Advanced Modern Macroeconomics

Investment and Banking Productivity

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- Consumer gives savings to financial intermediary
- Financial intermediary lends capital to firm.
- Intermediation cost increases capital marginal product.
- Uses financial intermediation production with labor.
- Stated within dynamic $AS - AD$ exogenous zero growth.
- Bank productivity affects capital marginal product.
- Financial crisis: as significant fall in bank productivity,
  - capital stock, employment, output decrease.
- Application to international bank insurance.
- Appendix $A16$ shows closed-form solution.

Building on the Last Chapters

- Uses same bank production approach,
  - now applied to savings, investment, intermediation.
  - Deposits invested by consumer in bank;
  - instead of insurance across states,
  - now intertemporal investment for firms.
  - Costly transfer now cost of transfer across time.
- Extension of zero exogenous growth economy
  - of baseline dynamic model of Part 4.
- Solution methodology modifies that of Chapter 10.
  - Cost of banking affects calculation,
  - through marginal product of capital,
  - giving different capital stock.
- Using endogenous growth as in Part 5:
  - left as extension not presented here.

Learning Objective

- Costly intermediation adds to per unit cost of capital.
    - reflected in loan rate,
    - cost of intermediation raises loan rate,
    - raises marginal product of capital,
    - lowers capital stock.
- Banking crisis -led recessions:
    - represent as one simple comparative static,
    - decrease in bank productivity.
- Bank crisis: simple comparative static productivity change,
    - compares to analysis of business cycles, growth trends,
    - with productivity change for goods, human capital production.
    - Now one additional productivity change: banking.
- Eliminates need to rely on fixed prices in Chapters 3, 6, 9
    - to generate deeper recession than normal.
    - Now presented more fundamentally from bank sector,
    - as in Great Depression, and 2007-2010.

Who Made It Happen

- Hicks 1936 suggests banking in general equilibrium,
  - Pesek, Saving 1968, Sealey, Lindley 1977,
  - financial intermediation approach to banking.
- Application of banking in macroeconomics
  - usually within monetary economics,
  - although loan rate, deposit rates often exogenous.
- Berk and Green 2004 banking costs of intermediation
  - for mutual funds investment, with partial equilibrium.
- Goodfriend, McCallum 2007 use production function approach,
  - for credit used in exchange.
- Clark 1984 approach uses constant returns to scale
  - with deposits as a factor of production.
  - Also found in Gillman, Kejak 2010, for exchange credit.

Savings-Investment Intermediation

- Consumer deposits d_{t+1} in bank, receives return $\left(1+R_t^d\right)d_t$,
  - R_t^d is dividend return to consumer as owner of bank.
  - $R_t^d d_t$ profit of bank equivalent to interest earned.
  - Consumer gets one share of ownership with each dollar deposited.
  - Share fixed at one, but variable profits possible.
- Consumer time t bank savings deposit is d_{t+1}
  - receiving $\left(1+R_t^d\right)d_t$ at time t; d_t is state variable.
  - Also works for bank and chooses l_{Ft} :

$$
\begin{aligned}
V\left(d_t\right) &= \underset{c_t^d, x_t, l_t^s, l_{Ft}, d_{t+1}}{Max} : \ln c_t^d + \alpha \ln x_t + \beta V\left(d_{t+1}\right); \\
c_t &= w_t\left(l_t^s + l_{Ft}\right) - d_{t+1} + d_t\left(1+R_t^d\right), \\
1 &= l_t^s + l_{Ft}^s + x_t.
\end{aligned}
$$

Consumer Equilibrium Conditions

$$V(d_t) = \underset{l_t^s, d_{t+1}}{Max} \ln\left[w_t(1-x_t) - d_{t+1} + d_t\left(1+R_t^d\right)\right] + \alpha \ln(1-l_t) + \beta V(d_{t+1}).$$

$$w_t = \frac{\alpha c_t}{x_t};\ \frac{1}{c_t^d} = \beta \frac{\partial V(d_{t+1})}{\partial d_{t+1}},\ \frac{\partial V(d_t)}{\partial d_t} = \frac{1}{c_t^d}\left(1+R_t^d\right);$$

$$\frac{c_t^d}{\beta c_{t-1}^d} = 1 + R_t^d.$$

- R_t^d is now consumer discount rate.

Goods Producer Problem

- Firm invests in capital: $-k_{t+1}+k_t\left(1-\delta_k\right)$
- Uses loans from bank q_{t+1} at time t; paid back: $q_t\left(1+R_t^q\right)$:

$$\begin{aligned}\Pi_t &= A l_t^{\gamma} k_t^{1-\gamma}-w_t l_t-k_{t+1}+k_t\left(1-\delta_k\right)\\ &\quad +q_{t+1}-q_t\left(1+R_t^q\right),\end{aligned}$$

- Assume all new k_t investment from loans q_t, from $t=0$, for all t :

$$\begin{aligned} i_t &= k_{t+1}-k_t=q_{t+1}-q_t;\\ &\Longrightarrow\; k_t=q_t.\end{aligned}$$

- Reduces firm problem to a static one:

$$\begin{aligned}\underset{l_t,k_t}{Max} &: \;\Pi=A l_t^{\gamma} k_t^{1-\gamma}-w_t l_t-k_t\left(R_t^q+\delta_k\right);\\ \left(1-\gamma\right)A\left(\frac{l_t}{k_t}\right)^{\gamma} &= R_t^q+\delta_k,\\ w_t &= \gamma A_G\left(\frac{l_t}{k_t}\right)^{\gamma-1}.\end{aligned}$$

Bank Technology for Producing Loans

$$\begin{aligned} q_t &= A_F \left(l_{Ft} \right)^{\kappa} d_t^{1-\kappa}, \\ \kappa &\in [0, 1). \end{aligned}$$

- $\kappa = 0$, $q_t = A_F d_t$; and if $A_F = 1$, $q_t = d_t$.
- WIth costs, $\kappa > 0$, $A_F < 1$, still possible to have $q_t = d_t$,

$$\begin{aligned} \frac{q_t}{d_t} &= A_F \left(\frac{l_{Ft}}{d_t} \right)^{\kappa} = 1, \\ &\implies \frac{l_{Ft}}{d_t} = \frac{1}{\left(A_F \right)^{\frac{1}{\kappa}}}. \end{aligned}$$

