Advanced Modern Macroeconomics

Taxes and Growth

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- Effect of taxes, on capital, labor and goods,
  - with focus on BGP growth rate effect.
  - Uses endogenous growth dynamic model,
  - extended from Part 5 baseline.
- Capital income tax added,
  - with new government budget constraint.
  - Modified $AS - AD$ approach; AS, AD shift back, w falls.
  - Growth rate falls, capital ratio falls.
  - Labor demand shifts back, supply shifts out, employment falls.
- Extended to labor tax: growth fall compared to capital tax.
  - Labor tax causes capital to human capital ratio to rise,
  - as substitution from taxed human capital towards physical capital.
  - Opposite of effect with capital tax.
- Consumption tax also analyzed: similar to labor tax.

Building on the Last Chapters

- Chapter 3, 6 : static models with labor capital taxes.
  - Chapter 9 has labor tax in dynamic model but exogenous growth,
  - in which can be no growth effect.
- Chapters 3 ,9 : labor tax decreases output, employment;
  - here employment falls but output per h rises
  - as substitute from human to physical capital,
  - while growth rate falls.
- Solution methodology modification of Chapter 12.
  - Still a closed form solution, with quadratic solution equation.

Learning Objective

- All taxes decrease balanced path equilibrium growth rate.
  - Major taxes: on labor, capital, goods, and money use in Chapter 20.
  - Fundamental element of macroeconomic government policy.
  - International policy emphasis on high growth for last 25 years,
  - requires understanding how taxes affect growth.
- Taxes uniformily decrease employment with endogenous growth.
- Difference between capital, labor taxes:
  - taxing physical capital versus taxing human capital.
  - Explains changes in capital ratios with taxation.

Who Made It Happen

- Frank Ramsey 1927 derived optimization analysis of taxes,
  - gave rise to specialized field Public Finance.
  - Lucas 1988 endogenous growth model allows tax extension.
- King, Rebelo 1990 "Public Policy and Economic Growth:
  - Developing Neoclassical Implications"; endogenous growth, taxes.
  - Rebelo 1991 "Long-Run Policy Analysis and Long-Run Growth,"
  - Stokey, Rebelo's 1995 "Growth Effects of Flat-Rate Taxes".
- Many such journal articles show how tax rates reduce growth.

Capital Income Tax

- Assume tax on capital income, τ_k, capital income $r_t k_t (1 - \tau_k)$;
  - government spending G_t equals capital tax revenue; lump sum transfer:

$$G_t = \tau_k r_t k_t.$$

  - Consumption of consumer budget constraint:

$$c_t = w_t l_t h_t + r_t k_t (1 - \tau_k) - k_{t+1} + k_t - \delta_k k_t + G_t.$$

- Endogenous growth dynamic model:

$$\begin{aligned} V(k_t, h_t) &= \underset{k_{t+1}, l_t, l_{Ht},}{Max} \\ &\ln [w_t l_t h_t + r_t k_t (1 - \tau_k) - k_{t+1} + k_t - \delta_k k_t + G_t] \\ &+\alpha \ln (1 - l_{Ht} - l_t) + \beta V [k_{t+1}, h_t (1 - \delta_h) + A_H l_{Ht} h_t]. \end{aligned}$$

- Tax: output, employment, investment rate, wage, after tax rental rate, g, all fall.

Equilibrium Conditions with Capital Income Tax

$$\begin{aligned}
k_{t+1} &: \quad \frac{1}{c_t}(-1) + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial k_{t+1}} = 0, \\
Envelope\ k_t &: \quad \frac{\partial V(k_t, h_t)}{\partial k_t} = \frac{1}{c_t}\left[1 + r_t(1-\tau_k) - \delta_k\right], \\
l_t &: \quad \frac{1}{c_t}(w_t h_t) + \frac{\alpha}{x_t}(-1) = 0, \\
l_{Ht} &: \quad \frac{\alpha}{x_t}(-1) + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial h_{t+1}}(A_H h_t) = 0, \\
Envelope\ h_t &: \\
\frac{\partial V(k_t, h_t)}{\partial h_t} &= \frac{1}{c_t}(w_t l_t) + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial h_{t+1}}(1 + A_H l_{Ht} - \delta_H); \\
\frac{c_{t+1}}{c_t} &= \frac{1 + r_t(1-\tau_k) - \delta_k}{1+\rho} = \frac{1 + A_H(1 - x_t) - \delta_h}{1+\rho}.
\end{aligned}$$

