, with spillover.
- Maffezzoli shows good international business cycle facts
  - with two countries model, compared to data,
  - 0.97 simulated correlation savings/investment within a country.
  - Supports Feldstein Horioka 1980 results.

Autarky

- Agent/country 1, agent/country 2, same log utility.
- Same parameters as Example 12.1, for country $1, 2$,
- except: $A_{1H} = 0.189$, $A_{2H} = (1.15)\,0.189 = 0.21735$.
- New $AS - AD$ now with subscripts 1 and 2 for each country.
- Different growth g_1, g_2, wages w_1, w_2, interest r_1, r_2.

Autarky Aggregate Supply, Demand Functions

$$y_{1t}^{d} = \frac{w_{1t}h_{1t}\left(1-\frac{g_1+\delta_H}{A_{1H}}\right)+k_{1t}\left[\rho\left(1+g_1\right)+\left(1+\alpha\right)\left(\delta_k+g_1\right)\right]}{1+\alpha},$$

$$y_{2t}^{d} = \frac{w_{2t}h_{2t}\left(1-\frac{g_2+\delta_H}{A_{2H}}\right)+k_{2t}\left[\rho\left(1+g_2\right)+\left(1+\alpha\right)\left(\delta_k+g_2\right)\right]}{1+\alpha};$$

$$y_{1t}^{s} = \left(A_{1G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_{1t}}\right)^{\frac{\gamma}{1-\gamma}}k_{1t},$$

$$y_{2t}^{s} = \left(A_{2G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_{2t}}\right)^{\frac{\gamma}{1-\gamma}}k_{2t}.$$

Inverted to Solve for Relative Price 1/w

$$AD_1 \quad : \quad \frac{1}{w_{1t}} = \frac{h_{1t}\left(1-\frac{g_1+\delta_H}{A_{1H}}\right)}{y_{1t}^d\left(1+\alpha\right)-k_{1t}\left[\rho\left(1+g_1\right)+\left(1+\alpha\right)\left(\delta_k+g_1\right)\right]},$$

$$AD_2 \quad : \quad \frac{1}{w_{2t}} = \frac{h_{2t}\left(1-\frac{g_2+\delta_H}{A_{2H}}\right)}{y_{2t}^d\left(1+\alpha\right)-k_{2t}\left[\rho\left(1+g_2\right)+\left(1+\alpha\right)\left(\delta_k+g_2\right)\right]};$$

$$AS_1 \quad : \quad \frac{1}{w_{1t}} = \frac{1}{\gamma\left(A_{1G}\right)^{\frac{1}{\gamma}}}\left(\frac{y_{1t}^s}{k_{1t}}\right)^{\frac{1-\gamma}{\gamma}},$$

$$AS_2 \quad : \quad \frac{1}{w_{2t}} = \frac{1}{\gamma\left(A_{2G}\right)^{\frac{1}{\gamma}}}\left(\frac{y_{2t}^s}{k_{2t}}\right)^{\frac{1-\gamma}{\gamma}}.$$

Example 14. Autarky

- $\alpha = 1$, $\delta_k = 0.05$, $\delta_h = 0.015$, $\beta = \frac{1}{1+\rho} = 0.95$, $\rho = \left(\frac{1}{0.95} - 1\right) = 0.052632$, $A_{1G} = A_{2G} = 0.282\,24$; $A_{1H} = 0.189$, $A_{2H} = (0.189)(1.15) = (0.217\,35)$.
- $g_1 = 0.02$, and $\frac{k_{1t}}{h_{1t}} = 1$; $g_2 = 0.05271$ and $\frac{k_{2t}}{h_{2t}} = 0.429\,84$.
- Excess output demand function for country 2 :

$$
\begin{aligned}
0 &= Y\left(w_{2t}, g_2; \frac{k_{2t}}{h_{2t}}\right) = y_{2t}^d - y_{2t}^s = \\
&= \frac{w_{2t}h_{2t}\left(1 - \frac{g_2+\delta_H}{A_{2H}}\right) + k_{2t}\left[\rho\left(1+g_2\right) + (1+\alpha)\left(\delta_k + g_2\right)\right]}{1+\alpha} \\
&\quad - \left(A_{2G}\right)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_{2t}}\right)^{\frac{\gamma}{1-\gamma}} k_{2t};
\end{aligned}
$$

Appendix A12 Solution for Country 2 Growth Rate

$$\begin{aligned} Y(g_2) &= (1+g_2+\beta(\delta_k-1))\gamma[(A_{2H}-(g_2+\delta_H))] \\ &+(1+g_2)(1-\beta)(1-\gamma)[\rho(1+g_2)+(1+\alpha)(\delta_k+g_2)] \\ &-(1+\alpha)(1+g_2)\rho(1+g_2+\beta(\delta_k-1))=0 \end{aligned}$$

Calibrated as

$$\begin{aligned} & Y(g_2) \\ = & \frac{(0.95)}{3}(1+g_2+0.95(0.05-1))[(0.217)-(0.95)(g_2+0.015)] \\ & +\frac{2(1+g_2)}{3}(1-0.95)[(0.05)(1+g_2)+2(0.95)(0.05+g_2)] \\ & -2(1+g_2)(1-0.95)(1+g_2+0.95(0.05-1)). \end{aligned}$$

Graphed Solution Equation for Country 2 Growth Rate

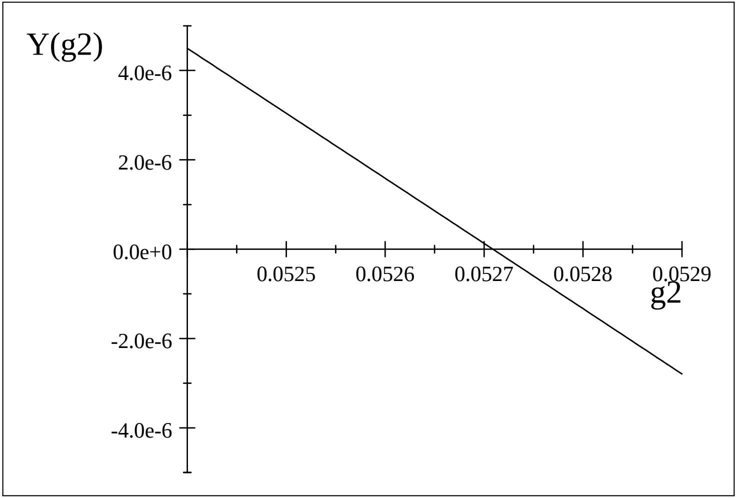


Figure 14.1. Normalized Country 2 Excess Demand as a Function of g_2 in Example 14.1.

- $Ag^2 + Bg + C = 0,$

$$\begin{aligned} \text{Quadratic term:} \quad & A = -0.3517, \\ \text{Linear term:} \quad & B = -0.0067, \\ \text{Constant term:} \quad & C = 0.0013. \\ \Longrightarrow \quad & g_2 = 0.05271. \end{aligned}$$

$$\begin{aligned} l_{2t}^{s} &= \frac{(1+g_2)(1-\beta)}{A_{2H}\beta} = \frac{(1.05271)(1-0.95)}{(0.217\,35)\,0.95} = 0.25492, \\ l_{2Ht} &= \frac{g_{2t}+\delta_H}{A_{2H}} = \frac{0.05271+0.015}{0.21735} = 0.31153, \\ \frac{k_{2t}}{h_{2t}} &= \left(\frac{\beta(1-\gamma)A_{1G}}{1+g_2+\beta(\delta_k-1)}\right)^{\frac{1}{\gamma}} l_{2t}^{d}; \;\; l_{2t}^{s} = l_{2t}^{d}. \\ \frac{k_{2t}}{h_{2t}} &= \left(\frac{(0.95)\,\frac{2}{3}\,(0.28224)}{(1.05271)+(0.95)(0.05-1)}\right)^{3} (0.2549) = 0.4296. \end{aligned}$$

Autarky AS-AD for Country 1 (Red), 2 (Blue)

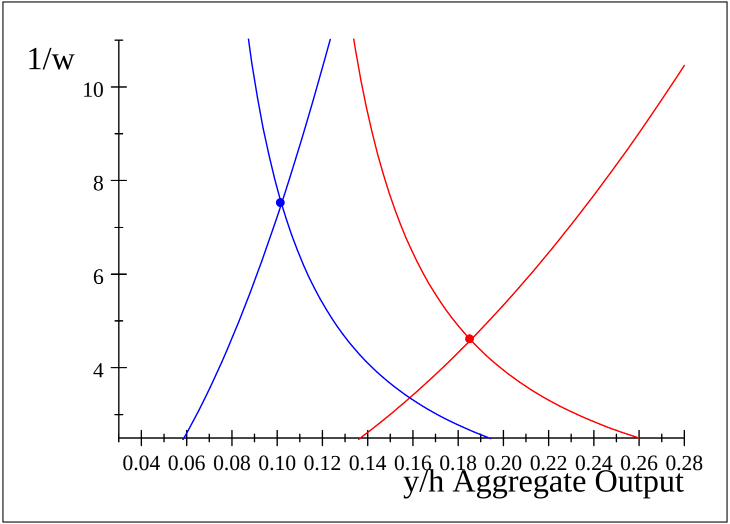


Figure 14.2. Autarky $AS - AD$; Country 1 (red) and Country 2 (blue) in Example 14.1.

