Advanced Modern Macroeconomics

Incomplete Markets and Banking

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- Extends uncertainty over two states of nature
  - to allow incomplete markets,
  - with cost of transfering income across states.
  - Special case: no transference cost, complete markets.
- With costs introduced no perfect consumption smoothing:
  - using microeconomic banking sector,
  - "financial intermediation approach to banking",
  - using labor in banking to for interstate income transfer.
- Consumption "tilted" towards good state when transfer cost.
  - As labor share in banking rises, consumption more tilted.
  - As labor share goes to zero, consumption less tilted.
- Set out graphically with good/bad state consumption; plus
  - with amount invested in transfering income across states,
  - and bad state consumption.

Building on the Last Chapters

- Previous chapters: smoothing between work and leisure
  - smoothing goods consumption across time
  - using physical and human capital investment.
- Now smoothing across uncertain states of nature
  - using "investment" in inter-state transfers.
  - Uncertainty in simplest way of two states of nature.
- Same log utility tool, and Cobb-Douglas output in bad state
  - through banking sector;
  - general equilibrium graph with utility level curve, budget line,
  - production of bad state consumption through banking.

Learning Objective

- Why consumption is smooth across states
  - with no cost of transferring income;
- Good state consumption exceeds bad state consumption
  - when cost of transferring income.
- Complete versus incomplete markets concept:
  - degree of incompleteness with cost of transference.
- Banking production to introduce costs,
  - with investment across states of nature,
  - in terms of how actual economy insurance.

Who Made It Happen

- Arrow and Debreu 1954 equilibrium across states of nature.
- Two state analysis with production across states:
  - Hirshleifer 1970 *Investment, Interest and Capital*
  - Ehrlich and Becker 1972
  - "Market Insurance, Self-Insurance, and Self-Protection."
  - Representative agent production called "self-insurance".
- "Self-production" becomes market production
  - by banking industry, suggested by Hicks 1935,
  - "A Suggestion for Simplifying the Theory of Money":
  - "So as far as banking theory is concerned ... my suggestion is that we ought to regard every individual in the community as being, on a small scale, a bank."
- Clark 1984: Cobb-Douglas production for financial intermediation
  - in terms of labor, capital, deposits of financial capital,
  - empirical support by Hancock 1985.
  - Berger (2003) uses this approach in modern work;
  - in banking textbooks: Matthews and Thompson 2008.

Two State Analysis in General Equilibrium

- $p_g \in [0,1]$, $p_b \in [0,1]$, $p_g + p_b = 1$.
- Good, bad state consumption c_g, c_b;
- income endowment y_g; y_b; with $y_g > y_b$.
- Expected utility, with E denoting expectations,

$$E\left[u\left(c_g, c_b\right)\right] = p_g u\left(c_g\right) + p_b u\left(c_b\right).$$

- Budget constraints:
  - d amount of funds invested at insurance company.
  - y paid out in the bad state by insurance company.
  - Good state constraint:

$$c_g + d = y_g.$$

  - Bad state constraint:

$$c_b = y + y_b.$$

Transfer with Perfect Consumption Smoothing

- Let there be a "social resource constraint" such that
  - expected consumption equals the expected income:

$$p_g c_g + p_b c_b = p_g y_g + p_b y_b.$$

  - This is similar to a competive insurance industry.
- Substituting in budget constraints:

$$\begin{aligned} p_g \left(y_g - d\right) + p_b \left(y + y_b\right) &= p_g y_g + p_b y_b; \\ y &= \frac{p_g d}{p_b}. \end{aligned}$$

- Bad state transfer is investment factored by probability ratio.

Equilibrium with Zero Cost of Transfer

- Utility maximization problem with $y = \frac{p_g d}{p_b}$:

$$\begin{aligned}
\underset{d}{Max}\ u &= p_g u\left(y_g - d\right) + p_b u\left(y + y_b\right) = \\
\underset{d}{Max}\ u &= p_b u\left(y_g - d\right) + p_b u\left[\frac{p_g d}{p_b} + y_b\right];
\end{aligned}$$

$$\begin{aligned}
0 &= p_g \frac{\partial u\left(c_g\right)}{\partial c_g}(-1) + p_b \frac{\partial u\left(c_b\right)}{\partial c_b}\frac{p_g}{p_b}. \\
&\Longrightarrow \frac{p_b \frac{\partial u(c_B)}{\partial c_B}}{p_g \frac{\partial u(c_g)}{\partial c_g}} = \frac{p_b}{p_g}; \\
\frac{\partial u\left(c_b\right)}{\partial c_b} &= \frac{\partial u\left(c_g\right)}{\partial c_g}, \\
c_b &= c_g.
\end{aligned}$$

- Perfect consumption smoothing across states.

Costly Transfer of Income Across States

-

$$\begin{aligned} c_g &= y_g - d, \\ c_b &= y + y_b, \end{aligned}$$

- But now a function of d, $f(d)$, rather than d, is transferred;

$$f(d) \leq d.$$

- Now amount distributed y is

$$y = \frac{p_g f(d)}{p_b}.$$

- Consider a cost $(1 - A_F) d$, with $A_F \leq 1$;
  - transfer to the bad state is

$$d - (1 - A_F) d = A_F d.$$

  - Implies the form of $f(d)$:

$$f(d) = A_F d, \; y = A_F d (p_g / p_b).$$

Utility Maximization with Costly Transfer

$$
\begin{aligned}
\underset{d}{Max}\ u &= p_g u\left(c_g\right) + p_B u\left(c_b\right) = \\
\underset{d}{Max}\ u &= p_b u\left(y_g - d\right) + p_b u\left[\frac{p_g f\left(d\right)}{p_b} + y_b\right]; \\
0 &= p_G \frac{\partial u\left(c_G\right)}{\partial c_G}\left(-1\right) + p_B \frac{\partial u\left(c_B\right)}{\partial c_B}\frac{p_G f\left(d\right)}{p_B}. \\
\frac{p_B \frac{\partial u(c_B)}{\partial c_B}}{p_G \frac{\partial u(c_G)}{\partial c_G}} &= \frac{p_B}{p_G \frac{\partial f(d)}{\partial d}} = \frac{p_B}{p_G A_F}; \\
\frac{\frac{\partial u(c_G)}{\partial c_G}}{\frac{\partial u(c_B)}{\partial c_B}} &= \frac{\partial f\left(d\right)}{\partial d} = A_F.
\end{aligned}
$$

- Log expected utility, consumption tilting if $A_F < 1$:

$$c_b = A_F c_g.$$

Complete Consumption Smoothing Special Case

$$\begin{aligned} c_G &= c_B, \\ &\Longrightarrow u(c_g) = u(c_b); \\ &\Longrightarrow \frac{\partial u(c_g)}{\partial c_g} = \frac{\partial u(c_b)}{\partial c_b}. \end{aligned}$$

$$\begin{aligned} 1 &= \frac{\frac{\partial u(c_G)}{\partial c_G}}{\frac{\partial u(c_B)}{\partial c_B}} = \frac{\partial f(d)}{\partial d}; \\ 1 &= \frac{\partial (A_F d)}{\partial d} = A_F \end{aligned}$$

- Perfect smoothing if productivity factor $A_F = 1$.
- Corresponds implicitly to zero cost of transfer:
- Cost $(1 - A_F)\, d = 0$ if $A_F = 1$.

Log-utilty, Linear Production in General

- $f(d) = A_F d$:

$$\begin{aligned}
\underset{d_g}{Max} u &= p_g \ln (y_g - d) + p_b \ln \left(\frac{p_g A_F d}{p_b} + y_B \right), \\
0 &= p_g \frac{-1}{(y_g - d)} + p_b \frac{A_F \frac{p_g}{p_b}}{\left(\frac{p_g A_F d}{p_b} + y_b \right)}, \\
\frac{p_g A_F d}{p_b} + y_b &= A_F (y_g - d), \\
c_b &= A_F c_g
\end{aligned}$$

Full Solution for Economy

- Solve for d, y, c_b, c_g from equilibrium conditions :

$$
\begin{aligned}
A_F d + \frac{p_g A_F d}{p_b} &= A_F d \left(1 + \frac{p_g}{p_b}\right) = A_F y_g - y_b; \\
d &= \frac{A_F y_g - y_b}{A_F \left(1 + \frac{p_g}{p_b}\right)},
\end{aligned}
$$

$$
\begin{aligned}
y &= \frac{p_g}{p_b} A_F d = \frac{p_g}{p_b} \left(\frac{A_F y_g - y_b}{1 + \frac{p_g}{p_b}} \right); \\
c_b &= y + y_b = \frac{p_g}{p_b} \left(\frac{A_F y_g - y_b}{1 + \frac{p_g}{p_b}} \right) + y_b; \\
c_g &= y_g - d = y_g - \frac{A_F y_g - y_b}{A_F \left(1 + \frac{p_g}{p_b}\right)}; c_b = A_F c_g .
\end{aligned}
$$

Example 15.1 Baseline

- $A_F = 1$; $p_g = 0.9$, $p_b = 0.1$, $y_g = 1$, $y_b = 0$.

$$
\begin{aligned}
d &= \frac{A_F y_g - y_b}{A_F \left(1 + \frac{p_g}{p_b}\right)} = \frac{1}{1 + \frac{0.9}{0.1}} = 0.1 \\
y &= \frac{p_g}{p_b} \left(\frac{A_F y_g - y_b}{1 + \frac{p_g}{p_b}} \right) = \frac{0.9}{0.1} \frac{1}{1 + \frac{0.9}{0.1}} = 0.9; \\
c_b &= y + y_b = y + 0 = 0.9; \\
c_g &= y_g - d = 1 - d = 1 - 0.1 = 0.9; \\
c_b &= c_g = 0.9.
\end{aligned}
$$

- Investment $d = 0.1$, and consumption is smoothed.

$$
\begin{aligned}
-0.105\,36 &= u = 0.9\ln(c_g) + 0.1\ln(c_b) \\
-0.105\,36 &= 0.9\ln(0.9) + 0.1\ln(0.9), \\
c_b &= \left(\frac{e^{-0.105\,36}}{(c_g)^{0.9}}\right)^{\frac{1}{0.1}}.
\end{aligned}
$$

$$
\begin{aligned}
c_b &= y_b + y = y_b + \frac{p_g f(d)}{p_b} = y_b + \frac{p_g A_F d}{p_b}; \\
\implies & d = \frac{p_b(c_b - y_b)}{A_F p_g}; \\
c_g &= y_g - d = y_g - \frac{p_b(c_b - y_b)}{A_F p_g}, \\
c_b &= \frac{A_F p_g y_G}{p_b} + y_b - \frac{A_F p_g c_g}{p_b} = \frac{1(0.9)1}{(0.1)} + 0 - \frac{1(0.9)c_g}{(0.1)}.
\end{aligned}
$$