Growth Rate Conditions with Capital Tax

$$1+g = \frac{1+r_t\left(1-\tau_k\right)-\delta_k}{1+\rho},$$
$$1+g = \frac{1+A_H\left(1-x_t\right)-\delta_H}{1+\rho}.$$

-
  - where $r_t = (1-\gamma) A_G \left(\frac{k_t}{h_t l_t}\right)^{-\gamma}$, and for x_t :

$$MRS_{c,x} : \frac{\alpha c_t}{x_t} = w_t h_t.$$

- Tax rate τ_k directly reduces after tax marginal product of capital
  - and so growth rate; implies return to human capital must fall
  - $A_H\left(1-x_t\right)$ must fall as τ_k increases, so x_t must rise,
  - as substitute from goods to leisure.

AS-AD: Consumption Demand with Capital Tax

- Taxes paid, government spending cancel out in equilibrium
  - same consumption demand function as Chapter 12:

$$\begin{aligned} c_t &= w_t l_t h_t + r_t k_t - k_{t+1} + k_t - \delta_k k_t, \\ c_t^d &= \frac{w_t h_t \left(1 - \frac{g+\delta_H}{A_H}\right) + k_t \left(r_t - \delta_k - g\right)}{1+\alpha}. \end{aligned}$$

- Difference is that term r_t affected by tax:

$$\begin{aligned} 1+g &= \frac{1 + r_t \left(1 - \tau_k\right) - \delta_k}{1+\rho}, \ r_t = \frac{\left(1+g\right)\left(1+\rho\right) - 1 + \delta_k}{\left(1-\tau_k\right)}, \\ c_t^d &= \frac{w_t h_t \left(1 - \frac{g+\delta_H}{A_H}\right) + k_t \left(\frac{\rho(1+g) + (g+\delta_k)\tau_k}{(1-\tau_k)}\right)}{1+\alpha}. \end{aligned}$$

AS-AD with Capital Tax

- Aggregate demand AD :
- add investment $k_t\,(g+\delta_k)$ to consumption demand function.
- AS unchanged from Chapters $8-13$.

$$y_t^d = \frac{w_t h_t\left(1-\frac{g+\delta_H}{A_H}\right)+k_t\left(\frac{\rho(1+g)+(g+\delta_k)[1+\alpha(1-\tau_k)]}{(1-\tau_k)}\right)}{1+\alpha},$$

$$y_t^s = A_G\left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t.$$

Solution Methodology with Capital Tax

- Excess aggregate output demand $Y(w_t, h_t, k_t, g) = 0$:

$$Y(w_t, h_t, k_t, g) = y_t^d - y_t^s = \frac{\left[w_t h_t \left(1 - \frac{g+\delta_H}{A_H}\right) + k_t \left(\frac{\rho(1+g)+(g+\delta_k)[1+\alpha(1-\tau_k)]}{(1-\tau_k)}\right)\right]}{1+\alpha}$$

$$-A_G \left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t = 0$$

  - Divide by $w_t h_t$:

$$0 = \frac{\left[\left(1 - \frac{g+\delta_H}{A_H}\right) + \frac{k_t}{w_t h_t} \left(\frac{\rho(1+g)+(g+\delta_k)[1+\alpha(1-\tau_k)]}{(1-\tau_k)}\right)\right]}{1+\alpha}$$

$$-A_G \left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} \frac{k_t}{w_t h_t}.$$

- Solve for $\frac{k_t}{w_t h_t}$, w_t in terms of g, get one equation in g.