Country 2 Interest Rate, Consumption/Output

$$r = \frac{1 + g + \beta\left(\delta_k - 1\right)}{\beta} = \frac{(1.05271) + 0.95\left(0.05 - 1\right)}{0.95} = 0.158.$$

$$\begin{aligned}
c_t &= \left(\frac{1}{1+\alpha}\right)\left[w_{2t}h_{2t}\left(1 - \frac{g_2 + \delta_H}{A_{2H}}\right) + k_{2t}\rho\left(1 + g_1\right)\right], \\
&= 0.5\left[0.133\left(1 - \frac{0.05271 + 0.015}{(0.21735)}\right) + 0.4296\left(0.0526\right)1.0527\right], \\
&= 0.057764. \\
y_t &= \left(A_{2G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_{2t}}\right)^{\frac{\gamma}{1-\gamma}} k_{2t} = (0.28224)^{1.5}\left(\frac{1}{3\left(0.133\right)}\right)^{0.5} 0.4296, \\
&= 0.10189.
\end{aligned}$$

$$\frac{c_t}{y_t} = \frac{0.057764}{0.10189} = 0.56693.$$

Autarky Labor Markets

$$
\begin{aligned}
l_{1t}^{s} &= 1-\frac{\alpha}{1+\alpha}\left[1+\frac{k_{1t}}{w_{1t}h_{1t}}\rho\left(1+g_{1}\right)\right]-\frac{g_{1}+\delta_{H}}{A_{1H}\left(1+\alpha\right)},\\
l_{2t}^{s} &= 1-\frac{\alpha}{1+\alpha}\left[1+\frac{k_{2t}}{w_{2t}h_{2t}}\rho\left(1+g_{2}\right)\right]-\frac{g_{2}+\delta_{H}}{A_{2H}\left(1+\alpha\right)};
\end{aligned}
$$

$$
\begin{aligned}
l_{1t}^{d} &= \left(\frac{\gamma A_{1G}}{w_{t}}\right)^{\frac{1}{1-\gamma}}k_{1t},\\
l_{2t}^{d} &= \left(\frac{\gamma A_{2G}}{w_{t}}\right)^{\frac{1}{1-\gamma}}k_{2t}.
\end{aligned}
$$

Autarky Labor Markets Inversely Solved for w

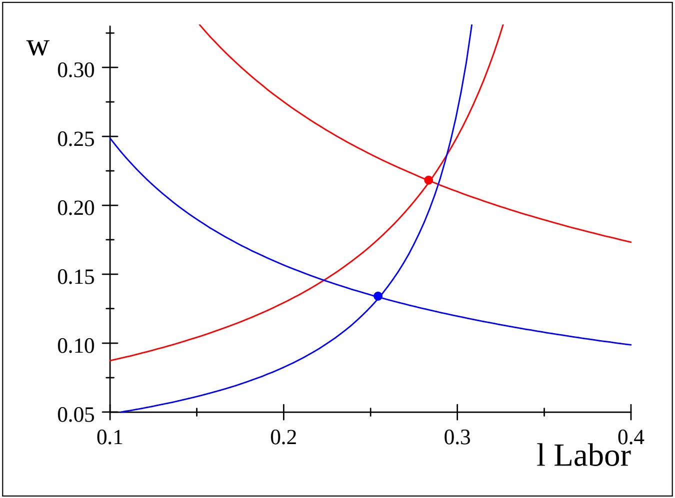
$$w_{1t} = \frac{\alpha\rho\left(1+g_1\right)\left(\frac{k_{1t}}{h_{1t}}\right)}{1-\left(1+\alpha\right)l_{1t}^{s}-\frac{\left(g_1+\delta_H\right)}{A_{1H}}} = \frac{1\left(0.0526\right)\left(1+0.02\right)1}{1-\left(1+1\right)l_{1t}^{s}-\frac{\left(0.02+0.015\right)}{0.189}},$$

$$w_{2t} = \frac{\alpha\rho\left(1+g_2\right)\left(\frac{k_{t}}{h_{t}}\right)}{1-\left(1+\alpha\right)l_{2t}^{s}-\frac{\left(g_2+\delta_H\right)}{A_{2H}}} = \frac{1\left(0.0526\right)\left(1.05271\right)\left(0.43\right)}{1-\left(1+1\right)l_{2t}^{s}-\frac{\left(0.05268+0.015\right)}{0.21735}};$$

$$w_{1t} = \gamma A_{1G}\left(\frac{k_{1t}}{h_{1t}l_{1t}^{d}}\right)^{1-\gamma} = \frac{\left(0.28224\right)}{3}\left(\frac{1}{l_{1t}^{d}}\right)^{\frac{2}{3}},$$

$$w_{2t} = \gamma A_{2G}\left(\frac{k_{2t}}{h_{2t}l_{2t}^{d}}\right)^{1-\gamma} = \frac{\left(0.28224\right)}{3}\left(\frac{0.42984}{l_{2t}^{d}}\right)^{\frac{2}{3}}.$$

Country 2 Wage Rate is 0.133



Fitgure 14.3. Labor Market with Autarky of Country 1 (red) and 2 (blue) in Example 14.1.

Trade: World Market Clearing, Single Growth Rate

$$y_{1t}^d + y_{2t}^d = y_{1t}^s + y_{2t}^s.$$

$$0 = Y\left(w_t, g; k_{1t}, k_{2t}, h_{1t}, h_{2t}\right) = y_{1t}^d + y_{2t}^d - y_{1t}^s - y_{2t}^s.$$

$$\begin{aligned} & Y\left(w_t, g; k_{1t}, k_{2t}, h_{1t}, h_{2t}\right) \\ = \quad & \frac{w_t h_{1t}\left(1 - \frac{g+\delta_H}{A_{1H}}\right) + k_{1t}\left[\rho\left(1+g\right) + \left(1+\alpha\right)\left(\delta_k + g\right)\right]}{1+\alpha} \\ & + \frac{w_t h_{2t}\left(1 - \frac{g+\delta_H}{A_{2H}}\right) + k_{2t}\left[\rho\left(1+g\right) + \left(1+\alpha\right)\left(\delta_k + g\right)\right]}{1+\alpha} \\ & - \left(A_{1G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_{1t} - A_{2G}^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_{2t}. \end{aligned}$$

Trade Solution Methodology

Divide by h_{1t} and write $\frac{k_{2t}}{h_{1t}}$ as $\frac{k_{2t}}{k_{1t}}\frac{k_{1t}}{h_{1t}}$:

$$\begin{aligned}
& \frac{Y\left(w_t, g; k_{1t}, k_{2t}, h_{1t}, h_{2t}\right)}{h_{1t}} \\
= \quad & \frac{w_t\left(1-\frac{g+\delta_H}{A_{1H}}\right)+\frac{k_{1t}}{h_{1t}}\left[\rho\left(1+g\right)+\left(1+\alpha\right)\left(\delta_k+g\right)\right]}{1+\alpha} \\
& +\frac{w_t\frac{h_{2t}}{h_{1t}}\left(1-\frac{g+\delta_H}{A_{2H}}\right)+\frac{k_{2t}}{k_{1t}}\frac{k_{1t}}{h_{1t}}\left[\rho\left(1+g\right)+\left(1+\alpha\right)\left(\delta_k+g\right)\right]}{1+\alpha} \\
& -\left(A_{1G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}}\frac{k_{1t}}{h_{1t}}-\left(A_{2G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}}\frac{k_{2t}}{k_{1t}}\frac{k_{1t}}{h_{1t}}.
\end{aligned}$$

Rearranging Excess Output Demand

- Normalize $\frac{h_{2t}}{h_{1t}} = 1$, substitute in that $A_{1G} = A_{2G}$:

$$\frac{Y\left(w_t, g; k_{1t}, k_{2t}, h_{1t}\right)}{h_{1t}}$$
$$= \left(\frac{1}{1+\alpha}\right) w_t \left(2 - \frac{\left(A_{1H}+A_{2H}\right)\left(g+\delta_H\right)}{A_{1H}A_{2H}}\right)$$
$$+ \left(\frac{1}{1+\alpha}\right)\left[\rho\left(1+g\right)+\left(1+\alpha\right)\left(\delta_k+g\right)\right]\frac{k_{1t}}{h_{1t}}\left(1+\frac{k_{2t}}{k_{1t}}\right)$$
$$- \left(A_{1G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}}\frac{k_{1t}}{h_{1t}}\left(1+\frac{k_{2t}}{k_{1t}}\right) = 0.$$

- Implies solution in terms of
  - g, w_t, and
  - $\frac{k_{1t}}{h_{1t}}\left(1+\frac{k_{2t}}{k_{1t}}\right)$.