Bank Profit Problem

- Bank gives out loans, takes in deposits, pays labor cost.
- Revenue $q_t\left(1+R_t^q\right)+d_{t+1}$, costs $q_{t+1}+d_t\left(1+R_t^d\right)+w_t l_{Ft}$.
- Dividends paid out, $d_t R_t^d$, leave zero residual profit:

$$0=\Pi_{Ft}=-q_{t+1}+q_t\left(1+R_t^q\right)+d_{t+1}-d_t\left(1+R_t^d\right)-w_t l_{Ft}.$$

- Profit Π_{Ft} is subject to production technology for loans:

$$q_t=A_F\left(l_{Ft}\right)^{\kappa} d_t^{1-\kappa}.$$

Bank Equilibrium Conditions

$$\Pi_F(q_t, d_t) = \underset{q_{t+1}, d_{t+1}, l_{Ft}}{Max} \Big\{ -q_{t+1} + q_t(1 + R_t^q) + d_{t+1} - d_t\left(1 + R_t^d\right) - w$$
$$-\lambda_t\left[q_t - A_F(l_{Ft})^\kappa d_t^{1-\kappa}\right] + z_t \Pi_F(q_{t+1}, d_{t+1})\Big\}.$$

$$0 = -1 + z_t \frac{\partial \Pi_F(q_{t+1}, d_{t+1})}{\partial q_{t+1}},$$
$$0 = 1 + z_t \frac{\partial \Pi_F(q_{t+1}, d_{t+1})}{\partial d_{t+1}},$$
$$\lambda_t = \frac{w_t}{\kappa A_F\left(\frac{l_{Ft}}{d_t}\right)^{\kappa-1}}.$$

$$q_t : \frac{\partial \Pi_F(q_t, d_t)}{\partial q_t} = (1 + R_t^q) - \lambda_t;$$
$$d_t : \frac{\partial \Pi_F(q_t, d_t)}{\partial d_t} = -\left(1 + R_t^d\right) + \lambda_t(1-\kappa)A_F\left(\frac{l_{Ft}}{d_t}\right)^\kappa.$$

Deriving Marginal Cost of Credit using Interest Differential

$$
\begin{aligned}
z_t &= \frac{1}{\frac{\partial \Pi_F(q_{t+1}, d_{t+1})}{\partial q_{t+1}}} = -\frac{1}{\frac{\partial \Pi_F(q_{t+1}, d_{t+1})}{\partial d_{t+1}}}, \\
(1 + R_t^q) - \lambda_t &= \frac{\partial \Pi_F(q_t, d_t)}{\partial q_t} = \\
-\frac{\partial \Pi_F(q_t, d_t)}{\partial d_t} &= \left(1 + R_t^d\right) - \lambda_t (1 - \kappa) A_F \left(\frac{l_{Ft}}{d_t}\right)^{\kappa},
\end{aligned}
$$

$$
\Longrightarrow \lambda_t = \frac{R_t^q - R_t^d}{1 - (1 - \kappa) A_F \left(\frac{l_{Ft}}{d_t}\right)^{\kappa}}.
$$

- $1 - (1 - \kappa) A_F \left(\frac{l_{Ft}}{d_t}\right)^{\kappa} \leq 1$, marginal cost λ_t a multiple $R_t^q - R_t^d$.
  - Labor equilibrium condition gives another form for marginal cost

$$
\frac{w_t}{\kappa A_F \left(\frac{l_{Ft}}{d_t}\right)^{\kappa - 1}} = \lambda_t = \frac{R_t^q - R_t^d}{1 - (1 - \kappa) A_F \left(\frac{l_{Ft}}{d_t}\right)^{\kappa}}.
$$

Solution with Deposits equal Loans

- Assume all deposits turned into loans; $q_t = d_t$.
  - $d_t - q_t$ could be positive amount of capital requirements
  - but here there are no capital reserves for simplicity.
- $q_t = d_t$ implies from the production function

$$\begin{aligned} \frac{l_{Ft}}{d_t} &= \frac{1}{(A_F)^{\frac{1}{\kappa}}}\left(\frac{q_t}{d_t}\right)^{\frac{1}{\kappa}} = \frac{1}{(A_F)^{\frac{1}{\kappa}}}, \\ &\Longrightarrow R_t^q - R_t^d = \frac{w_t}{(A_F)^{\frac{1}{\kappa}}}, \\ \frac{R_t^q - R_t^d}{\kappa} &= \lambda_t = \frac{w_t}{\kappa\,(A_F)^{\frac{1}{\kappa}}}\left(\frac{q_t}{d_t}\right)^{\frac{1-\kappa}{\kappa}} = \frac{1}{(A_F)^{\frac{1}{\kappa}}}. \end{aligned}$$

Example 16.1 Marginal Cost and Bank Supply

- Assume $w_t = 0.0111$, $R_t^d = \rho = 0.0525$, $\kappa = 0.333$, $A_F = 0.95$.
  - $w_t = 0.0111$, $R_t^q = 0.0655$ are solution Example 16.2 below.

$$MC_{qt} = \frac{w_t}{\kappa \left(A_F\right)^{\frac{1}{\kappa}}} \left(\frac{q_t}{d_t}\right)^{\frac{1-\kappa}{\kappa}} = \frac{0.0111}{0.333 \left(0.95\right)^{\frac{1}{0.333}}} \left(\frac{q_t}{d_t}\right)^2 .$$

- At $\frac{q_t}{d_t} = 1$,

$$MC_{qt} = \frac{0.0111}{0.333 \left(0.95\right)^{\frac{1}{0.333}}} = 0.389,$$

  - marginal cost as $\frac{R_t^q - R_t^d}{\kappa}$:

$$\begin{aligned} MC_{qt} &= \frac{R_t^q - R_t^d}{\kappa} = \frac{0.06555 - 0.0526}{0.333} = 0.0389, \\ R_t^q - R_t^d &= 0.01295. \end{aligned}$$