Utility Level, Linear Production, Consumption Smoothing

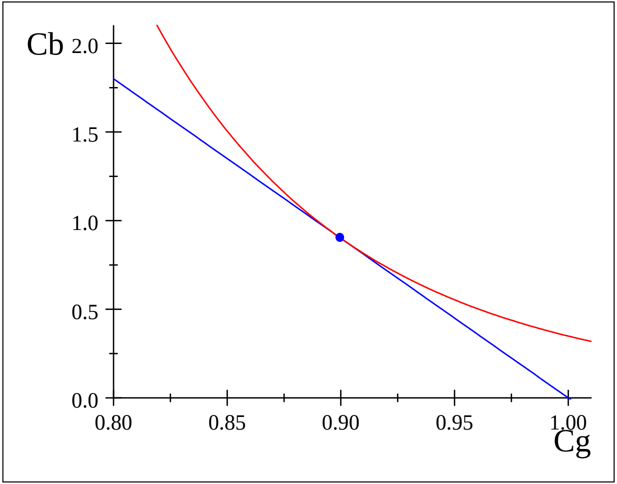

Figure 15.1. Good State and Bad State Consumption with Consumption Smoothing in Example 15.1

Alternative Graph: Bad State Consumption, Investment d

- $(c_b : d)$ space instead of $(c_b : c_g)$ space.
- Use constraint $c_g = y_g - d$, Utility level :

$$c_b = \left(\frac{e^{-0.105\,36}}{(c_g)^{0.9}} \right)^{\frac{1}{0.1}} = \left(\frac{e^{-0.105\,36}}{(1-d)^{0.9}} \right)^{\frac{1}{0.1}} .$$

Production

$$\begin{aligned} c_b &= \frac{A_F p_g y_g}{p_b} + y_b - \frac{A_F p_g c_g}{p_b}, \\ c_b &= \frac{A_F p_g y_G}{p_b} + y_b - \frac{A_F p_g (y_g - d)}{p_b}, \\ c_b &= \frac{1\,(0.9)\,1}{(0.1)} + 0 - \frac{1\,(0.9)\,(1-d)}{(0.1)} = 9d. \end{aligned}$$

Equilibrium Investment with Linear Production

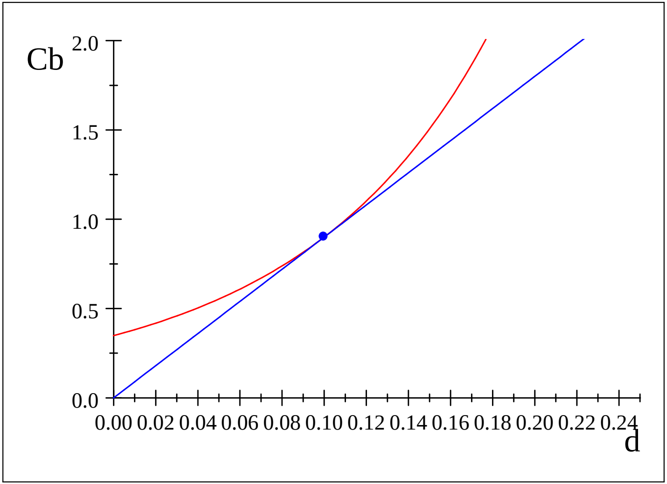

Figure 15.2. Utility and Production in Example 15.1, with Investment d.

Example 15.2 Consumption Tilting

- Assume $p_g = 0.9$, $p_b = 0.1$, $y_g = 1$, $y_b = 0$,
- and less bank productivity: $A_F = 0.8$, instead of $A_F = 1$.
- Transfering income across states now costly.

$$\begin{aligned} d &= \frac{0.8}{0.8\left(1+\frac{0.9}{0.1}\right)} = 0.1 \\ y &= 0.8\left(\frac{0.9}{0.1}\right)\frac{0.8}{0.8\left(1+\frac{0.9}{0.1}\right)} = 0.72; \\ c_b &= y = 0.72; \\ c_g &= 1-d = 1-0.1 = 0.9. \end{aligned}$$

- $c_b = 0.72 < c_g = 0.9$: consumption "tilted" good state.
- Tilting: general result if costly to transfer income across states.

Utility Level, Linear Production with Consumption Tilting

$$
\begin{aligned}
u &= 0.9 \ln\left(c_g\right) + 0.1 \ln\left(c_b\right) \\
-0.127\,67 &= 0.9 \ln\left(0.9\right) + 0.1 \ln\left(0.72\right), \\
e^{-0.127\,67} &= \left(c_g\right)^{0.9} \left(c_b\right)^{0.1}, \\
c_b &= \left(\frac{e^{-0.127\,67}}{\left(c_g\right)^{0.9}}\right)^{\frac{1}{0.1}}.
\end{aligned}
$$

$$
\begin{aligned}
c_b &= \frac{A_F p_g y_g}{p_b} + y_b - \frac{A_F p_g c_g}{p_b}; \\
c_b &= \frac{(0.8)\,(0.9)\,1}{(0.1)} + 0 - \frac{(0.8)\,(0.9)\,c_g}{(0.1)}.
\end{aligned}
$$

Shift Down in Bad State Consumption with Less Bank Productivity

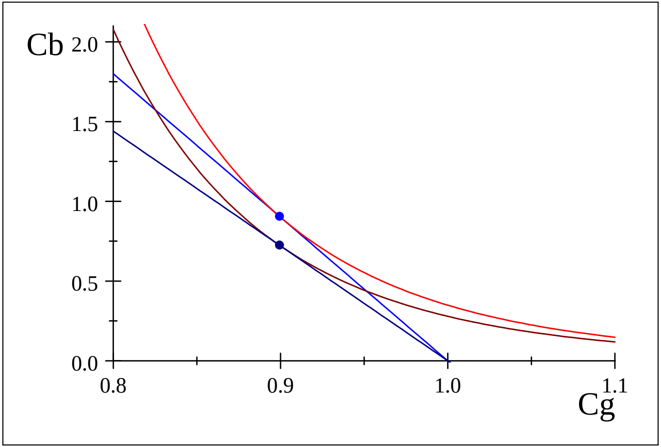

Figure 15.3. Good State and Bad State Consumption in Example 15.1 (lighter red, blue) and Example 15.2 (darker red, blue).