Solve for r, Capital Ratio, w, in Terms of g

$$
\begin{aligned}
1+g &= \frac{1+r_t-\delta_k}{1+\rho} = \beta\left(1+r_t-\delta_k\right),\ r_t = \frac{1+g+\beta\left(\delta_k-1\right)}{\beta}; \\
r_t &= \left(1-\gamma\right)A_{1G}\left(\frac{k_{1t}}{h_{1t}l_{1t}^d}\right)^{-\gamma},\ \frac{k_{1t}}{h_{1t}l_{1t}^d} = \left(\frac{\left(1-\gamma\right)A_{1G}}{r_t}\right)^{\frac{1}{\gamma}}, \\
\Longrightarrow &\ \frac{k_{1t}}{h_{1t}l_{1t}^d} = \left(\frac{\beta\left(1-\gamma\right)A_{1G}}{1+g+\beta\left(\delta_k-1\right)}\right)^{\frac{1}{\gamma}}.
\end{aligned}
$$

$$
w_t = \gamma A_{1G}\left(\frac{k_{1t}}{h_{1t}l_{1t}^d}\right)^{1-\gamma} = \gamma A_{1G}\left(\frac{\beta\left(1-\gamma\right)A_{1G}}{1+g+\beta\left(\delta_k-1\right)}\right)^{\frac{1-\gamma}{\gamma}}.
$$

Solve for Time Allocations in Terms of g

$$
\begin{aligned}
1+g &= 1+A_{1H}l_{1Ht}-\delta_H = 1+A_{2H}l_{2Ht}-\delta_H; \\
l_{1Ht} &= \frac{g+\delta_H}{A_{1H}},\ l_{2Ht} = \frac{g+\delta_H}{A_{2H}}, \\
1-x_{1t}-l_{1Ht} &= l^s_{1t};\ 1-x_{2t}-l_{2Ht} = l^s_{2t}.
\end{aligned}
$$

$$
1+g = \beta\left[1+A_{1H}\left(1-x_{1t}\right)-\delta_H\right] = \beta\left[1+A_{2H}\left(1-x_{2t}\right)-\delta_H\right],
$$

$$
\begin{aligned}
1-x_{1t} &= \frac{\left(1-\delta_H\right)\left(1-\beta\right)+g+\delta_H}{A_{1H}\beta}, \\
1-x_{2t} &= \frac{\left(1-\delta_H\right)\left(1-\beta\right)+g+\delta_H}{A_{2H}\beta}; \\
1-x_{1t}-l_{1Ht} &= l^s_{1t} = \frac{\left(1+g\right)\left(1-\beta\right)}{A_{1H}\beta}, \\
1-x_{2t}-l_{2Ht} &= l^s_{2t} = \frac{\left(1+g\right)\left(1-\beta\right)}{A_{2H}\beta}.
\end{aligned}
$$

Solve for Labor Supply, Demand in Terms of g

$$
\begin{aligned}
l_{1t}^{d} + l_{2t}^{d} &= l_t = l_{1t}^{s} + l_{2t}^{s} = \frac{(1+g)(1-\beta)}{A_{1H}\beta} + \frac{(1+g)(1-\beta)}{A_{2H}\beta}, \\
l_t &= \frac{(A_{1H} + A_{2H})(1+g)(1-\beta)}{A_{1H}A_{2H}\beta}.
\end{aligned}
$$

$$
\begin{aligned}
l_t &= l_{1t}^{d} + l_{2t}^{d} \\
&= \left(\frac{\gamma A_{1G}}{w_t}\right)^{\frac{1}{1-\gamma}} k_{1t} + \left(\frac{\gamma A_{2G}}{w_t}\right)^{\frac{1}{1-\gamma}} k_{2t} \\
&= \left(\frac{\gamma A_{1G}}{w_t}\right)^{\frac{1}{1-\gamma}} \frac{k_{1t}}{h_{1t}} \left(1 + \left(\frac{A_{2G}}{A_{1G}}\right)^{\frac{1}{1-\gamma}} \frac{\frac{k_{2t}}{h_{2t}}}{\frac{k_{1t}}{h_{1t}}}\right) \\
&= \left(\frac{\gamma A_{1G}}{w_t}\right)^{\frac{1}{1-\gamma}} \frac{k_{1t}}{h_{1t}} \left(1 + \frac{k_{2t}}{k_{1t}}\right).
\end{aligned}
$$

Labor Supply, Demand, Imply Capital Ratio

$$
\begin{aligned}
l_t &= \frac{(A_{1H}+A_{2H})(1+g)(1-\beta)}{A_{1H}A_{2H}\beta} \\
&= \left(\frac{1}{\left(\frac{\beta(1-\gamma)A_{1G}}{1+g+\beta(\delta_k-1)}\right)^{\frac{1-\gamma}{\gamma}}} \right)^{\frac{1}{1-\gamma}} \frac{k_{1t}}{h_{1t}}\left(1+\frac{k_{2t}}{k_{1t}}\right).
\end{aligned}
$$

- Now have used goods and labor market clearing
  - to solve for capital ratio term
  - to be substituted back into excess output demand:

$$
\begin{aligned}
& \frac{k_{1t}}{h_{1t}}\left(1+\frac{k_{2t}}{k_{1t}}\right) \\
&= \left(\frac{\beta(1-\gamma)A_{1G}}{1+g+\beta(\delta_k-1)}\right)^{\frac{1}{\gamma}} \frac{(A_{1H}+A_{2H})(1+g)(1-\beta)}{A_{1H}A_{2H}\beta}.
\end{aligned}
$$

Solve for Excess Output Demand in Terms of g

$$
\begin{aligned}
& Y(g) \\
= \quad & \left(\frac{\beta(1-\gamma)A_{1G}}{1+g+\beta(\delta_k-1)}\right)^{\frac{1}{\gamma}} \circ \\
& \left\{\frac{\gamma A_{1G}}{1+\alpha}\frac{1+g+\beta(\delta_k-1)}{\beta(1-\gamma)A_{1G}}\left(2-\frac{(A_{1H}+A_{2H})(g+\delta_H)}{A_{1H}A_{2H}}\right)\right. \\
& +\frac{[\rho(1+g)+(1+\alpha)(\delta_k+g)]}{1+\alpha}\frac{(A_{1H}+A_{2H})(1+g)(1-\beta)}{A_{1H}A_{2H}\beta} \\
& \left.-\left(\frac{(A_{1G})^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}}}{(\gamma A_{1G})^{\frac{\gamma}{1-\gamma}}\left(\frac{\beta(1-\gamma)A_{1G}}{1+g+\beta(\delta_k-1)}\right)}\right)\frac{(A_{1H}+A_{2H})(1+g)(1-\beta)}{A_{1H}A_{2H}\beta}\right\}.
\end{aligned}
$$

Single Solution Equation in g

- World excess output demand equation
- reduces to a single equation in g
- that can be solved analytically
- or graphically given a full calibration :

$$\begin{aligned} 0 &= (1+\alpha)\left[1+g+\beta\left(\delta_k-1\right)\right](1+g)(1-\beta) \\ &\quad -\gamma\beta\left(\frac{2A_{1H}A_{2H}}{\left(A_{1H}+A_{2H}\right)}-\left(g+\delta_H\right)\right)\left[1+g+\beta\left(\delta_k-1\right)\right] \\ &\quad -\left[(1-\beta)(1+g)+\beta(1+\alpha)\left(\delta_k+g\right)\right](1+g)(1-\beta)(1-\gamma) \end{aligned}$$

Analytic Solution of g

$Ag^2 + Bg + C = 0$

$$g = \frac{-B - \sqrt{B^2 - 4AC}}{2A},$$

$$\begin{aligned}
\text{Quadratic Term} \quad &: \quad A \equiv -\beta\gamma + (1-\beta)(1-\gamma)(1+\alpha\beta) - (1+\alpha)(1-\beta) \\
\text{Linear Term} \quad &: \quad B \equiv -\beta\gamma\left[1 + \beta(\delta_k - 1) - \frac{2A_{1h}A_{2h}}{A_{1h}+A_{2h}} + \delta_h\right] \\
&\quad + (1-\beta)(1-\gamma)\left[1 - \beta + \beta(1+\alpha)\delta_k + (1+\alpha\beta)\right] \\
&\quad - (1+\alpha)(1-\beta)\left[2 + \beta(\delta_k - 1)\right], \\
\text{Constant Term} \quad &: \quad C \equiv \beta\gamma\left(\frac{2A_{1h}A_{2h}}{A_{1h}+A_{2h}} - \delta_h\right)\left[1 + \beta(\delta_k - 1)\right] \\
&\quad + (1-\beta)(1-\gamma)\left[1 - \beta + \beta(1+\alpha)\delta_k\right] \\
&\quad - (1+\alpha)(1-\beta)\left[1 + \beta(\delta_k - 1)\right].
\end{aligned}$$