Marginal cost at q=d

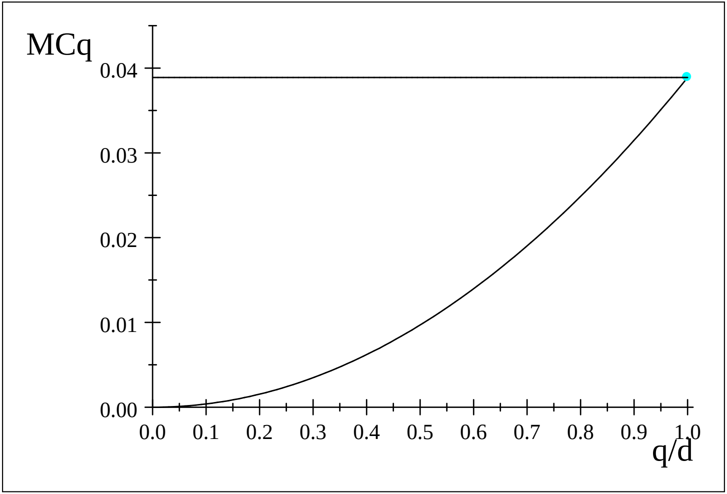

Figure 16.1. Marginal Cost of Banking: Loan Supply Curve

AS-AD with Banking: Consumption Demand

$$
\begin{aligned}
c_t &= w_t (1 - x_t) - d_{t+1} + d_t \left(1 + R_t^d\right), \\
c_t &= w_t (1 - x_t) + d_t \left(R_t^d - g\right).
\end{aligned}
$$

$$k_t = q_t = d_t.$$

$$
\begin{aligned}
c_t &= w_t (1 - x_t) + k_t \left(R_t^d - g\right), \\
1 + g &= \frac{1 + R_t^d}{1 + \rho},\ R_t^d - g = \rho (1 + g), \\
c_t &= w_t (1 - x_t) + \rho (1 + g) k_t,\ w_t = \frac{\alpha c_t}{x_t}, \\
c_t &= \frac{1}{1 + \alpha} \left[w_t + \rho (1 + g) k_t\right].
\end{aligned}
$$

Aggregate Output Demand and Supply

$$
\begin{aligned}
y_t^d &= c_t + i_t = \frac{1}{1+\alpha}\left[w_t + \rho\left(1+g\right)k_t\right] + \left(g+\delta_k\right)k_t, \\
y_t^d &= \frac{1}{1+\alpha}\left\{w_t + \left[\rho\left(1+g\right) + \left(g+\delta_k\right)\left(1+\alpha\right)\right]k_t\right\}; \\
y_t^s &= A_G^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t.
\end{aligned}
$$

- $g = 0,\ R_t^d = \rho,$

$$
\begin{aligned}
c_t &= \frac{1}{1+\alpha}\left[w_t + \rho k_t\right], \\
y_t^d &= \frac{1}{1+\alpha}\left\{w_t + \left[\rho + \delta_k\left(1+\alpha\right)\right]k_t\right\}.
\end{aligned}
$$

-

$$
\left(1-\gamma\right)A\left(\frac{l_t}{k_t}\right)^{\gamma} - \delta_k = R_t^q = R_t^d + \frac{w_t}{\left(A_F\right)^{\frac{1}{\kappa}}}.
$$

Example 16.2 Baseline Calibration with Banking

- $A_G = 0.1$, $\gamma = \kappa = 0.333$, $\delta_k = 0.05$, $\alpha = 0.5$, $\beta = 0.95$, $A_F = 0.95$.
- Appendix $A16: k_t = 0.0838$.

$$\frac{1}{w_t} = \frac{1}{y_t^d\left(1+\alpha\right) - k_t\left[\rho + \left(1+\alpha\right)\delta_k\right]},$$

$$\frac{1}{w_t} = \frac{1}{y_t^d\left(1.5\right) - \left(0.0838\right)\left(0.0526 + \left(1.5\right)0.05\right)},$$

$$\frac{1}{w_t} = \frac{\left(y_t^s\right)^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{1-\gamma}}\left(k_t\right)^{\frac{1-\gamma}{\gamma}}} = \frac{\left(y_t^s\right)^{\frac{1-0.333}{0.333}}}{0.333\left(0.1\right)^{\frac{1}{0.333}}\left(0.0838\right)^{\frac{1-0.333}{0.333}}}.$$

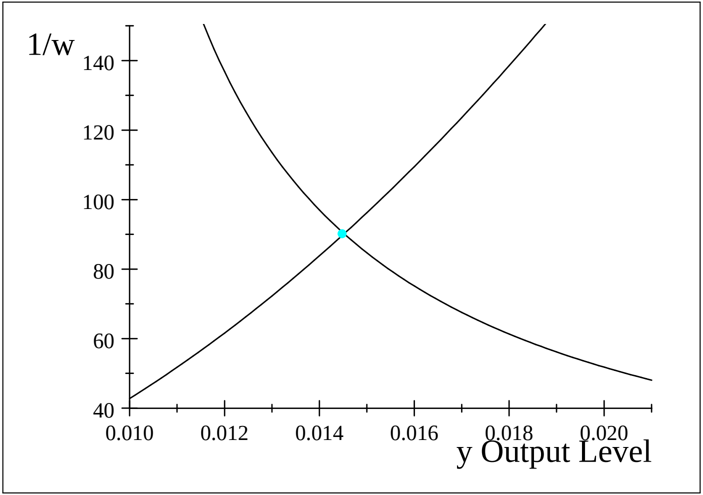

Figure 16.2. $AS - AD$ Baseline with Intermediation of Investment/Savings in Example 16.2.

$$
\begin{aligned}
y_t^s &= A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t, \\
y_t^s &= (0.1)^{\frac{1}{1-0.333}} \left(\frac{0.333}{0.011081}\right)^{\frac{0.333}{1-0.333}} (0.0838) = 0.0145.
\end{aligned}
$$

$$
R^q = 1.3\% + \rho = 1.3\% + 5.26\% = 6.56\%
$$

Example 16.3 Bank Productivity, Bank Crises

- Assume A_F falls from 0.95 down to "crisis" level 0.7
  - Appendix $A16: k_t = 0.0552$, a 34% drop in value
  - 34.1% fall in U.S. Dow Jones Industrial, $13,058$ to 8599
  - May 2, 2008, to January 9, 2009 during bank crisis.
- Output drops to

$$\begin{aligned} y_t &= \frac{1}{1+\alpha}\left(w_t + \rho k_t\right) + \delta_k k_t, \\ y_t &= \frac{1}{1.5}\left(0.008927 + (0.0526)\,0.0552\right) + (0.05)\,(0.0552) = 0.01065. \end{aligned}$$