Investment, Bad State Consumption with Tilting

$$c_b = \left(\frac{e^{-0.12767}}{(1-d)^{0.9}} \right)^{\frac{1}{0.1}} ; \; c_b = 7.2d.$$

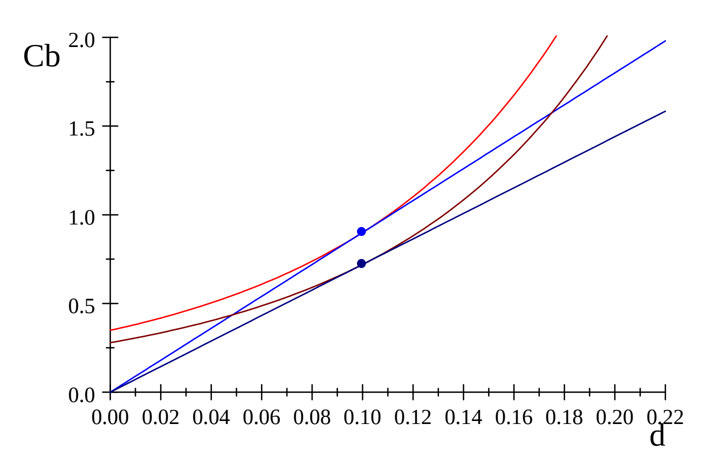

Figure 15.4. Utility and Production in Examples 15.1 and 15.2.

Example 15.3 Change in Transfer Productivity

- More general when bad state endowmentis $y_b > 0$:
  - at $y_b = 0$, d constant as A_F changes: $d = \frac{A_F y_g - y_b}{A_F\left(1+\frac{p_g}{p_b}\right)} = \frac{y_g}{1+\frac{p_g}{p_b}}$.
  - at $y_b > 0$, d falls as A_F falls: $d = \frac{y_g - \frac{y_b}{A_F}}{1+\frac{p_g}{p_b}}$
- Let $y_b = 0.1$; other parameters as in Example 15.2 :
  - $A_F = 0.8$, $p_g = 0.9$, $p_b = 0.1$, $y_g = 1$:

$$d = \frac{y_g - \frac{y_b}{A_F}}{1 + \frac{p_g}{p_b}} = \frac{1 - \frac{0.1}{0.8}}{1+9} = 0.0875.$$

With $A_F = 0.6$, d is lower:

$$d = \frac{y_g - \frac{y_b}{A_F}}{1 + \frac{p_g}{p_b}} = \frac{1 - \frac{0.1}{0.6}}{1+9} = 0.08333.$$

Investment d, Bank Productivity, Positive Bad State Endowment

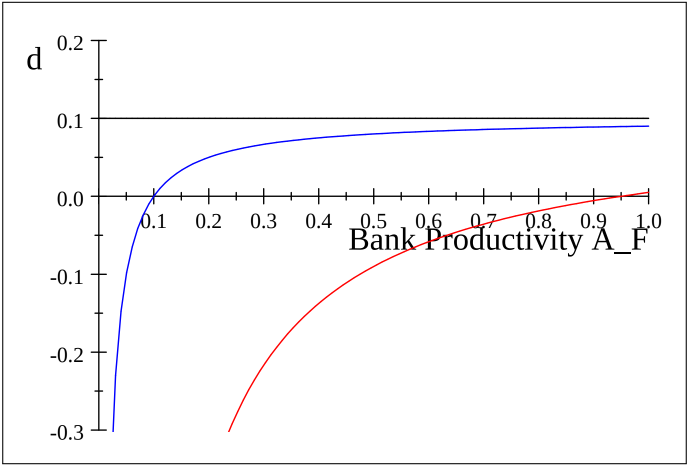

Figure 15.5. Productivity A_F and Investment d; Blue Line $y_b = 0.1$, Red Line $y_b = 0.95$.

Decentralized Problem: Consumer

$$E\left[u\left(c_g, c_b\right)\right] = p_g u\left(c_g\right) + p_b u\left(c_b\right),$$
$$c_g - d = y_g;\ c_b = \left(1 + R^d\right) d + y_b.$$

$$\underset{d}{Max}\ E\left[u\left(c_g, c_b\right)\right] = p_g u\left(y_g - d\right) + p_b u\left(\left(1 + R^d\right) d + y_b\right),$$
$$\frac{p_g \frac{\partial u(c_g)}{\partial c_g}}{p_b \frac{\partial u(c_b)}{\partial c_b}} = 1 + R^d.$$

- $1 + R^d$ is now a market determined price,
- In equilibrium, "gross return" $1 + R^d$ is ratio of MRS_{c_g, c_b}
- If also equals probability ratio, $\frac{p_b}{p_g}$,
  - then consumption smoothing results,
  - $c_g = c_b$.
- Price $1 + R^d$ determined by firm marginal productivity.