Trade Example 14.2

- $\alpha = 1$, $\delta_k = 0.05$, $\delta_h = 0.015$, $\beta = \frac{1}{1+\rho} = 0.95$, $A_{1G} = A_{2G} = 0.282\,24$, $A_{1H} = 0.189$, $A_{2H} = 0.217\,35$.
- Solution equation give that reduces to $0 =$

$$
\begin{aligned}
&2\left(1+g+0.95\left(0.05-1\right)\right)\left(1+g\right)\left(1-0.95\right) \\
&\left(\frac{-0.95}{3}\right)\left(\frac{2\left(0.189\right)\left(0.217\,35\right)}{\left(0.189+0.217\,35\right)}-\left(g+0.015\right)\right)\left(1+g-0.9025\right) \\
&-\frac{2}{3}\left[\left(1-0.95\right)\left(1+g\right)+\left(0.95\right)2\left(0.05+g\right)\right]\left(1+g\right)\left(1-0.95\right)
\end{aligned}
$$

- $Ag^2 + Bg + C = 0$ with $g = 0.0358$:

$$
\begin{aligned}
\text{Quadratic Term:} \quad & A = -0.3517, \\
\text{Linear Term:} \quad & B = -0.0115, \\
\text{Constant Term:} \quad & C = 0.00086.
\end{aligned}
$$

Trade Example Excess Output Demand

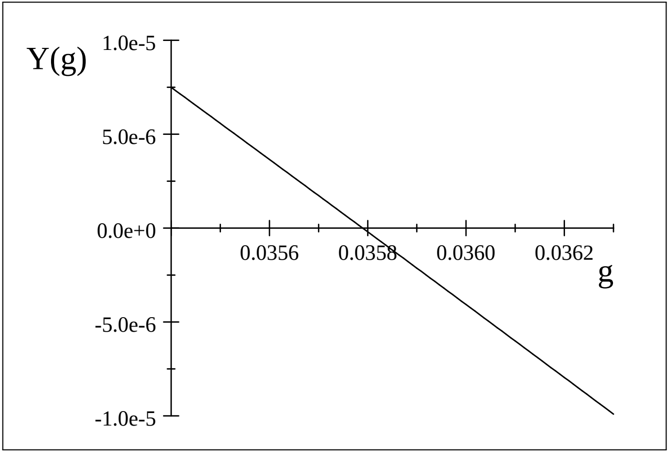


Figure 14.4. Excess World Output Demand $Y(g)$ and Equilibrium Growth g in Example 14.2.

Labor Market Equilibrium Values

$$w_t = \frac{(0.282\,24)}{3}\left(\frac{(0.95)\,\frac{2}{3}\,(0.282\,24)}{(1.0358)+(0.95)\,(0.05-1)}\right)^2 = 0.16918.$$

$$l_{1t}^s = \frac{(1+g)\,(1-\beta)}{A_{1H}\beta} = \frac{(1.0358)\,(1-0.95)}{(0.189)\,0.95} = 0.28844;$$

$$l_{2t}^s = \frac{(1+g)\,(1-\beta)}{A_{2H}\beta} = \frac{(1.0358)\,(1-0.95)}{(0.217\,35)\,0.95} = 0.25082.$$

$$\begin{aligned} l_{1t}^s &= 1-0.5\left[1+\frac{k_{1t}}{(0.16918)\,h_{1t}}(0.052632)\,(1.0358)\right]-\frac{0.0358+0.015}{(0.189)\,(2)} \\ &= 0.28844, \\ l_{2t}^s &= 1-0.5\left[1+\frac{k_{2t}}{(0.16918)\,h_{2t}}(0.052632)\,(1.0358)\right]-\frac{0.0358+0.015}{(0.217\,35)\,(2)} \\ &= 0.25082. \end{aligned}$$

Capital Ratio Equilibrium Value

- Trade causes country 1, $\frac{k_{1t}}{h_{1t}}$, to fall from 1,
- country 2, $\frac{k_{2t}}{h_{2t}}$, to rise from autarky 0.4296

$$
\begin{aligned}
\frac{k_{1t}}{h_{1t}} &= \frac{\left(2\left(1-0.28844-\left(\frac{0.0358+0.015}{(0.189)(2)}\right)\right)-1\right)(0.16918)}{(0.052632)(1.0358)} \\
&= 0.47895, \\
\frac{k_{2t}}{h_{2t}} &= \frac{\left(2\left(1-0.25082-\left(\frac{0.0358+0.015}{(0.21735)(2)}\right)\right)-1\right)(0.16918)}{(0.052632)(1.0358)} \\
&= 0.82124.
\end{aligned}
$$

Graphing AS-AD with g=0.0358

$$\frac{1}{w_t} = \frac{\left(1-\frac{0.0358+0.015}{0.189}\right)}{y_{1t}^d(1+1)-0.47895\left[\left(\frac{1}{0.95}-1\right)(1.0358)+(2)(0.0858)\right]},$$

$$\frac{1}{w_t} = \frac{\left(1-\frac{0.0358+0.015}{(0.217\,35)}\right)}{y_{2t}^d(1+1)-0.82124\left[\left(\frac{1}{0.95}-1\right)(1.0358)+(2)(0.0858)\right]};$$

$$\frac{1}{w_t} = \frac{3}{(0.282\,24)^3}\left(\frac{y_{1t}^s}{0.47895}\right)^2,$$

$$\frac{1}{w_t} = \frac{3}{(0.282\,24)^3}\left(\frac{y_{2t}^s}{0.82124}\right)^2.$$

Aggregate World Output Demand

$$\begin{aligned} y_t^d &= y_{1t}^d + y_{2t}^d = \frac{w}{1+\alpha}\left[2-(g+\delta_H)\left(\frac{1}{A_{1H}}+\frac{1}{A_{2H}}\right)\right] \\ &+ \frac{(k_{1t}+k_{2t})}{1+\alpha}\left[\rho(1+g)+(1+\alpha)(\delta_k+g)\right]; \end{aligned}$$

Inversely

$$\begin{aligned} \frac{1}{w_t} &= \frac{2-(g+\delta_H)\left(\frac{1}{A_{1H}}+\frac{1}{A_{2H}}\right)}{\left(y_t^d\right)(1+\alpha)-(k_{1t}+k_{2t})\left[\rho(1+g)+(1+\alpha)(\delta_k+g)\right]} \\ &= \frac{2-(0.0358+0.015)\left(\frac{1}{0.189}+\frac{1}{(0.217\,35)}\right)}{\left(y_{1t}^d+y_{2t}^d\right)(2)-(1.299)\left(0.0526(1.0358)+2(0.0858)\right)} \end{aligned}$$

Aggregate World Output Supply

$$y_t^s = y_{1t}^s + y_{2t}^s = (A_{1G})^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_{1t} + (A_{2G})^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_{2t}.$$

Inversely

$$\frac{1}{w_t} = \frac{\left(y_t^s\right)^{\frac{1-\gamma}{\gamma}}}{\gamma \left(A_{1G}\right)^{\frac{1}{\gamma}} k_{1t}^{\frac{1-\gamma}{\gamma}} \left(1 + \left(\frac{A_{2G}}{A_{1G}}\right)^{\frac{1}{1-\gamma}} \frac{k_{2t}}{k_{1t}}\right)^{\frac{1-\gamma}{\gamma}}},$$

$$= \frac{3\left(y_t^s\right)^2}{(0.282\,24)^3 (0.47895)^2 \left(1 + \frac{0.82124}{0.47895}\right)^2}.$$

Graphing Aggregate World Goods Market

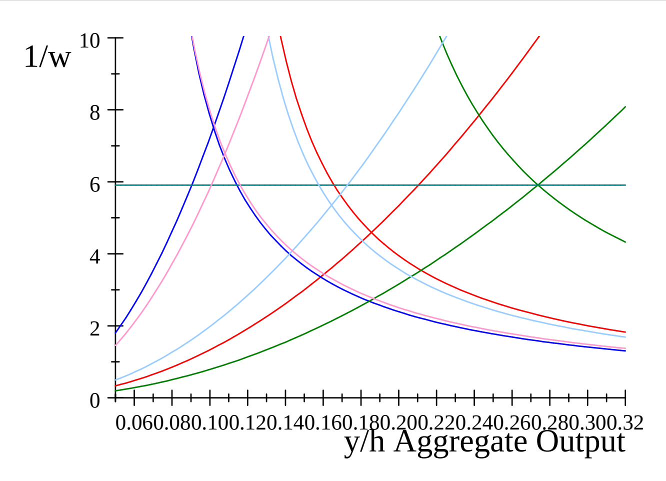


Figure 14.5. $AS - AD$ under Autarky (dark red and blue) and Trade (light red and blue), with World Equilibrium (green) in Example 14.2.