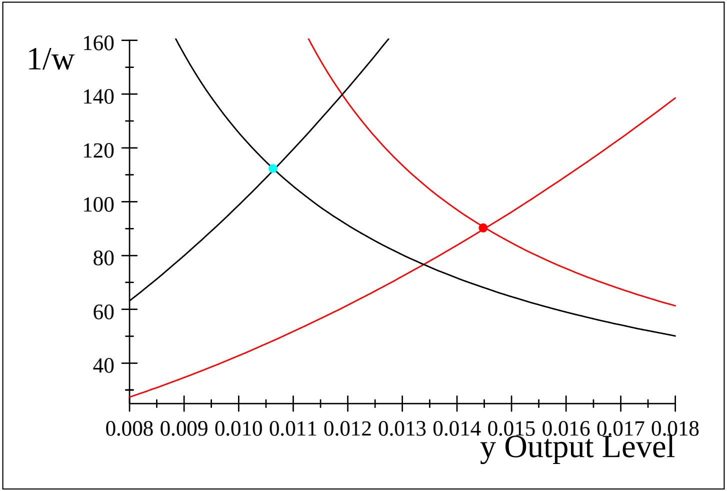

Figure 16.3. $AS - AD$ Bank Crisis fall in Bank Producitivity to $A_F = 0.7$

Interest Rate Spread

- Before bank productivity decrease:

$$R_t^q - R_t^d = \frac{w_t}{(A_F)^{\frac{1}{\kappa}}} = \frac{0.01108}{(0.95)^{\frac{1}{0.333}}} = 0.0129,$$

- After bank productivity decrease:

$$R_t^q - R_t^d = \frac{w_t}{(A_F)^{\frac{1}{\kappa}}} = \frac{0.00893}{(0.7)^{\frac{1}{0.333}}} = 0.0261.$$

  - 2.61%, a doubling of spread.

Labor Market and Bank Crisis

$$l_t^d = \left(\frac{\gamma A_G}{w_t} \right)^{\frac{1}{1-\gamma}} k_t.$$

$$x_t = \frac{\alpha c_t}{w_t},\ l_{Ft} = \frac{k_t}{(A_F)^{\frac{1}{\kappa}}},\ k_t = d_t,\ c_t = \frac{1}{1+\alpha}(w_t + \rho k_t),$$

$$\begin{aligned}
1 &= l_t + l_{Ft} + x_t, \\
1 &= l_t + \frac{k_t}{(A_F)^{\frac{1}{\kappa}}} + \frac{\alpha c_t}{w_t}, \\
1 &= l_t + \frac{k_t}{(A_F)^{\frac{1}{\kappa}}} + \frac{\alpha}{1+\alpha} \frac{[w_t + \rho k_t]}{w_t}, \\
l_t^s &= 1 - \frac{\alpha}{1+\alpha} \left[1 + \left(\frac{\rho}{w_t} + \frac{1+\alpha}{\alpha (A_F)^{\frac{1}{\kappa}}} \right) k_t \right].
\end{aligned}$$

Labor Market Graph

$$w_t = \gamma A_G \left(\frac{k_t}{l_t^d}\right)^{1-\gamma},$$

$$w_t = \frac{\alpha\rho k_t}{(1+\alpha)\left(1 - l_t^s - \frac{k_t}{(A_F)^{\frac{1}{\kappa}}}\right) - \alpha};$$

$$w_t = 0.333\,(0.1)\left(\frac{0.0838}{l_t^d}\right)^{1-0.333};$$

$$w_t = \frac{(0.5)\,(0.0526)\,(0.0838)}{(1.5)\left(1 - l_t^s - \frac{(0.0838)}{(0.95)^{\frac{1}{0.333}}}\right) - 0.5}.$$

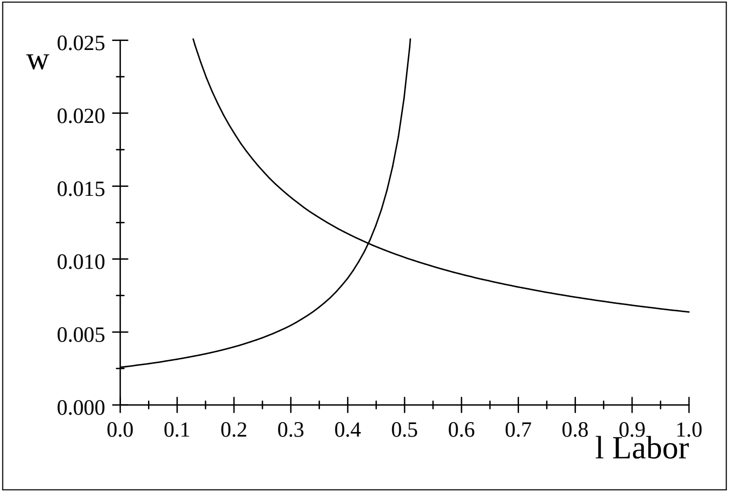

Figure 16.4. The Labor Market in Example 16.2.

Labor Market with Bank Crisis

- Capital stock falls, and :

$$w_t = 0.333\,(0.1)\left(\frac{0.0552}{l_t^d}\right)^{1-0.333},$$

$$w_t = \frac{(0.5)\,(0.0526)\,(0.0552)}{(1.5)\left(1 - l_t^s - \frac{(0.0552)}{(0.7)^{\frac{1}{0.333}}}\right) - 0.5}.$$

$$\frac{(0.5)\,(0.0526)\,(0.0552)}{(1.5)\left(1 - l_t^s - \frac{(0.0552)}{(0.7)^{\frac{1}{0.333}}}\right) - 0.5} = 0.333\,(0.1)\left(\frac{0.0552}{l_t^d}\right)^{1-0.333},$$

$$l = 0.3973.$$

- Employment falls $\frac{0.436-0.3973}{0.436}$, or 8.9%.