Solving for Variables in Terms of g with Capital Tax

- From Chapter Appendix $A12$, $l_t = \frac{(1+g)(1-\beta)}{A_H \beta}$.
  - $\frac{k_t}{h_t l_t}$ can be solved from marginal product of capital

$$r_t = (1-\gamma) A_G \left(\frac{k_t}{h_t l_t}\right)^{-\gamma}, \quad \frac{k_t}{h_t l_t} = \left(\frac{(1-\gamma) A_G}{r_t}\right)^{\frac{1}{\gamma}},$$

$$r_t = \frac{(1+g)(1+\rho) - 1 + \delta_k}{(1-\tau_k)}, \quad \frac{k_t}{h_t l_t} = \left(\frac{(1-\gamma) A_G}{\frac{(1+g)(1+\rho)-1+\delta_k}{(1-\tau_k)}}\right)^{\frac{1}{\gamma}}.$$

  - Marginal product of labor

$$w_t = \gamma A_G \left(\frac{k_t}{h_t l_t}\right)^{1-\gamma} = \gamma A_G \left(\frac{(1-\gamma) A_G}{\frac{(1+g)(1+\rho)-1+\delta_k}{(1-\tau_k)}}\right)^{\frac{1-\gamma}{\gamma}},$$

$$\frac{k_t}{w_t h_t} = \left(\frac{k_t}{h_t l_t}\right) \frac{l_t}{w_t} = \left(\frac{(1-\gamma) A_G}{\frac{(1+g)(1+\rho)-1+\delta_k}{(1-\tau_k)}}\right) \frac{\frac{(1+g)(1-\beta)}{A_H \beta}}{\gamma A_G}.$$

Normalized Excess Demand Function in Terms of g

$$
\begin{aligned}
0 \;=\; & \left(\frac{1}{1+\alpha}\right)\left(1-\frac{g+\delta_H}{A_H}\right)-\frac{(1+g)(1-\beta)}{\gamma A_H \beta}+ \\
& \frac{(1-\gamma)A_G(1+g)(1-\beta)\left(\rho(1+g)+(g+\delta_k)\left[1+\alpha(1-\tau_k)\right]\right)}{(1+\alpha)\gamma A_G A_H \beta(1+g)(1+\rho)-1+\delta_k}.
\end{aligned}
$$

$$
\begin{aligned}
\Longrightarrow\quad & 0=\beta(A_H-g-\delta_H)\gamma\left[(1+g)+\beta(\delta_k-1)\right] \\
& +\beta(1-\gamma)(1+g)(1-\beta)\left(\rho(1+g)+(g+\delta_k)\left[1+\alpha(1-\tau_k)\right]\right) \\
& -(1+\alpha)(1+g)(1-\beta)\left[(1+g)+\beta(\delta_k-1)\right].
\end{aligned}
$$

- If $\tau_k = 0$, same solution equation as in Chapter 12.

Example 19.1 Baseline Calibration with Capital Tax

- Assume same parameters as Example 13.2
  - $\gamma = \frac{1}{3}$, $\alpha = 1$, $A_h = 0.20$, $\delta_k = 0.05$, $\delta_h = 0.015$, $\beta = \frac{1}{1+\rho} = 0.95$, $A_G = 0.282\,24$; $\Longrightarrow g = 0.0333$, $\tau_k = 0$.
  - Assume government spending a constant share of output; plus $\tau_k = 0.30$;

$$\frac{G_t}{y_t} = \frac{\tau_k r_t k_t}{y_t} = \tau_k (1-\gamma) = 0.30\,(\,0.67) = 0.201.$$

  - 20% of output.
- Solve for g from excess demand solution equation:

$$\begin{aligned} 0 &= (0.95)(0.20 - g - 0.015)\frac{1}{3}((1+g) + 0.95(0.05-1)) + \\ &\quad \frac{(0.95)\,2\,(1+g)(0.05)\left[0.0526(1+g) + (g+0.05)(2-0.3)\right]}{3} \\ &\quad -2(1+g)(1-0.95)((1+g) + 0.95(0.05-1)). \end{aligned}$$