Trade Flows

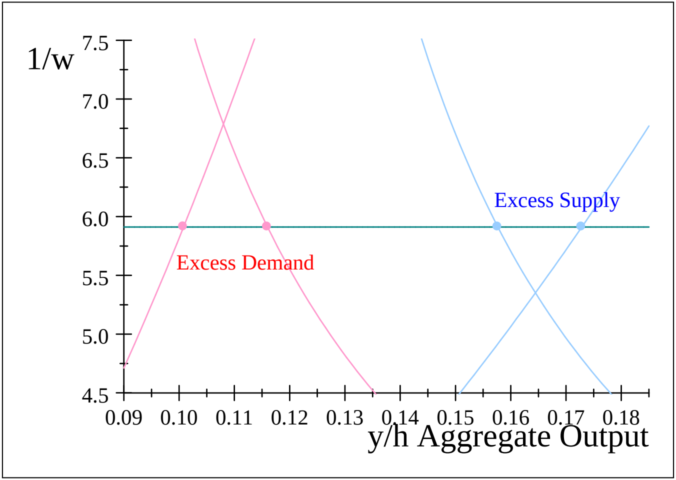


Figure 14.6. $AS - AD$ Output Trade Flow with Excess Demand and Excess Supply in Example 14.2.

Excess Output Demand Trade Flow Computed

$$
\begin{aligned}
y_{1t}^{d} &= \frac{w_t h_{1t}\left(1-\frac{g+\delta_H}{A_{1H}}\right)+k_{1t}\left[\rho\left(1+g\right)+\left(1+\alpha\right)\left(\delta_k+g\right)\right]}{1+\alpha}, \\
&= 0.5\left(0.169\,18\right)\left(1-\frac{0.0358+0.015}{0.189}\right) \\
&\quad +0.5\left(0.47895\right)\left(\left(\frac{1}{0.95}-1\right)\left(1.0358\right)+2\left(0.0858\right)\right) \\
&= 0.116.
\end{aligned}
$$

$$
\begin{aligned}
y_{1t}^{s} &= \left(A_{1G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_{1t}, \\
&= \left(0.282\,24\right)^{1.5}\left(\frac{1}{3\left(0.16918\right)}\right)^{0.5}\left(0.47895\right) \\
&= 0.100\,81;
\end{aligned}
$$

Excess Output Supply Trade Flow Computed

$$
\begin{aligned}
y_{2t}^{d} &= \frac{w_t h_{2t}\left(1-\frac{g+\delta_H}{A_{2H}}\right)+k_{2t}\left[\rho\left(1+g\right)+\left(1+\alpha\right)\left(\delta_k+g\right)\right]}{1+\alpha}, \\
&= 0.5\left(0.169\,18\right)\left(1-\frac{0.0358+0.015}{0.21735}\right) \\
&\quad +0.5\left(0.82124\right)\left(\frac{1}{0.95}-1\right)\left(1.0358\right)+2\left(0.0858\right) \\
&= 0.15767.
\end{aligned}
$$

$$
\begin{aligned}
y_{2t}^{s} &= \left(A_{2G}\right)^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_{2t}, \\
&= \left(0.282\,24\right)^{1.5}\left(\frac{1}{3\left(0.16918\right)}\right)^{0.5}\left(0.82124\right) \\
&= 0.172\,85.
\end{aligned}
$$

$$
\textit{Excess Supply}\ \ (2): y_{2t}^{s}-y_{2t}^{d}=0.17285-0.15767=0.0152.
$$

Trade in Labor Market: World Demand

$$
\begin{aligned}
l_{1t}^d + l_{2t}^d &= 1 - \frac{\alpha}{1+\alpha}\left[1 + \frac{k_{1t}}{w_t h_{1t}}\rho\left(1+g\right)\right] - \frac{g+\delta_H}{A_{1H}\left(1+\alpha\right)} \\
&\quad +1 - \frac{\alpha}{1+\alpha}\left[1 + \frac{k_{2t}}{w_t h_{2t}}\rho\left(1+g\right)\right] - \frac{g+\delta_H}{A_{2H}\left(1+\alpha\right)} \\
&= \frac{2}{1+\alpha} - \frac{\alpha\rho\left(1+g\right)}{\left(1+\alpha\right)w_t}\left(\frac{k_{1t}}{h_{1t}} + \frac{k_{2t}}{h_{2t}}\right) - \frac{g+\delta_H}{\left(1+\alpha\right)}\left(\frac{1}{A_{1H}} + \frac{1}{A_{2H}}\right);
\end{aligned}
$$

Inversely:

$$
\begin{aligned}
w_t &= \frac{\alpha\rho\left(1+g\right)\left(\frac{k_{1t}}{h_{1t}} + \frac{k_{2t}}{h_{2t}}\right)}{\left(1+\alpha\right)\left(\frac{2}{1+\alpha} - \frac{g+\delta_H}{\left(1+\alpha\right)}\left(\frac{1}{A_{1H}} + \frac{1}{A_{2H}}\right) - \left(l_{1t}^d + l_{2t}^d\right)\right)}, \\
&= \frac{1\left(\frac{1}{0.95} - 1\right)\left(1+0.0358\right)\left(0.47895 + 0.82124\right)}{\left(1+1\right)\left(\frac{2}{1+1} - \frac{0.0358+0.015}{\left(1+1\right)}\left(\frac{1}{0.189} + \frac{1}{0.217\,35}\right) - \left(l_{1t}^d + l_{2t}^d\right)\right)}
\end{aligned}
$$

Trade in Labor Market: World Supply

$$l_{1t}^{s} + l_{2t}^{s} = \left(\frac{\gamma A_{1G}}{w_t}\right)^{\frac{1}{1-\gamma}} k_{1t} + \left(\frac{\gamma A_{2G}}{w_t}\right)^{\frac{1}{1-\gamma}} k_{2t}$$

$$= \left(\frac{\gamma}{w_t}\right)^{\frac{1}{1-\gamma}} \left((A_{1G})^{\frac{1}{1-\gamma}} k_{1t} + (A_{21G})^{\frac{1}{1-\gamma}} k_{2t}\right);$$

Inversely:

$$w_t = \left(\frac{(\gamma)^{\frac{1}{1-\gamma}} \left((A_{1G})^{\frac{1}{1-\gamma}} k_{1t} + (A_{21G})^{\frac{1}{1-\gamma}} k_{2t}\right)}{(l_{1t}^{s} + l_{2t}^{s})}\right)^{1-\gamma}$$

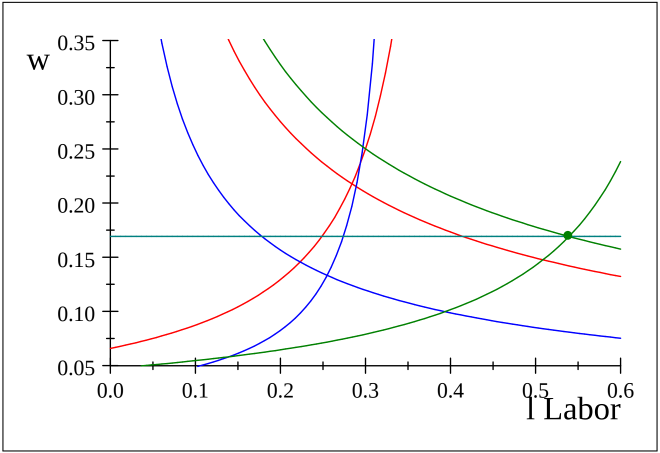
$$= \left(\frac{\left(\frac{1}{3}\right)^{1.5} \left((0.28224)^{1.5} (0.47895) + (0.282)^{1.5} (0.82124)\right)}{(l_{1t}^{s} + l_{2t}^{s})}\right)^{\frac{2}{3}}.$$

Each Country's Labor Supply, Demand

$$
\begin{aligned}
w_t &= \frac{\alpha\rho(1+g)\left(\frac{k_{1t}}{h_{1t}}\right)}{1-(1+\alpha)l_{1t}^s-\frac{(g+\delta_H)}{A_{1H}}} = \frac{1(0.052632)(1.036)(0.47895)}{1-(1+1)l_{1t}^s-\frac{(0.036+0.015)}{0.189}}, \\
w_t &= \frac{\alpha\rho(1+g)\left(\frac{k_t}{h_t}\right)}{1-(1+\alpha)l_{2t}^s-\frac{(g+\delta_H)}{A_{2H}}} = \frac{1(0.052632)(1.036)(0.82124)}{1-(1+1)l_{2t}^s-\frac{(0.036+0.015)}{0.217\,35}};
\end{aligned}
$$