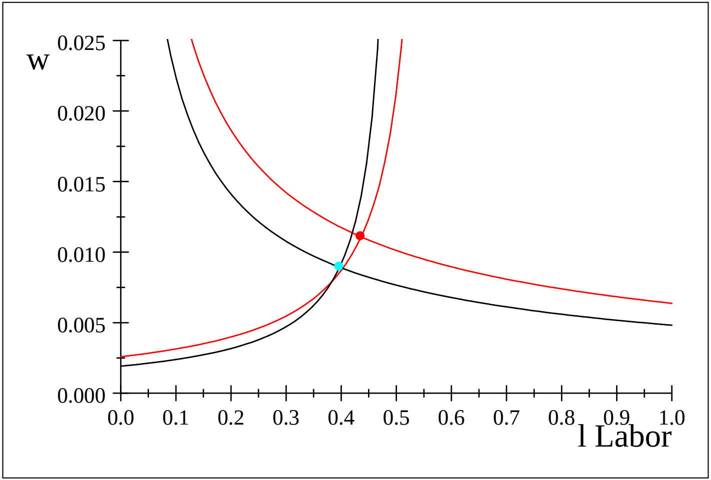

Figure 16.5. Lower Employment and Wage Rate during Bank Crisis in Example 16.3 (black) compared to Example 16.2 (red).

Isocost, Isoquant, Factor Inputs before Bank Crisis

Isocost

$$\begin{aligned} y_t &= w_t l_t + \left(R_t^q + \delta_k\right) k_t, \\ 0.014531 &= (0.011081)\, l_t + (0.06565 + 0.05)\, k_t, \\ k_t &= \frac{0.014531}{0.06565 + 0.05} - \frac{(0.011081)\, l_t}{0.06565 + 0.05}. \end{aligned}$$

Isoquant

$$\begin{aligned} 0.014531 &= y_t^s = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} = (0.1) \left(l_t^d\right)^{\frac{1}{3}\gamma} (k_t)^{\frac{2}{3}}; \\ k_t &= \left(\frac{0.014531}{0.1 \left(l_t^d\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}} = \frac{\left(\frac{0.014531}{0.1}\right)^{\frac{3}{2}}}{\left(l_t^d\right)^{\frac{1}{2}}}. \end{aligned}$$

Factor input ratio

$$\frac{k_t}{l_t} = \frac{0.08382}{0.436\,3} = 0.19212.$$

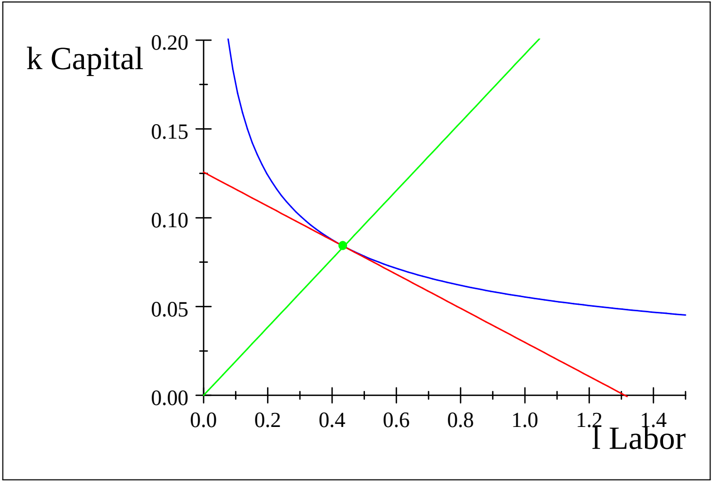

Figure 16.6. Factor Market Equilibrium in Example 16.2.

Production, Utility, Budget Line before Bank Crisis

Production

$$c_t^d = y_t^s - i_t = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} - \delta_k k_t,$$
$$c_t^d = (0.10) \left(l_t^d\right)^{\frac{1}{3}} (0.08382)^{\frac{2}{3}} - (0.05)(0.08382).$$

Utility

$$u = \ln c_t + \alpha \ln x_t = \ln c_t + \alpha \ln (T_t - l_t),$$
$$-4.9538 = \ln 0.01034 + 0.5 \ln (1 - 0.098 - 0.4363),$$
$$c_t = \frac{e^{-4.9538}}{(1 - 0.098 - l_t)^{0.5}}.$$

Budget Line

$$c_t^d = w_t \left(l_t^s + l_{Ft}^s\right) + \rho k_t^s = w_t l_t^s + w_t l_{Ft}^s + \rho k_t^s$$
$$c_t^d = (0.011)\, l_t^s + (0.011)(0.098) + (0.0526)(0.084).$$

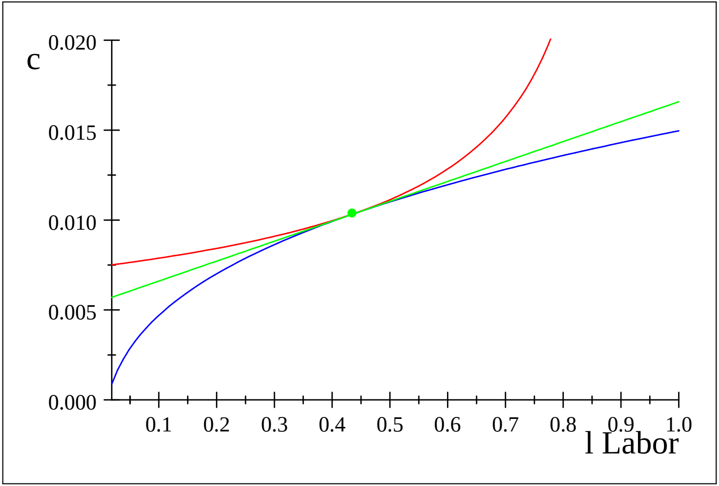

Figure 16.7. General Equilibrium Consumption and Utility Levels in Example 16.2.