Decentralized Problem: Firm

$$f(d) = A_F d,$$

$$\begin{aligned} \underset{d}{Max}\ E\Pi &= p_g f(d) - p_b \left(1 + R^d\right) d, \\ &= p_g \left[d - (1 - A_F) d\right] - p_b \left(1 + R^d\right) d, \\ &= p_g A_F d - p_b \left(1 + R^d\right) d. \end{aligned}$$

$$1 + R^d = \frac{p_g A_F}{p_b}.$$

Market Clearing Equilibrium

$$\frac{p_g \frac{\partial u(c_g)}{\partial c_g}}{p_b \frac{\partial u(c_b)}{\partial c_b}} = 1 + R^d = \frac{p_g A_F}{p_b}.$$

- $A_F = 1, \ c_g = c_b :$

$$\frac{\frac{\partial u(c_b)}{\partial c_b}}{\frac{\partial u(c_g)}{\partial c_g}} = 1.$$

- $A_F < 1, \ A_F c_g = c_b.$

$$\frac{\frac{\partial u(c_b)}{\partial c_b}}{\frac{\partial u(c_g)}{\partial c_g}} = A_F.$$

The Financial Intermediation Approach

- Use labor, physical capital, along with deposits to transfer income.
- Represents financial industry, people, machines, buildings:
  - value added measured as labor, physical capital cost.

$$f(d, k_F, l_F) = A_F d^{1-\kappa_1-\kappa_2} (k_F)^{\kappa_1} (l_F)^{\kappa_2},$$

  - $\kappa_1 + \kappa_2 < 1$.
- Ignoring physical capital for simplification,

$$f(d, l_F) = A_F d^{1-\kappa} l_F^{\kappa},$$

  - $\kappa < 1$.

Banking Across States of Nature with Time Allocation

- Output produced linearly with labor l; good state:

$$y_g = A_g l.$$

- Time allocation: time endowment of 1 goods, bank production.

$$l = 1 - l_F.$$

- Bad state, all time used for goods production:

$$y_b = A_b l = A_b,$$

- assuming $A_g > A_b$.

Consumer Problem

- Consumer receives wages from bank in good state;

$$
\begin{aligned}
c_g &= A_g l + w l_F - d = A_g (1 - l_F) + w l_F - d; \\
c_b &= A_b + \left(1 + R^d\right) d.
\end{aligned}
$$

- Choses deposits d and bank time l_F

$$
\begin{aligned}
& \underset{d, l_F}{Max} E\left[u\left(c_g, c_b\right)\right] \\
&= p_g u\left[A_g (1 - l_F) + w l_F - d\right] + p_b u\left[\left(1 + R^d\right) d + A_b\right], \\
\frac{p_g \frac{\partial u(c_g)}{\partial c_g}}{p_b \frac{\partial u(c_b)}{\partial c_b}} &= 1 + R^d, \\
w &= A_g.
\end{aligned}
$$

Bank Problem

$$f\left(d, l_F\right) = A_F d^{1-\kappa} l_F^{\kappa}.$$

$$\begin{aligned}
\underset{d, l_F}{Max}\ E\Pi &= p_g\left(A_F d^{1-\kappa} l_F^{\kappa} - w l_F\right) - p_b\left(1 + R^d\right) d, \\
1 + R^d &= \frac{p_g\left(1-\kappa\right) A_F \left(\frac{l_F}{d}\right)^{\kappa}}{p_b}, \\
w &= \kappa A_F \left(\frac{l_F}{d}\right)^{\kappa - 1}; \Longrightarrow \frac{l_F}{d} = \left(\frac{\kappa A_F}{w}\right)^{\frac{1}{1-\kappa}}, \\
1 + R^d &= \frac{p_g\left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{w}\right)^{\frac{\kappa}{1-\kappa}}}{p_b}.
\end{aligned}$$

Market Clearing

$$\frac{p_b \frac{\partial u(c_b)}{\partial c_b}}{p_g \frac{\partial u(c_g)}{\partial c_g}} = \frac{1}{1+R^d} = \frac{p_b}{p_g \left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{w}\right)^{\frac{\kappa}{1-\kappa}}},$$

$$A_g = w,$$

$$\frac{p_b \frac{\partial u(c_b)}{\partial c_b}}{p_g \frac{\partial u(c_g)}{\partial c_g}} = \frac{1}{1+R^d} = \frac{p_b}{p_g \left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}}}.$$

Log Utility

- $u(c) = \ln c$,
$$\frac{c_g}{c_b} = \frac{1}{(1-\kappa) A_F \left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}}}.$$
- If $\kappa > 0$, $A_F \leq 1$, $A_g \leq 1$; $(1-\kappa) A_F \left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}} < 1$,
$$\begin{aligned} c_g (1-\kappa) A_F \left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}} &= c_b; \\ c_g &> c_b. \end{aligned}$$
- If $\kappa = 0$, then $A_F c_g = c_b$; if also $A_F = 1$, $c_g = c_b$.