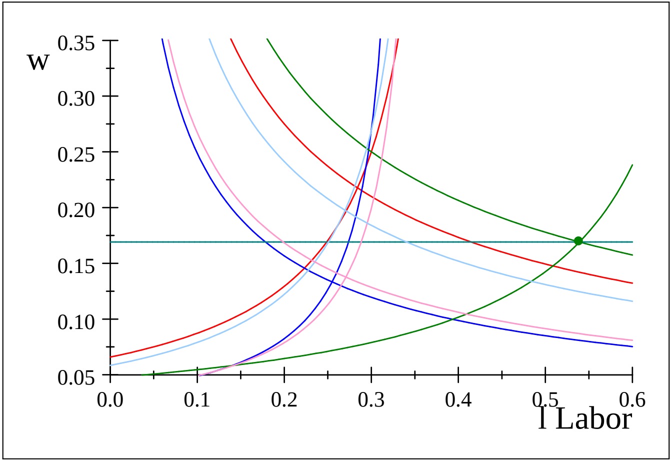
$$
\begin{aligned}
w_t &= \gamma A_{1G}\left(\frac{k_{1t}}{h_{1t}l_{1t}^d}\right)^{1-\gamma} = \frac{(0.282\,24)}{3}\left(\frac{0.47895}{l_{1t}^d}\right)^{\frac{2}{3}}, \\
w_t &= \gamma A_{2G}\left(\frac{k_{2t}}{h_{2t}l_{2t}^d}\right)^{1-\gamma} = \frac{(0.282\,24)}{3}\left(\frac{0.82124}{l_{2t}^d}\right)^{\frac{2}{3}}.
\end{aligned}
$$

Autarky Labor Market plus World Supply, Demand



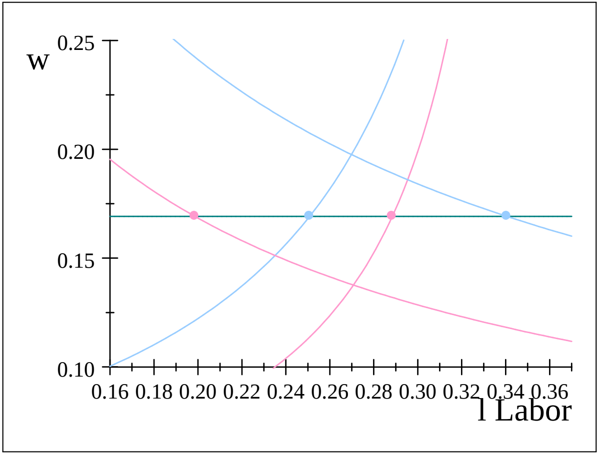
Fitgure 14.7. Labor Market with Autarky (red and blue) and World Equilibrium (green).

Labor Market with Trade: Light Red, Blue



Fitgure 14.8. Labor Market with Autarky (dark red and blue), Trade (light red and light blue), and World Equilibrium (green) in Example 14.2.

Focus on Excess Labor Supply and Demand



Fitgure 14.9. Labor Market with Trade (light red and light blue), and Equilibrium Wage (green) in Example 14.2.

Excess Labor Supply of Country 1

$$
\begin{aligned}
l_{1t}^{s} &= 1 - \frac{\alpha}{1+\alpha}\left[1 + \frac{k_{1t}}{w_t h_{1t}}\rho\left(1+g\right)\right] - \frac{g+\delta_H}{A_{1H}\left(1+\alpha\right)}, \\
&= 1 - 0.5\left(1 + \frac{0.47895}{(0.16918)}\left(0.052632\right)\left(1.0358\right)\right) - \frac{0.0508}{0.189\left(2\right)} \\
&= 0.28844; \\
l_{1t}^{d} &= \left(\frac{\gamma A_{1G}}{w_t}\right)^{\frac{1}{1-\gamma}} k_{1t} = \left(\frac{(0.28224)}{3\left(0.16918\right)}\right)^{1.5} 0.47895 \\
&= 0.19862,
\end{aligned}
$$

$$
Excess\ Supply\ \left(1\right) : l_{1t}^{s} - l_{1t}^{d} = 0.28844 - 0.19862 = 0.09,
$$

Excess Labor Demand of Country 2

$$
\begin{aligned}
l_{2t}^{d} &= \left(\frac{\gamma A_{2G}}{w_t}\right)^{\frac{1}{1-\gamma}} k_{2t} = \left(\frac{(0.28224)}{3\,(0.16918)}\right)^{1.5} 0.82124 \\
&= 0.34056. \\
l_{2t}^{s} &= 1 - \frac{\alpha}{1+\alpha}\left[1 + \frac{k_{2t}}{w_t h_{2t}}\rho\,(1+g)\right] - \frac{g+\delta_H}{A_{2H}\,(1+\alpha)}, \\
&= 1 - 0.5\left(1 + \frac{0.82124}{(0.16918)}\,(0.052632)\,(1.0358)\right) - \frac{0.0508}{0.21735\,(2)} \\
&= 0.25082;
\end{aligned}
$$

$$
\textit{Excess Demand}\ \ (2) : l_{2t}^{d} - l_{2t}^{s} = 0.34056 - 0.25082 = 0.09.
$$

- Country 2 demands excess labor supply from Country 1.

Factor Price Equalization: Interest Rate

- From intertemporal consumer margin,

$$r = \frac{1 + g + \beta\left(\delta_k - 1\right)}{\beta} = \frac{(1.0358) + 0.95\left(0.05 - 1\right)}{0.95} = 0.1403.$$

- Check using marginal product of capital

$$r_t = (1-\gamma) A_{1G} \left(\frac{h_{1t} l_{1t}^d}{k_{1t}}\right)^{\gamma} = \frac{2}{3}(0.282\,24)\left(\frac{0.47895}{0.19862}\right)^{-\frac{1}{3}} = 0.1403$$

$$r_t = (1-\gamma) A_{2G} \left(\frac{h_{2t} l_{2t}^d}{k_{2t}}\right)^{\gamma} = \frac{2}{3}(0.282\,24)\left(\frac{0.82124}{0.34056}\right)^{-\frac{1}{3}} = 0.1403$$

- Rises from autarky of 0.1237 in country 1,
- falls from autarky of 0.15812 in country 2.
- More human capital productive Country 2 has fall in r
  - similar to more productive country B in Chapter 7
  - having a fall in r when trade opens.

Factor Price Equalization: Wage Rate, Input Ratios

- Wage rate from marginal products:

$$w_t = \gamma A_{1G}\left(\frac{k_{1t}}{h_{1t}l_{1t}^d}\right)^{1-\gamma} = \frac{(0.282\,24)}{3}\left(\frac{0.47895}{0.19862}\right)^{1-\frac{1}{3}} = 0.169\,18,$$

$$w_t = \gamma A_{2G}\left(\frac{k_{2t}}{h_{2t}l_{2t}^d}\right)^{1-\gamma} = \frac{(0.282\,24)}{3}\left(\frac{0.82124}{0.34056}\right)^{1-\frac{1}{3}} = 0.169\,18.$$

- Capital to effective labor ratios equalization :

$$\frac{k_{1t}}{h_{1t}l_{1t}^d} = \frac{k_{2t}}{h_{2t}l_{2t}^d},$$

$$\frac{0.47895}{0.19862} = \frac{0.82124}{0.34056} = 2.4114.$$

Isocost line :

$$\begin{aligned} 0.10189 &= y_{2t}^{s} = w_{2t} l_{2t}^{d} h_{2t} + r_{2t} k_{2t}, \\ \frac{k_{2t}}{h_{2t}} &= \frac{0.101\,89}{(0.158\,12)\, h_{2t}} - \frac{(0.133\,23)\, l_{2t}^{d}}{0.158\,12}. \end{aligned}$$

Isoquant :

$$\begin{aligned} 0.10189 &= y_{2t}^{s} = A_{2G} \left(l_{2t}^{d} h_{2t} \right)^{\gamma} (k_{2t})^{1-\gamma}; \\ \frac{k_{2t}}{h_{2t}} &= \left(\frac{0.101\,89}{(0.282\,24)\, h_{2t} \left(l_{2t}^{d} \right)^{\frac{1}{3}}} \right)^{\frac{3}{2}} = \frac{\left(\frac{0.101\,89}{(0.282\,24) h_{2t}} \right)^{\frac{3}{2}}}{\left(l_{2t}^{d} \right)^{\frac{1}{2}}}. \end{aligned}$$

Factor input ratio :

$$\frac{k_{2t}}{h_{2t} l_{2t}^{d}} = \frac{0.42960}{0.25492} = 1.6852.$$

Country 2 with Lower Employment in Autarky

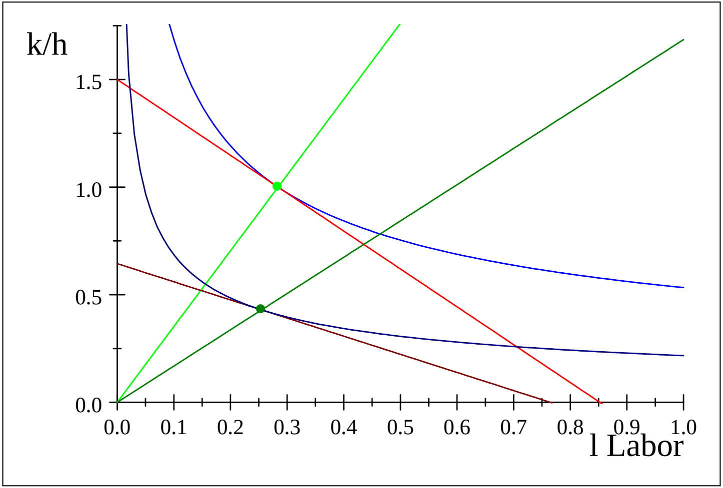


Figure 14.11. Factor Market Equilibrium with Autarky, Country 1 (lighter red, blue, green) and Country 2 (darker red, blue, green) in Example 14.1.