Example 19.1: Graphical Solution for g with Capital Tax

$g = 0.0121.$

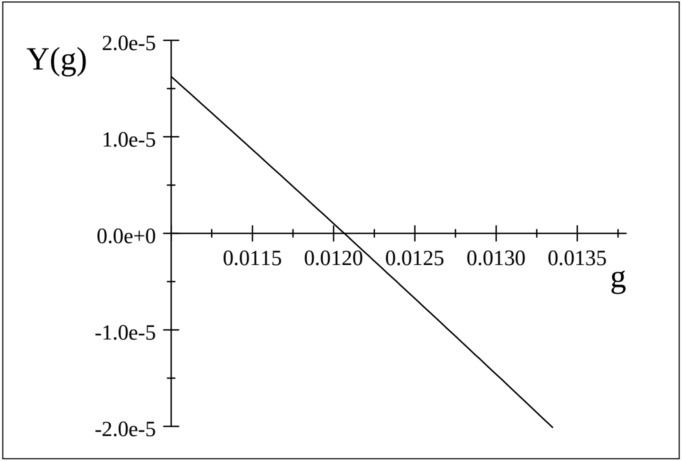


Figure 19.1. Excess Output Demand with Capital Income Tax $\tau_k = 0.30$ in Example 19.1.

Analytic Solution for g with Capital Tax

- Quadratic solution for g: $Ag^2 + Bg + C = 0$,

$$
\begin{aligned}
& A \\
\equiv \; & -\beta\gamma + \beta(1-\beta)(1-\gamma)\left[1+\alpha(1-\tau_k)+\rho\right] - (1+\alpha)(1-\beta), \\
& B \\
\equiv \; & -\beta\gamma\left[1+\beta(\delta_k - 1) - A_H + \delta_H\right] \\
& -(1+\alpha)(1-\beta)\left[2+\beta(\delta_k - 1)\right] + \\
& \beta(1-\beta)(1-\gamma)\left\{\rho + \delta_k\left[1+\alpha(1-\tau_k)\right] + 1 + \alpha(1-\tau_k) + \rho\right\}, \\
& C \\
\equiv \; & \beta\gamma(A_H - \delta_H)\left[1+\beta(\delta_k - 1)\right] - (1+\alpha)(1-\beta)\left[1+\beta(\delta_k - 1)\right] \\
& +\beta(1-\beta)(1-\gamma)\left\{\rho + \delta_k\left[1+\alpha(1-\tau_k)\right]\right\}.
\end{aligned}
$$

- Calibration: $A = -0.36117$, $B = -0.02218$, $C = 0.00032021$; $g = 0.0121$.

Example 19.1 AS-AD with 30% Capital Tax

- $\frac{k_t}{h_t} = \left(\frac{k_t}{h_t l_t}\right) l_t = (1.488)(0.26634) = 0.396\,32;\ \tau_k = 0,\ \frac{k_t}{h_t} = 0.694.$
- $AD - AS:$

$$\frac{1}{w_t} = \frac{\left(1 - \frac{g+\delta_H}{A_H}\right)}{\frac{y_t^d}{h_t}(1+\alpha) - \frac{k_t}{h_t}\left(\frac{\rho(1+g)+(g+\delta_k)[1+\alpha(1-\tau_k)]}{(1-\tau_k)}\right)}$$

$$= \frac{\left(1 - \frac{0.0121+0.015}{0.20}\right)}{2y - (0.396)\left[\frac{(0.0526)(1+0.0121)+(0.0121+0.05)(1+1(1-0.30))}{1-0.30}\right]};$$

$$\frac{1}{w_t} = \frac{\left(y_t^s\right)^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{\gamma}}\left(k_t\right)^{\frac{1-\gamma}{\gamma}}} = \frac{3y_t^2}{0.282\,24\left[(0.282\,24)(0.396\,32)\right]^2}$$

- Equilibrium wage: marginal product of labor with $\frac{k_t}{h_t l_t} = 1.\,488:$

$$w_t = \gamma A_G\left(\frac{k_t}{h_t l_t}\right)^{1-\gamma} = \frac{1}{3}(0.282\,24)(1.\,488)^{\frac{2}{3}} = 0.12262.$$