General Solution, Financial Intermediation Production

$$c_g = A_g(1-l_F) + wl_F - d = A_g - d,$$

$$c_b = \left(1+R^d\right)d + A_b = \left(\frac{p_g(1-\kappa)A_F\left(\frac{\kappa A_F}{A}\right)^{\frac{\kappa}{1-\kappa}}}{p_b}\right)d + A_b;$$

$$c_b = (1-\kappa)A_F\left(\frac{\kappa A_F}{A_G}\right)^{\frac{\kappa}{1-\kappa}}c_g = (1-\kappa)A_F\left(\frac{\kappa A_F}{A_G}\right)^{\frac{\kappa}{1-\kappa}}(A_g - d)$$

$$c_b = \frac{p_g}{p_b}(1-\kappa)A_F\left(\frac{\kappa A_F}{A}\right)^{\frac{\kappa}{1-\kappa}}d + A_b,$$

$$\Longrightarrow \quad d = \frac{A_g(1-\kappa)A_F\left(\frac{\kappa A_F}{A_G}\right)^{\frac{\kappa}{1-\kappa}} - A_b}{(1-\kappa)A_F\left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}}\left(1+\frac{p_g}{p_b}\right)}.$$

Different State Probabilities

- Example 15.4 Equal Probabilities of States: $p_b = p_g$.
  - Let $\kappa = 0, A_F = 1$, then $R^d = \frac{p_g}{p_b} - 1 = 0$.
- Example 15.5 Good State More Probable: $p_g > p_b$.
  - Let $p_G = 0.55$, $p_B = 0.45$; $\kappa = 0$ and $A_F = 1$.

$$R^d = \frac{p_g}{p_b} - 1 = \frac{0.55}{0.45} - 1 = 0.22.$$

  - Let $p_G = 0.55$, $p_B = 0.45$; $\kappa = 0.05$, $A_F = 0.95, A_g = 0.15$

$$\begin{aligned} R^d &= \frac{p_g \left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{w}\right)^{\frac{\kappa}{1-\kappa}}}{p_b} - 1 \\ &= \frac{0.55 \left(1-0.05\right) 0.95 \left(\frac{(0.05)0.95}{0.15}\right)^{\frac{0.05}{1-0.05}}}{0.45} - 1 = 0.038. \end{aligned}$$

  - Cost of producing transfer lowers the return R^d.

Example 15.6 Rare Bad State

- $p_g = 0.9, \; p_b = 0.1,$
  - $\kappa = 0, A_F = 1:$

$$R^d = \frac{p_g}{p_b} - 1 = \frac{0.9}{0.1} - 1 = 8.0.$$

  - $\kappa = 0.05, \; A_F = 0.95, \; A_g = 1.0, \; A_b = 0.60$; lower R^d :

$$\begin{aligned}
R^d &= \frac{p_g (1-\kappa) A_F \left(\frac{\kappa A_F}{w}\right)^{\frac{\kappa}{1-\kappa}}}{p_b} - 1 \\
&= \frac{0.9 (1-0.05)\, 0.95 \left(\frac{(0.05)0.95}{1}\right)^{\frac{0.05}{1-0.05}}}{0.1} - 1 = 5.919; \\
d &= \frac{A_g (1-\kappa) A_F \left(\frac{\kappa A_F}{A_G}\right)^{\frac{\kappa}{1-\kappa}} - A_b}{(1-\kappa) A_F \left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}} \left(1 + \frac{p_g}{p_b}\right)} = \frac{0.769 - 0.6}{0.769\,(10)} = 0.022.
\end{aligned}$$

Example 15.6: Big Consumption Tilt

- $d = 0.022$ and $R = 5.92$;

$$\begin{aligned} c_g &= 1 - d = 1 - 0.021954 = 0.978. \\ c_b &= \left(1 + R^d\right) d + A_b = (6.919)\,(0.022) + 0.60 = 0.7519. \end{aligned}$$

$$\begin{aligned} l_F &= d\left(\frac{\kappa A_F}{A_g}\right)^{\frac{1}{1-\kappa}} = (0.021954)\left(\frac{0.05\,(0.95)}{1.0}\right)^{\frac{1}{1-0.05}} \\ &= 0.00089 \end{aligned}$$

- $d = 2.2\%$ of output invested in good state, 0.978 consumed
- Only 0.75 consumed in the bad state.
- And 0.089% of time used in banking.

Example 15.6: Utility Level, Production, Budget Line

$$
\begin{aligned}
-0.048\,49 &= 0.9 \ln (0.978\,05) + 0.1 \ln (0.751\,9), \\
c_b &= \left(\frac{e^{-0.048\,49}}{(c_g)^{0.9}} \right)^{\frac{1}{0.1}}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{p_g}{p_b} \left(A_F d^{1-\kappa} l_F^{\kappa} - w l_F \right) &= \left(1 + R^d\right) d = c_b - A_b, \\
c_b &= \frac{p_g}{p_b} \left(A_F (1 - c_g)^{1-\kappa} l_F^{\kappa} - w l_F \right) + A_b;
\end{aligned}
$$

$$
c_b = 9 \left(0.95 (1 - c_g)^{1-0.05} (0.00089)^{0.05} - 0.00089 \right) + 0.6.
$$

$$
\begin{aligned}
c_b &= \left(1 + R^d\right) d + A_b = \left(1 + R^d\right) (1 - c_g) + A_b, \\
c_b &= 6.92 (1 - c_g) + 0.60.
\end{aligned}
$$

Blue Concave Production, Green Budget Line, Red Utility

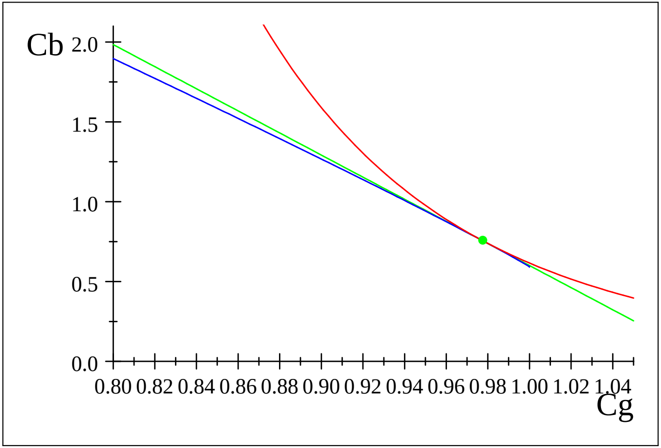

Figure 15.6. Good State and Bad State Consumption in Example 15.6

Formulated in Bad State, Investment Space

- Utility level

$$c_b = \left(\frac{e^{-0.04849}}{(1-d)^{0.9}} \right)^{\frac{1}{0.1}},$$

- Production function

$$c_b = \frac{0.9}{0.1} \left((0.95)\, d^{1-0.05} (0.00089)^{0.05} - 0.00089 \right) + 0.6,$$