Country 2 Autarky Production, Utility, Budget

Production:

$$\begin{aligned} c^s_{2t} &= y^s_{2t} - i_{2t} = A_{2G}\left(l^d_{2t} h_{2t}\right)^{\gamma}(k_{2t})^{1-\gamma} - (g+\delta_k)\,k_{2t}, \\ \frac{c^s_{2t}}{h_{2t}} &= (0.282\,24)\left(l^d_{2t}\right)^{\frac{1}{3}}(0.429\,60)^{\frac{2}{3}} - (0.10271)\,(0.429\,60). \end{aligned}$$

Utility

$$\begin{aligned} u &= \ln c^d_{2t} + \alpha \ln x_{2t} = \ln c^d_{2t} + \alpha \ln\left(1 - l_{2Ht} - l^s_{2t}\right), \\ -3.6871 &= \ln 0.057764 + 1 \ln\left(1 - 0.311\,53 - 0.254\,92\right), \\ c^d_{2t} &= \frac{e^{-3.687\,1}}{\left(1 - 0.311\,53 - l^s_{2t}\right)}. \end{aligned}$$

Budget line

$$\begin{aligned} c^d_{2t} &= w_t l^s_{2t} h_{2t} + \rho\,(1+g)\,k^s_{2t}, \\ \frac{c^d_{2t}}{h_{2t}} &= (0.133\,23)\,l^s_{2t} + 0.052632\,(1.05271)\,(0.4296). \end{aligned}$$

Country 2 Autarky

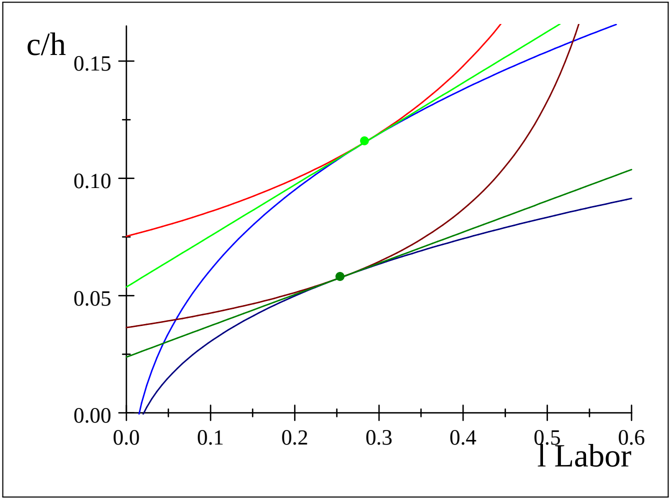


Figure 14.12. General Equilibrium Goods and Labor with Autarky of Country 1 (lighter red, blue, green), Country 2 (darker red, blue, green).

Trade Isocosts, Isoquants, Budget Lines

$$Isocost \quad : \quad \frac{k_{1t}}{h_{1t}} = \frac{0.10081}{(0.14032)\, h_{1t}} - \frac{(0.169\,18)\, l_{1t}}{0.14032};$$

$$Isocost \quad : \quad \frac{k_{2t}}{h_{2t}} = \frac{0.17285}{(0.14032)\, h_{2t}} - \frac{(0.169\,18)\, l_{2t}^{d}}{0.14032}.$$

Isoquants:

$$\frac{k_{1t}}{h_{1t}} = \left(\frac{0.10081}{(0.282\,24)\, h_{1t} \left(l_{1t}^{d}\right)^{\frac{1}{3}}} \right)^{\frac{3}{2}} = \frac{\left(\frac{0.10081}{(0.28224) h_{1t}} \right)^{\frac{3}{2}}}{\left(l_{1t}^{d}\right)^{\frac{1}{2}}};$$

$$\frac{k_{2t}}{h_{2t}} = \left(\frac{0.17285}{(0.282\,24)\, h_{2t} \left(l_{2t}^{d}\right)^{\frac{1}{3}}} \right)^{\frac{3}{2}} = \frac{\left(\frac{0.17285}{(0.28224) h_{2t}} \right)^{\frac{3}{2}}}{\left(l_{2t}^{d}\right)^{\frac{1}{2}}}.$$

Factor input ratios:

$$\frac{k_{1t}}{h_{1t}} = \frac{0.47895}{0.10862} l_{1t}^{d} = (2.411)\, l_{1t}^{d} = \frac{0.82124}{0.34056} l_{2t}^{d} = \frac{k_{2t}}{h_{2t}}.$$

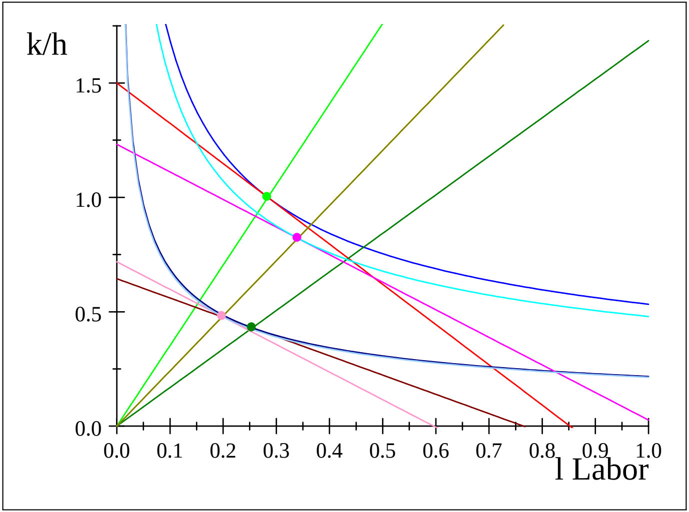


Figure 14.13. Factor Market Equilibrium with Autarky and Trade in Example 14.2.

Trade Production, Utility, Budget Lines

Production:

$$\frac{c_{1t}^{s}}{h_{1t}} = (0.282\,24)\left(l_{1t}^{d}\right)^{\frac{1}{3}}(0.47895)^{\frac{2}{3}} - ((0.0358)+0.05)(0.47895),$$
$$\frac{c_{2t}^{s}}{h_{2t}} = (0.282\,24)\left(l_{2t}^{d}\right)^{\frac{1}{3}}(0.82124)^{\frac{2}{3}} - ((0.0358)+0.05)(0.82124).$$

Utility levels:

$$c_{1t}^{d} = \frac{e^{-3.4062}}{(1-0.26878-l_{1t})},$$
$$c_{2t}^{d} = \frac{e^{-3.1022}}{(1-0.23372-l_{2t})}.$$

Budget lines:

$$\frac{c_{1t}^{d}}{h_{1t}} = (0.16918)\, l_{1t}^{s} + 0.052632\,(1+0.0358)(0.47895),$$
$$\frac{c_{2t}^{d}}{h_{2t}} = (0.16918)\, l_{2t}^{s} + 0.052632\,(1+0.0358)(0.82124).$$

Production Curve Shifts Down for 1, Up for 2

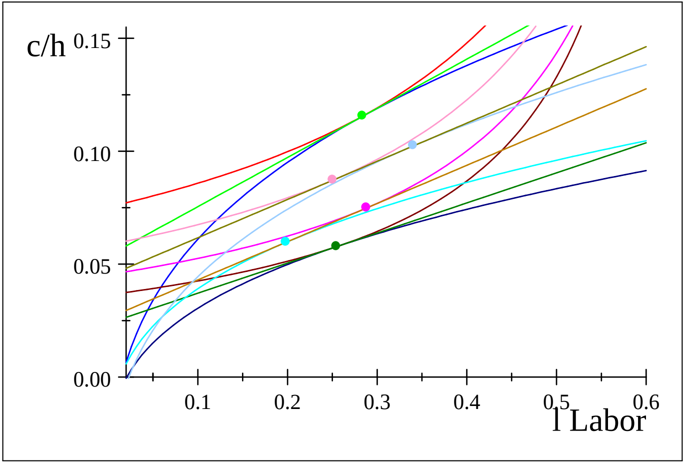


Figure 14.14. General Equilibrium with Autarky and Trade of Country 1 (upper lower-middle curves), Country 2 (lower and upper-middle curves) in Example 14.2.