Isocost, Isoquant, Factor Inputs after Bank Crisis

Isocost

$$\begin{aligned}
y_t &= w_t l_t + \left(R_t^q + \delta_k\right) k_t, \\
0.010\,65 &= (0.00893)\, l_t + (0.02606 + 0.052632 + 0.05)\, k_t, \\
k_t &= \frac{0.01065}{(0.026 + 0.0526 + 0.05)} - \frac{(0.00893)\, l_t}{(0.026 + 0.0526 + 0.05)}.
\end{aligned}$$

Isoquant

$$\begin{aligned}
0.010\,65 &= y_t^s = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} = (0.1) \left(l_t^d\right)^{\frac{1}{3}} (k_t)^{\frac{2}{3}}, \\
k_t &= \left(\frac{0.01065}{0.1 \left(l_t^d\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}} = \frac{\left(\frac{0.01065}{0.1}\right)^{\frac{3}{2}}}{\left(l_t^d\right)^{\frac{1}{2}}}.
\end{aligned}$$

Factor input ratio

$$\frac{k_t}{l_t} = \frac{0.0552}{0.397\,3} = 0.139.$$

Bank Crisis: Decrease in Isoquant, Isocost, Factor Input

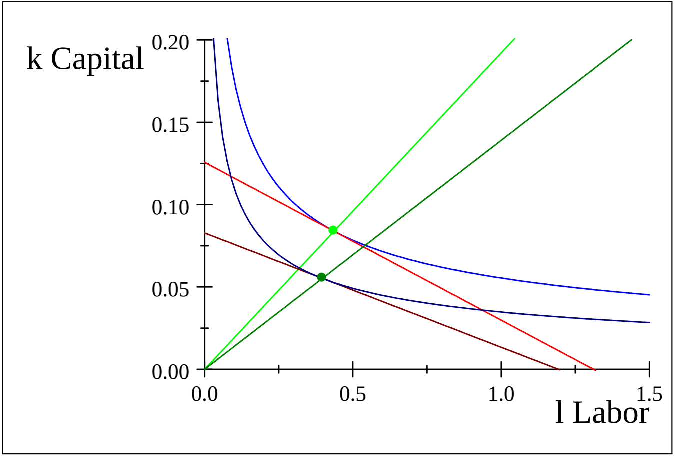

Figure 16.8 Factor Market Equilibrium in Baseline Example 16.2 (lighter red, blue, green) and Crisis Example 16.3 (darker red, blue, green).

Production, Utility, Budget Line after Bank Crisis

Production:

$$
\begin{aligned}
c_t^d &= y_t^s - i_t = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} - \delta_k k_t, \\
c_t^d &= (0.10) \left(l_t^d\right)^{\frac{1}{3}} (0.0552)^{\frac{2}{3}} - (0.05)(0.0552).
\end{aligned}
$$

Utility level :

$$
\begin{aligned}
u &= \ln c_t + \alpha \ln x_t = \ln c_t + \alpha \ln (T_t - l_t), \\
-5.2505 &= \ln 0.0078902 + 0.5 \ln (1 - 0.161 - 0.39713), \\
-5.2505 &= \ln c_t + 0.5 \ln (1 - 0.161 - l_t), \\
c_t &= \frac{e^{-5.2505}}{(1 - 0.161 - l_t)^{0.5}}.
\end{aligned}
$$

Budget line :

$$
\begin{aligned}
c_t^d &= w_t \left(l_t^s + l_{Ft}^s\right) + \rho k_t^s = w_t l_t^s + w_t l_{Ft}^s + \rho k_t^s \\
c_t^d &= (0.00893)\, l_t^s + (0.00893)(0.161) + (0.052632)(0.0552).
\end{aligned}
$$

Bank Crisis: Decrease in Production, Utility, Budget Line

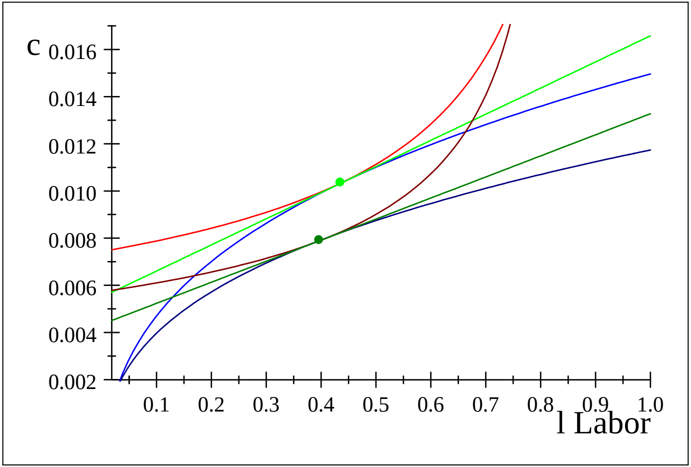

Figure 16.9. General Equilibrium Consumption and Utility Levels in Baseline Example 16.2 (lighter red, blue, green) and Crisis Example 16.3 (darker red, blue, green).

2007-2010 Crash: Viewed as Bank Productivity Fall

- Huge debt in US during wartime in Iraq, Afganistan.
  - Threat of future taxes; can decrease growth, asset prices.
  - Unexpected asset price fall can lead to bank crisis,
  - if asset prices serve as collateral for loans.
- 2007-2009: housing prices fell as world asset prices fell.
  - As Interest rates rose, so did loan repayments,
  - but house prices were down, and could not just sell house.
- Bankruptcies ensued: banks lost assets, collapsed worldwide.
  - Banks then provided financial intermediation services,
  - but less efficiently in wake of widespread collapse of sector.
  - As if a big fall in bank productivity.

Comparison of 1930s, 2007-2010 Crashes

- 1930s, 2007-2009 crises characterized by lack of insurance
  - against so-called aggregate risk arising from banking sector.
  - Lack of effective insurance in banking led to collapse;
  - can be viewed as large drop in bank productivity.
  - Would not have occured with working banking insurance.
- US in 1930s waited until 1933 to supply bank insurance
  - through new Federal Deposit Insurance Corporation (FDIC).
  - Governments acted more quickly in 2007-2009,
  - through a variety of measures to insure liabilities of banks,
  - but without a systematic finance industry insurance scheme.
- Ad hoc insurance 2007-2010 made recession less than 1930s.