Tax Shifts Back Both Supply, Demand: Output, Wage Falls

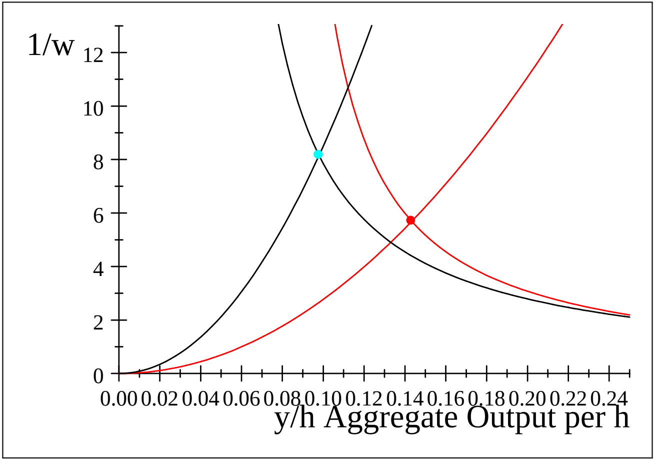


Figure 19.2. $AS - AD$ with a 30% Capital Income Tax in Example 19.1 (black) and a Zero Tax of Example 13.2 (red).

Consumption and Output

$$
\begin{aligned}
c_t^d &= (0.5)\,0.12262\left(1-\frac{0.0121+0.015}{0.20}\right) \\
&\quad +0.5\,(0.396\,32)\,\frac{(0.05\,2632\,(1.0121)+(0.0621)\,0.30)}{1-0.30}=0.073; \\
y_t^d &= (0.5)\,0.122\,62\left(1-\frac{0.0121+0.015}{0.20}\right)+ \\
&\quad \frac{(0.396\,32)\,(0.0526\,(1.0121)+(0.0621)\,(2-0.30))}{2\,(1-0.30)}=0.098; \\
\frac{c_t^d}{y_t^d} &= \frac{0.073356}{0.097968}=0.748\,78.
\end{aligned}
$$

- Substantial increase in $\frac{c_t^d}{y_t^d}$: was 0.5966 with zero tax rate, Example 13.2
  - Higher fraction consumed, less invested when tax on capital income.

Revenue and Interest Rate with Capital Tax

- Present value of government revenue:

$$\frac{G_t}{\rho(1+g)} = \frac{\frac{G_t}{y_t} y_t}{\rho(1+g)} = \frac{\frac{\tau_k r_t k_t}{y_t} y_t}{\rho(1+g)} = \frac{\tau_k (1-\gamma) y_t}{\rho(1+g)}$$

$$= \frac{(0.30)\frac{2}{3}(0.097968)}{0.052632(1+0.0120)} = 0.367\,86.$$

- Higher interest rate with tax:

$$r_t = \frac{(1+g)(1+\rho)-1+\delta_k}{(1-\tau_k)},$$

$$r_t = \frac{(1+0.0121)(1+0.052632)-1+0.05}{(1-0.30)} = 0.16481.$$

  - With no tax, $r_t = 0.1377$, in Example 13.2.
  - After tax rate is 11.5% :

$$r_t(1-\tau_k) = 0.16481(1-0.30) = 0.11537.$$

  - Explains lower capital ratio of 0.40 compared to 0.69 with no tax.

- Labor supply affected by capital tax; labor demand unaffected

$$\frac{c_t^d \alpha}{w_t h_t} = x_t = 1 - l_t^s - l_{Ht}, \; l_t^s = 1 - \frac{c_t^d \alpha}{w_t h_t} - l_{Ht},$$

$$\frac{c_t^d}{h_t} = \frac{w_t}{1+\alpha}\left(1 - \frac{g+\delta_H}{A_H}\right) + \frac{k_t\left(\frac{\rho(1+g)+(g+\delta_k)\tau_k}{(1-\tau_k)}\right)}{h_t\,(1+\alpha)},$$

$$l_t^s = 1 - \frac{\alpha}{1+\alpha}\left[1 + \frac{k_t\left(\frac{\rho(1+g)+(g+\delta_k)\tau_k}{(1-\tau_k)}\right)}{w_t h_t}\right] - \frac{g+\delta_H}{A_H\,(1+\alpha)};$$