Focus on Normalized Goods Trade Triangles

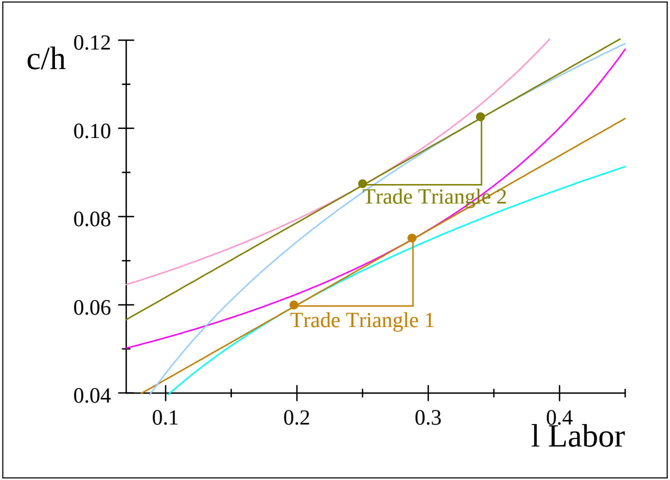


Figure 14.15. General Equilibrium Trade Triangles of Country 1 and Country 2 in Example 14.2.

Normalized Goods Trade Flow

$$\frac{c^d_{1t}}{h_{1t}} = (0.16918)\, l^s_{1t} + 0.052632\,(1.0358)\,(0.47895) = 0.074909,$$
$$\frac{c^s_{1t}}{h_{1t}} = (0.16918)\, l^d_{1t} + 0.0526\,(1.0358)\,(0.47895) = 0.059713.$$

$$\frac{c^s_{2t}}{h_{2t}} = (0.16918)\, l^d_{2t} + 0.0526\,(1.0358)\,(0.82124) = 0.10239,$$
$$\frac{c^d_{2t}}{h_{2t}} = (0.16918)\, l^s_{2t} + 0.052632\,(1.0358)\,(0.82124) = 0.087205.$$

$$\textit{Excess Demand}\ \ (1) : c^d_{1t} - c^s_{1t} = 0.074909 - 0.059713 = 0.0152.$$
$$\textit{Excess Supply}\ \ (2) : c^s_{2t} - c^d_{2t} = 0.10239 - 0.087205 = 0.0152.$$

- Country 2 supplies goods to Country 1.

Savings, Investment and Feldstein-Horioka

- Agent 1 excess supply of consumption goods
  - equals agent 1 excess supply of output,
  - both 0.0059.
- Implies zero excess demand, supply of investment
  - savings equals the investment of agent 1;
  - same result follows for agent 2.
- Value of labor trade equals value of goods trade.
- No trade in capital on balanced growth path.
- Similar to 0.97 correlation of savings-investment,
  - in Maffezzolli 2000,
  - supports empirical evidence of comovement
  - as in Feldstein and Horioka 1980.
- Result remains to be proved more generally.

Changes in Good Sector Productivities

- Assume $A_{1G} \neq A_{2G}$, while all other parameters the same.
- Factor price equalization implies

$$\frac{r_t}{w_t} = \frac{(1-\gamma) A_{1G} \left(\frac{h_{1t} l_{1t}^d}{k_{1t}}\right)^{\gamma}}{\gamma A_{1G} \left(\frac{h_{1t} l_{1t}^d}{k_{1t}}\right)^{\gamma-1}} = \frac{(1-\gamma) A_{2G} \left(\frac{h_{2t} l_{2t}^d}{k_{2t}}\right)^{\gamma}}{\gamma A_{2G} \left(\frac{h_{2t} l_{2t}^d}{k_{2t}}\right)^{\gamma-1}} = \frac{r_t}{w_t};$$

$$\Longrightarrow \frac{\left(\frac{h_{1t} l_{1t}^d}{k_{1t}}\right)^{\gamma}}{\left(\frac{h_{1t} l_{1t}^d}{k_{1t}}\right)^{\gamma-1}} = \frac{\left(\frac{h_{2t} l_{2t}^d}{k_{2t}}\right)^{\gamma}}{\left(\frac{h_{2t} l_{2t}^d}{k_{2t}}\right)^{\gamma-1}}; \Longrightarrow \frac{k_{1t}}{h_{1t} l_{1t}^d} = \frac{k_{2t}}{h_{2t} l_{2t}^d}.$$

- Yet for equal wage rates

$$\frac{w_t}{w_t} = 1 = \frac{\gamma A_{1G} \left(\frac{h_{1t} l_{1t}}{k_{1t}}\right)^{\gamma-1}}{\gamma A_{2G} \left(\frac{h_{2t} l_{2t}}{k_{2t}}\right)^{\gamma-1}}; \Longrightarrow A_{1G} = A_{2G}.$$

- Cannot have $A_{1G} \neq A_{2G}$; output good is single good.
- Need two goods in utility for $A_{1G} \neq A_{2G}$, more complex.

Change in Human Capital Investment Productivity

- Human capital productivities can be changed.
- If A_{1H}, A_{2H} move closer together, trade flows decrease.
  - If farther apart, trade flows increase.
- If A_{1H} is decreased only, world growth rate falls,
  - trade flows increase.
- If A_{2H} is increased only, growth rate rises,
  - trade flows increase.

Exogenous Growth

- No factor price equalization under exogenous growth
  - with only one output good in preferences.
- Exogenous growth rate fixes interest rate,
  - and capital to effective labor ratio,
  - $r_{1t} = (1-\gamma) A_{1G} \left(\frac{h_{1t} l_{1t}}{k_{1t}} \right)^{\gamma}$, so $\frac{h_{1t} l_{1t}}{k_{1t}}$ fixed by g_1.
  - since $1 + g_1 = \beta (1 + r_{1t} - \delta_k)$.
  - And r_{2t}, $\frac{h_{2t} l_{2t}}{k_{2t}}$, fixed by g_2 assumption.
  - Interest rates cannot converge if growth rates different.
- Wage rates also cannot converge,
  - since $w_{1t} = \gamma A_{1G} \left(\frac{h_{1t} l_{1t}}{k_{1t}} \right)^{\gamma - 1}$, and $\frac{h_{1t} l_{1t}}{k_{1t}}$ fixed;
  - and $w_{2t} = \gamma A_{2G} \left(\frac{h_{2t} l_{2t}}{k_{2t}} \right)^{\gamma - 1}$, and $\frac{h_{2t} l_{2t}}{k_{2t}}$ fixed.
- WIth only one good, only endogenous growth
  - with different A_H allows for factor price equalization.

Example 14.3 Numerous Agents

- Suppose there are 400 of agent 1, 600 of agent 2.
- Market clearing conditions imply

$$\begin{aligned} 400y^d_{1t} + 600y^d_{2t} &= 400y^s_{1t} + 600y^s_{2t}, \\ 400l^d_{1t} + 600l^d_{2t} &= 400l^s_{1t} + 600l^s_{2t}. \end{aligned}$$

- Gives new equilibrium world growth rate g,
  - and interest rate, wage rate,
  - following same methodology.
- Only the market clearing conditions change.
- All other equilibrium quantities can then be found.

Growth Convergence

- Convergence of countries within a region
    - in terms of their growth rates,
    - and even income levels,
    - Lucas 1988 endogenous growth stimulated literature.
    - Barro: empirical evidence of growth convergence
    - across different regions.
- Factor price equalization comes with growth convergence.
    - Empirical findings of growth convergence
    - support factor price equalization theory in part.
    - Includes moving towards same capital to effective labor ratios.
- Equalization of capital to effective labor ratios
    - behind Lucas 1990 question:
    - why does not capital simply flow
    - from rich to poor countries, and equalize factor prices?
- Factor prices only gradually equalize as growth rates converge
    - may take many years, for example in Africa.

Leontief Paradox

- Leontief paradox 1953:
  - U.S. exports labor intensive goods, while
  - comparative advantage is capital intensive goods.
  - Still exists today? U.S. exports services
  - said to be labor intensive.
- Paradox: view with human capital endogenous growth.
  - U.S. specializes in human capital intensive goods,
  - and exports these;
  - if service sector most human capital intensive.
  - then no Leontieff paradox.
- Exogenous growth theory:
  - services look like labor intensive goods.
  - Then paradox still unexplained.
  - A result of human capital not entering
  - production function in exogenous growth theory.
  - So not accounted for except through total factor productivity.

Feldstein-Horioka Puzzle

- Small open economies are said to borrow, lend;
  - logic says nations borrowing, lending
  - can have investment very different from savings.
- Feldstein and Horioka 1980 puzzle:
  - evidence shows high savings-investment correlation;
  - interpreted as implying world trade not significant.
- Obstfeld and Rogoff 2000 call this one of main puzzles.
- Levy 2004 argues theoretically that savings, investment
  - within countries engaged in trade
  - should move together empirically.
- Chapter 14 trade: both countries adjust capital ratios
  - so that growth rates are equal,
  - with factor price equalization ensuing.
  - Means trade in goods, labor such that
  - savings equal investment within each country.