Bank Insurance Policy to Internalize Aggregate Risk

- Resort to ad hoc bank insurance after collapse
  - gives very inefficient banking insurance system,
  - covering all types of financial intermediation,
  - from commercial banking to investment banking, to insurance:
  - even insurance company AIG was rescued by US.
- Also added car financing government aide
  - through subsidization of old car "scrappage" internationally.
  - Included partial nationalization of banks, worldwide,
  - increased bank capital requirements.

System Wide Bank Insurance

- De facto aggregate risk to entire financial sector
  - ultimately insured by governments.
- Suggests risk could have been insured in first place,
  - in more efficient, systematic, non-discriminatory fashion.
- Aggregate risk from banking a result of incomplete insurance.
- More complete insurance systems
  - could minimize residual aggregate risk from banking.

Deposit Insurance

- Main policy option: offset aggregate risk by
  - efficient systematic financial intermediation insurance.
  - Should decrease unexpected fall in effective bank productivity,
  - decrease need to supply ex post ad hoc residual insurance,
  - avoid moral hazard by internalizing true cost of bank insurance.
- US before 1933, had industry-formed "clearinghouses"
  - acted as means of insuring depositors at failing bank,
  - designed to minimize risk of failure spreading across banks.
- Market-based method of providing insurance against aggregate risk.

Bank Insurance with Risk-Based Premiums

- Establishment of US Federal Reserve Bank in 1913,
  - eliminated clearinghouse functions and private insurance.
  - Yet when banks failed in 1930s, US failed to insure them.
- Panic spread across banks, until new 1933 bank insurance law.
- Created flat-rate bank insurance premium system
- 2005 FDIC system reformed to have "risk-based premiums",
  - for bank insurance based on asset structure.
  - System has largely worked, except that financial institutions
  - not covered by FDIC
  - failed during 2007-2010 banking crisis.

Risk Based Assessments

- Lack of insurance over investment banks caused bank panic.
- No risk based premium system worldwide, even in Basel Accords:
- George Soros quote, of need to have risk-based assessment

"the Basel Accords made a mistake when they gave securities held by banks substantially lower risk ratings than regular loans: they ignored the systemic risks attached to concentrated positions in securities. This was an important factor aggravating the crisis. It has to be corrected by raising the risk ratings of securities held by banks. That will probably discourage the securitization of loans."

Global Bank Failure, Aggregate Risk, Moral Hazard

- International Monetary Fund (IMF) has long acted to intervene
  - to try to contain financial panics within countries, regions,
  - Latin America during 1980s; Asian during 1997.
- IMF criticism: inefficient insurance against aggregate financial risk.
  - IMF may increase probability of recurring bank failure,
  - through moral hazard: policy increases probability of bad state.
- No systematic international financial insurance.

International Finance Systemic Insurance

- US FDIC success: risk-based premium system can work
  - if applied to all financial institutions,
  - within global capital markets.
- Aggregate risk of bank failure could be largely eliminated.
- First issue: how to bring all US financial institutions
  - into FDIC type of insurance system,
  - including investment banks, insurance.

Analysis Used to Formulate International Insurance System

- Our analysis: compute expected payout $\left(1 + R^d\right) d$,
  - for every finance institution during economic downturn.
  - Base risk-based premium on R^d for each intermediary,
  - dependent on asset structure.
  - Mutual funds of investment banks, insurance funds,
  - pension funds, could all be so insured.
- Internationally, risk-based deposit insurance system
  - could be implemented as replacement for IMF operations.
  - Provide systematic insurance instead of ad hoc,
  - with less moral hazard resulting.

Appendix A16: Solution Methodology with Banking

- Use capital marginal product, bank interest differential,
  - plus intertemporal margin, and labor marginal product,
  - solve for $\frac{l_t}{k_t}$ implicitly.

$$
\begin{aligned}
(1-\gamma) A \left(\frac{l_t}{k_t}\right)^{\gamma} - \delta_k &= R_t^q = R_t^d + \frac{w_t}{(A_F)^{\frac{1}{\kappa}}} \\
R_t^d &= \rho, \; w_t = \gamma A_G \left(\frac{l_t}{k_t}\right)^{\gamma-1}, \\
(1-\gamma) A \left(\frac{l_t}{k_t}\right)^{\gamma} - \delta_k &= \rho + \frac{\gamma A_G \left(\frac{l_t}{k_t}\right)^{\gamma-1}}{(A_F)^{\frac{1}{\kappa}}}.
\end{aligned}
$$

- Use solution for $\frac{l_t}{k_t}$ to solve rest of variables.

Appendix A16: Solution for Capital Stock

- k_t as a function of $\frac{l_t}{k_t}$: use time constraint, $1 = x_t + l_t + l_{Ft}$,
  - substitute in l_{Ft} from bank problem: $\frac{l_{Ft}}{d_t} = \frac{1}{(A_F)^{\frac{1}{\kappa}}}$.
  - With $k_t = d_t$, plus goods-leisure margin, and c^d :

$$\begin{aligned}
l_{Ft} &= \frac{k_t}{(A_F)^{\frac{1}{\kappa}}},\ x_t = \frac{\alpha c_t}{w_t},\ c_t = \frac{1}{1+\alpha}\left(w_t + \rho k_t\right), \\
1 &= x_t + l_t + l_{Ft} = \frac{\alpha}{1+\alpha}\left[1 + \frac{\rho k_t}{w_t}\right] + l_t + \frac{k_t}{(A_F)^{\frac{1}{\kappa}}}, \\
w_t &= \gamma A_G \left(\frac{l_t}{k_t}\right)^{\gamma-1},
\end{aligned}$$

$$\Longrightarrow k_t = \frac{1}{\left[\frac{\alpha\rho\left(\frac{l_t}{k_t}\right)^{1-\gamma}}{\gamma A_G} + (1+\alpha)\frac{l_t}{k_t} + \frac{(1+\alpha)}{(A_F)^{\frac{1}{\kappa}}}\right]}.$$