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} \frac{k_t}{h_t}.$$

Inverted Labor Supply, Demand with Capital Tax

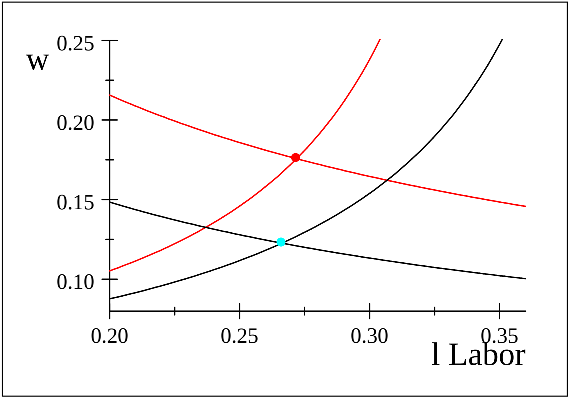
$$w_t = \frac{\alpha\left[\rho\left(1+g\right)+\left(g+\delta_k\right)\tau_k\right]\left(\frac{k_t}{h_t}\right)}{\left(1-\tau_k\right)\left[1-\left(1+\alpha\right)l_t^s-\frac{\left(g+\delta_H\right)}{A_H}\right]}$$

$$w_t = \frac{\left[\left(0.0526\right)\left(1.0121\right)+\left(0.0621\right)0.30\right]\left(0.396\right)}{\left(1-0.30\right)\left[1-2l_t^s-\frac{\left(0.0271\right)}{0.2}\right]};$$

$$w_t = \gamma A_G\left(\frac{k_t}{h_t l_t^d}\right)^{1-\gamma} = \frac{\left(0.282\,24\right)}{3}\frac{\left(0.396\,32\right)^{\frac{2}{3}}}{\left(l_t^d\right)^{\frac{2}{3}}}.$$

Labor Supply Shifts Out, Labor Market Shifts Back

- Employment falls to 0.26634 from 0.27192 with no tax; 2% fall.



Fitgure 19.3. Labor Market with 30% Capital Income Tax in Example 19.1 (in black) and Zero Tax (in red).

Isocost, Isoquant, Factor Input Ratio with Capital Tax

Isocost line:

$$\begin{aligned} 0.097968 &= y_t = w_t l_t h_t + r_t k_t, \\ \frac{k_t}{h_t} &= \frac{0.097968}{0.16481 h_t} - \frac{(0.122\,62)\, l_t}{0.16481}. \end{aligned}$$

Isoquant curve:

$$\begin{aligned} 0.097968 &= y_t^s = A_G \left(l_t^d h_t \right)^{\gamma} (k_t)^{1-\gamma}, \\ \frac{k_t}{h_t} &= \left(\frac{0.097968}{(0.28224)\, h_t \left(l_t^d \right)^{\frac{1}{3}}} \right)^{\frac{3}{2}} = \frac{\left(\frac{0.097968}{(0.28224) h_t} \right)^{\frac{3}{2}}}{\left(l_t^d \right)^{\frac{1}{2}}}. \end{aligned}$$

Factor input ratio:

$$\frac{k_t}{h_t l_t} = \frac{0.396\,32}{0.26634} = 1.488.$$

Shift Down of Isocost, Isoquant, Input Ratio

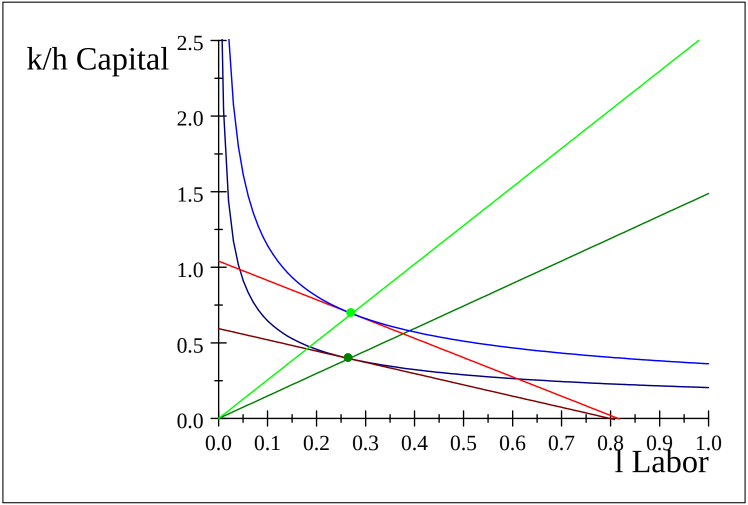


Figure 19.4. Factor Market Equilibrium with Endogenous Growth and Capital Income Tax of $\tau_k = 0.30$

Production, Utility, Budget Line with Capital Tax

Production

$$c_t^d = y_t^s - i_t = A_G \left(l_t^d h_t \right)^{\gamma} (k_t)^{1-\gamma} - (g + \delta_k)\, k_t,$$

$$\frac{c_t^d}{h_t} = (0.28224) \left(l_t^d \right)^{\frac{1}{3}} (0.396\,32)^{\frac{2}{3}} - (0.0621)\,(0.396\,32)\,.$$

Utility

$$-3.126\,3 = u = \ln c_t + \alpha \ln x_t = \ln c_t + \alpha \ln (1 - l_{Ht} - l_t)\,,$$

$$= \ln 0.073356 + 1 \ln (1 - (0.26634 + 0.135\,5))\,,$$

$$c_t = \frac{e^{-3.\,126\,3}}{(1 - 0.135\,5 - l_t)}.$$

Budget line

$$c_t^d = w_t l_t^s h_t + k_t (r_t - \delta_k - g)\,;$$

$$\frac{c_t^d}{h_t} = (0.122\,62)\, l_t^s + (0.396\,32)\,((0.16481) - 0.05 - 0.0121)\,.$$

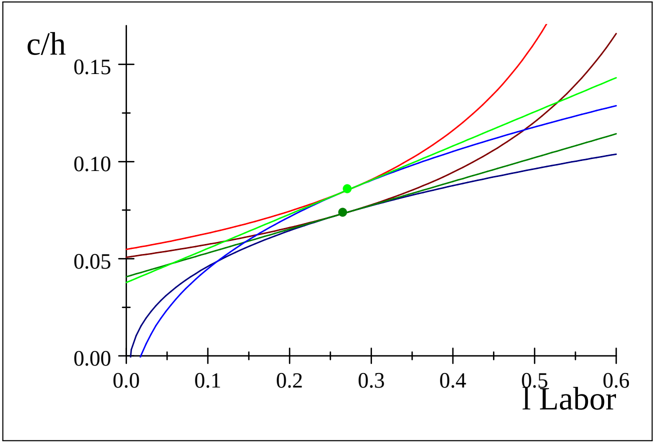


Figure 19.5. General Equilibrium Goods and Labor with Capital Income Tax of $\tau_k = 0.30$ in Example 19.1 Compared to $\tau_k = 0$ in Example 13.2.

Labor Income Tax

- Proportional tax on labor income τ_l with endogenous growth.
  - Consumer's wage income $w_t(1-\tau_l)\, l_t^s h_t$; government transfer G_t; government budget constraint

$$G_t = \tau_l w_t l_t h_t.$$

  - Consumer budget constraint

$$c_t^d = w_t h_t l_t^s (1-\tau_l) + r_t k_t + G_t - k_{t+1} + k_t (1-\delta_k).$$

- Consumer optimization problem

$$\begin{aligned} V(k_t, h_t) &= \underset{k_{t+1}, l_t, l_{Ht},}{Max} \\ & \ln\left[w_t h_t l_t (1-\tau_l) + r_t k_t - k_{t+1} + k_t - \delta_k k_t + G_t\right] \\ & + \alpha \ln (1 - l_{Ht} - l_t) + \beta V\left[k_{t+1}, h_t(1-\delta_h) + A_H l_{Ht} h_t\right]. \end{aligned}$$