- Budget line

$$c_b = 6.919\,(d) + 0.60.$$

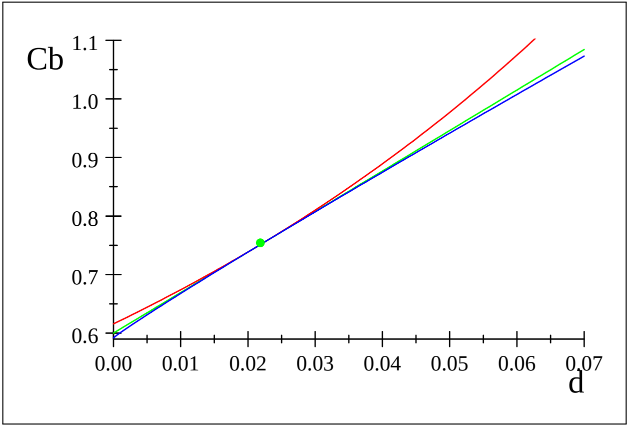

Figure 15.7. Bad State Consumption c_b and the Investment d, in Example 15.6.

Example 15.7 Change in Bank Productivity

- Decrease A_F to 0.80 from 0.95,
  - other parameters unchanged $p_g = 0.9$, $p_b = 0.1$, $\kappa = 0.05$, $A_F = 0.95$, $A_g = 1.0$, $A_b = 0.60$.

$$
\begin{aligned}
d &= \frac{A_g (1-\kappa) A_F \left(\frac{\kappa A_F}{A_G}\right)^{\frac{\kappa}{1-\kappa}} - A_b}{(1-\kappa) A_F \left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}} \left(1+\frac{p_g}{p_b}\right)} = \frac{0.642 - 0.6}{0.642\,(10)} = 0.0065, \\
c_g &= 1 - d = 1 - 0.0065 = 0.9935, \\
R^d &= \frac{p_g (1-\kappa) A_F \left(\frac{\kappa A_F}{w}\right)^{\frac{\kappa}{1-\kappa}}}{p_b} - 1 = \frac{0.577}{0.1} - 1 = 4.77, \\
c_b &= \left(1 + R^d\right) d + A_b = (5.77)\,(0.0065) + 0.6 = 0.64, \\
l_F &= d \left(\frac{\kappa A_F}{A_g}\right)^{\frac{1}{1-\kappa}} = 0.0065\,(0.04)^{\frac{1}{1-0.05}} = 0.00022.
\end{aligned}
$$

Example 15.7 Utility Curve, Production, Budget Line

$$\begin{aligned} u &= 0.9\ln\left(c_g\right) + 0.1\ln\left(c_b\right) \\ -0.050887 &= 0.9\ln\left(0.993\,52\right) + 0.1\ln\left(0.637\,4\right), \\ e^u &= \left(c_g\right)^{0.9}\left(c_b\right)^{0.1}, \\ c_b &= \left(\frac{e^{-0.050887}}{\left(1-d\right)^{0.9}}\right)^{\frac{1}{0.1}}; \end{aligned}$$

$$c_b = \frac{0.9}{0.1}\left(\left(0.80\right)d^{1-0.05}\left(0.00022\right)^{0.05} - 0.00022\right) + 0.6;$$

$$c_b = 5.774\left(d\right) + 0.60.$$

Shift Down in Investment, Bad State Consumption

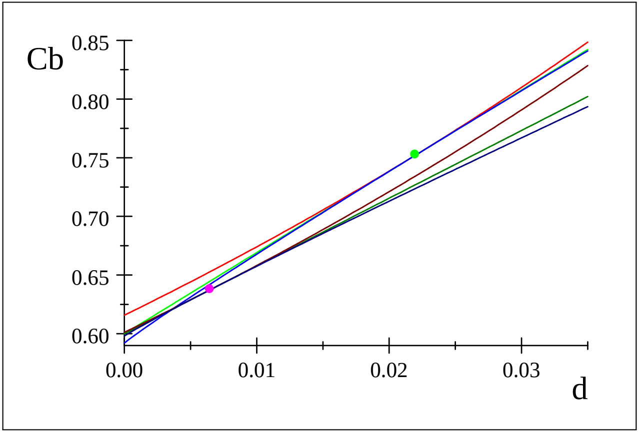

Figure 15.8. Lower Bad State Consumption c_b and Investment d, in Example 15.7.