Appendix A16: Example 16.2

- $A_G = 0.1$, $\gamma = \kappa = \frac{1}{3}$, $\delta_k = 0.05$, $\alpha = 1$, $\beta = 0.95$, $A_F = 0.95$.
- Define $Z\left(\frac{l_t}{k_t}\right) = 0$ from solution equation for $\frac{l_t}{k_t}$:

$$
\begin{aligned}
Z\left(\frac{l_t}{k_t}\right) &= (1-\gamma) A_G \left(\frac{l_t}{k_t}\right)^{\gamma} - \frac{\gamma A_G \left(\frac{l_t}{k_t}\right)^{\gamma-1}}{(A_F)^{\frac{1}{\kappa}}} - \delta_k - \rho = 0, \\
0 &= \left(1-\frac{1}{3}\right)(0.1)\left(\frac{l_t}{k_t}\right)^{\frac{1}{3}} - \frac{0.1\left(\frac{l_t}{k_t}\right)^{-\frac{2}{3}}}{3\,(0.95)^{3}} - 0.1026, \\
\Longrightarrow &\quad \frac{l_t}{k_t} = 5.205.
\end{aligned}
$$

Graph of Solution Equation for Labor/Capital

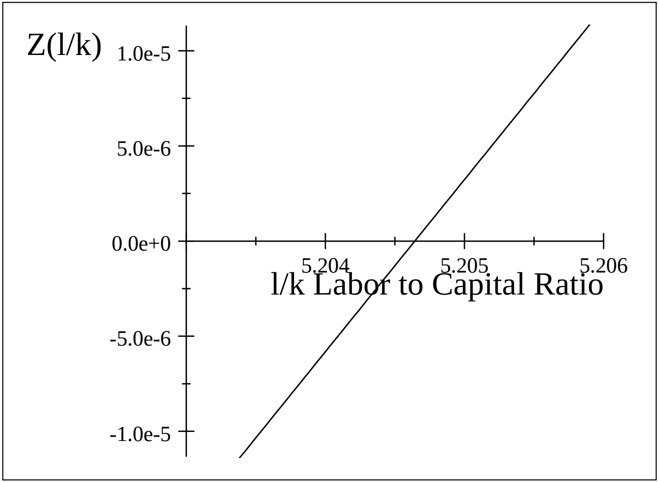

Figure 16.10. Labor to Capital Equilibrium Solution in Example 16.2.

Solution for Capital Stock, Other Variables

$$k_t = \frac{1}{\frac{3(0.5)(0.0526)(05.205)^{-\frac{2}{3}}}{(0.10)} + (1.5)(05.205) + \frac{(1.5)}{(0.95)^3}} = 0.08382.$$

- $w_t = 0.01108\,1$; $l_t = \left(\frac{l_t}{k_t}\right) k_t = (5.205)(0.08382) = 0.436,$
  - $l_{Ft} = \frac{k_t}{(A_F)^{\frac{1}{\kappa}}} = \frac{(0.08382)}{(0.95)^3} = 0.098,$
  - $x_t = \frac{0.5}{1+0.5}\left(1 + \frac{0.0526(0.08382)}{0.0111}\right) = 0.465\,7.$
  - $0.436\,3 + 0.098 + 0.465\,7 = 1.0$: time allocation.
  - $c_t = \frac{1}{1+\alpha}(w_t + \rho k_t) = \frac{1}{1.5}(0.0111 + 0.0526(0.08382)) = 0.01034.$
  - $y_t = \frac{1}{1+\alpha}(w_t + \rho k_t) + \delta_k k_t = 0.01034 + (0.05)(0.08382) =$ $0.014531.$
  - $R_t^q - R_t^d = \frac{w_t}{(A_F)^{\frac{1}{\kappa}}} = \frac{0.0111}{(0.95)^{\frac{1}{0.333}}} = 0.012948;$
  - $R^q = 0.012948 + 0.052632 = 0.06558.$

Example 16.3: Bank Crisis

- All parameters the same except $A_F = 0.7$:
  - $\Longrightarrow \frac{l_t}{k_t} = 7.1925.$

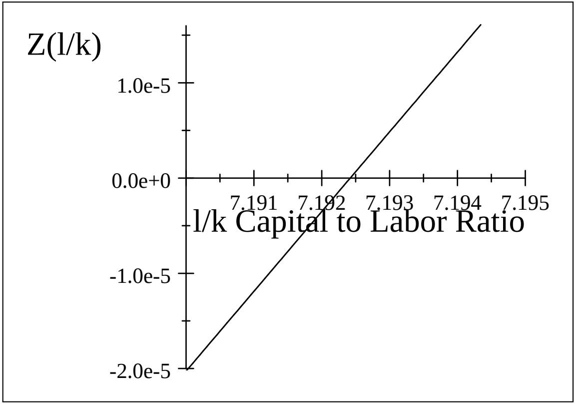

Figure 16.11. Labor to Capital Equilibrium Solution in Example 16.3.

Example 16.3: Other Variables

$$k_t = \frac{1}{\frac{(0.5)(0.0526)(7.193)^{1-0.333}}{(0.333)(.10)} + (1+(0.5))(7.193) + \frac{(1+(0.5))}{(0.7)^{\frac{1}{0.333}}}}$$
$$= 0.05521.$$

- $w_t = 0.333\,(0.10)\,(7.193)^{0.333-1} = 0.00893;$
  - $l_t = \frac{l_t}{k_t} k_t = (7.193)\,(0.05521) = 0.397;$
  - $l_{Ft} = \frac{k_t}{(A_F)^{\frac{1}{\kappa}}} = \frac{(0.05521)}{(0.7)^{\frac{1}{0.333}}} = 0.161;$
  - $x_t = \frac{\alpha}{1+\alpha}\left[1 + \frac{\rho k_t}{w_t}\right] = \frac{0.5}{1+0.5}\left(1 + \frac{0.0526(0.05521)}{0.00893}\right) = 0.44173;$
  - total time $1 = 0.397\,13 + 0.16113 + 0.44173;$
  - $R_t^q - R_t^d = \frac{w_t}{(A_F)^{\frac{1}{\kappa}}} = \frac{0.00893}{(0.7)^{\frac{1}{0.333}}} = 0.02606;$
  - $c_t = \frac{1}{1+0.5}\,(0.00893 + 0.0526\,(0.0552)) = 0.0079;$
  - $\delta_k k_t = (0.05)\,(0.05521) = 0.00276;$
  - $y_t = c_t + \delta_k k_t = 0.00789 + 0.00276 = 0.01065.$