Example 15.8 Larger Labor Costs

- Let κ rise to $\kappa = 0.10$ instead of 0.05, in Example 15.6.
  - with again $A_F = 0.95$, $p_g = 0.9$, $p_b = 0.1$, $A_g = 1.0$, $A_b = 0.60$;
  - investment d falls, R^d falls, c_b falls, relative to 15.6.

$$
\begin{aligned}
d &= \frac{A_g \left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{A_G}\right)^{\frac{\kappa}{1-\kappa}} - A_b}{\left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}} \left(1+\frac{p_g}{p_b}\right)} = \frac{0.66-0.6}{0.66\left(10\right)} = 0.0088, \\
c_g &= 1-d = 1-0.0088469 = 0.99115, \\
R^d &= \frac{p_g \left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{w}\right)^{\frac{\kappa}{1-\kappa}}}{p_b} - 1 = \frac{0.592}{0.1} - 1 = 4.92, \\
c_b &= \left(1+R^d\right) d + A_b = \left(5.92\right)\left(0.00885\right) + 0.6 = 0.652\,41, \\
l_F &= d \left(\frac{\kappa A_F}{A_g}\right)^{\frac{1}{1-\kappa}} = \left(0.00885\right)\left(0.095\right)^{\frac{1}{1-0.10}} = 0.000647.
\end{aligned}
$$

Example 15.9 High Labor Cost

- Let $\kappa = \frac{1}{3}$; with $A_b = 0$, $p_g = 0.9$, $p_b = 0.1$, $A_F = 0.95$, $A_g = 1$.

$$\begin{aligned}
d &= \frac{A_g \left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{A_G}\right)^{\frac{\kappa}{1-\kappa}} - A_b}{\left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{A_g}\right)^{\frac{\kappa}{1-\kappa}} \left(1+\frac{p_g}{p_b}\right)} = \frac{0.535}{0.535\left(10\right)} = 0.1, \\
c_g &= 1 - d = 1 - 0.1 = 0.9, \\
1 + R^d &= \frac{p_g \left(1-\kappa\right) A_F \left(\frac{\kappa A_F}{w}\right)^{\frac{\kappa}{1-\kappa}}}{p_b} = \frac{00.32076}{0.10} = 3.2076, \\
c_b &= \left(1 + R^d\right) d + A_b = \left(3.207\,6\right)\left(0.1\right) = 0.32076, \\
l_F &= d \left(\frac{\kappa A_F}{A_g}\right)^{\frac{1}{1-\kappa}} = \left(0.1\right) \left(\frac{\frac{1}{3}\left(0.95\right)}{1.0}\right)^{\frac{1}{1-\frac{1}{3}}} = 0.01782.
\end{aligned}$$

Example 15.9 Utility, Production, Budget Line

$$
\begin{aligned}
u &= 0.9\ln\left(c_g\right) + 0.1\ln\left(c_b\right) \\
-0.208\,53 &= 0.9\ln\left(0.9\right) + 0.1\ln\left(0.320\,76\right), \\
e^{u} &= \left(c_g\right)^{0.9}\left(c_b\right)^{0.1}, \\
c_b &= \left(\frac{e^{-0.208\,53}}{\left(1-d\right)^{0.9}}\right)^{\frac{1}{0.1}};
\end{aligned}
$$

$$
c_b = \frac{0.9}{0.1}\left(\left(0.95\right) d^{1-\frac{1}{3}}\left(0.01782\right)^{\frac{1}{3}} - 0.01782\right);
$$

$$
\begin{aligned}
c_b &= \left(1+R^{d}\right) d + A_b, \\
c_b &= 3.207\,6d.
\end{aligned}
$$

Example 15.9, with More Concave Production

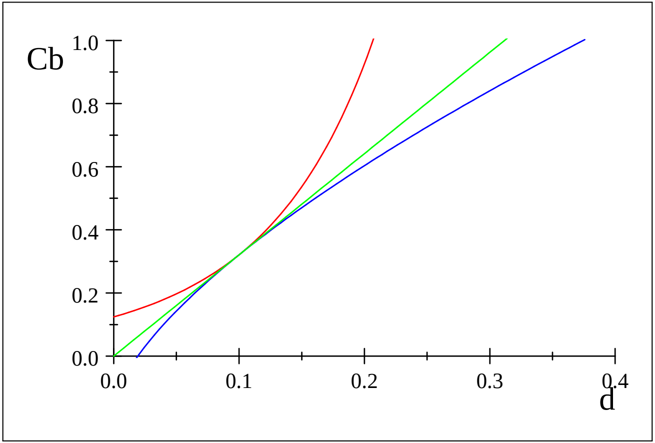

Figure 15.9. High Labor Costs and High Consumption Tilting in Example 15.9.

Aggregate Risk: Falling Bank Productivity in the Bad State

- Consider an aggregate shock
  - that causes banking productivity to fall during bad state,
  - after contracting for insurance in good state.
  - Fall in A_F to A_F' after contracting: less consumption smoothing.
- $\kappa = 0$:

$$1 + R^d = \frac{p_G A_F}{p_B}.$$

$$A_F > A_F'.$$

$$1 + R^d = \frac{p_G A_F'}{p_B}.$$

$$c_b > c_{b'}.$$

- Aggregate risk makes tilting even worse.
- Bank crises: failure of banks yields a market failure
  - due to a faulty bank insurance system perhaps,
  - occuring exactly when banks expected to pay out
  - insurance funds that smooth consumption during recessions.

Application: Unemployment and Health Insurance

- Productivity of government in supplying insurance
  - might have a lower A_F than private sector.
- Problem of supplying insurance is to apply insurance
  - with different probabilities for different people.
  - Private industry: profiles characteristics.
  - Government use less such profiling, out of "fairness".
- Profiling possible in the public sector
- Examples: additional fees for emergency health care
  - for drug, alcohol abuse problems,
  - to allow for normal emergency care service for others.
- Allowing unemployment insurance for limited duration,
  - with monitoring of efforts to find work,
  - with pilot programs to help train unemployed for work.
- Unemployment, health, old age pension schemes:
  - huge shares of government spending in all nations.
  - Reform with fair profiling enables big welfare gains.