Equilibrium and Envelope Conditions

$$
\begin{aligned}
k_{t+1} &: \quad \frac{1}{c_t}(-1) + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial k_{t+1}} = 0, \\
l_t &: \quad \frac{1}{c_t}\left[w_t h_t (1-\tau_l)\right] + \frac{\alpha}{x_t}(-1) = 0, \\
l_{Ht} &: \quad \frac{\alpha}{x_t}(-1) + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial h_{t+1}} (A_H h_t) = 0;
\end{aligned}
$$

$$
\begin{aligned}
k_t &: \quad \frac{\partial V(k_t, h_t)}{\partial k_t} = \frac{1}{c_t}(1 + r_t - \delta_k), \\
h_t &: \quad \frac{\partial V(k_t, h_t)}{\partial h_t} = \frac{1}{c_t}\left[w_t l_t (1-\tau_l)\right] \\
&\quad + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial h_{t+1}} (1 + A_H l_{Ht} - \delta_H).
\end{aligned}
$$

Labor Tax Affects Only Goods-Leisure Margin

- Same intertemporal margins as baseline endogenous growth.

$$1 + g = \frac{1 + r_t - \delta_k}{1 + \rho} = \frac{1 + A_H (1 - x_t) - \delta_k}{1 + \rho}$$

- Difference is tax affects goods-leisure margin, solved for x_t:

$$x_t = \frac{\alpha c_t}{w_t h_t (1 - \tau_l)}.$$

- Tax causes substitution from goods to leisure.

AS-AD Analysis with Labor Tax: Consumption Demand

- Tax affects c_t^d : $l_t = 1 - x_t - l_{Ht}$ $x_t = \frac{\alpha c_t}{w_t h_t (1-\tau_l)}$, $l_{Ht} = \frac{g+\delta_H}{A_H}$,

$$
\begin{aligned}
c_t^d &= w_t h_t l_t^s (1-\tau_l) + r_t k_t + G_t - k_{t+1} + k_t (1-\delta_k), \\
c_t^d &= w_t h_t l_t^s + r_t k_t - k_{t+1} + k_t (1-\delta_k), \\
c_t^d &= w_t h_t (1 - x_t - l_{Ht}) + r_t k_t - k_{t+1} + k_t (1-\delta_k), \\
c_t^d &= w_t h_t \left(1 - \frac{\alpha c_t}{w_t h_t (1-\tau_l)} - l_{Ht}\right) \\
&\quad + r_t k_t - k_{t+1} + k_t (1-\delta_k), \\
c_t^d &= \frac{w_t h_t \left(1 - \frac{g+\delta_H}{A_H}\right) + \rho (1+g) k_t}{\left(1 + \frac{\alpha}{(1-\tau_l)}\right)}.
\end{aligned}
$$

Add investment to consumption along BGP:

$$i_t = k_{t+1} - k_t\left(1-\delta_k\right) = k_t\left(1+g\right) - k_t\left(1-\delta_k\right) = k_t\left(g+\delta_k\right),$$

$$y_t^d = \frac{w_t h_t\left(1-\frac{g+\delta_H}{A_H}\right) + \rho\left(1+g\right)k_t}{\left(1+\frac{\alpha}{(1-\tau_l)}\right)} + k_t\left(g+\delta_k\right);$$

$$\frac{1}{w_t} = \frac{\left(1-\tau_l\right)\left(1-\frac{g+\delta_H}{A_H}\right)}{\frac{y_t^d}{h_t}\left(1+\alpha-\tau_l\right) - \frac{k_t}{h_t}\left[\left(1-\tau_l\right)\rho\left(1+g\right) + \left(g+\delta_k\right)\left(1+\alpha-\tau_l\right)\right]}.$$

$$y_t^s = A_G\left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t;\ \frac{1}{w_t} = \frac{\left(y_t^s\right)^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{\gamma}}\left(k_t\right)^{\frac{1-\gamma}{\gamma}}}.$$

Solution Methodology for Labor Tax

- Excess aggregate output demand $Y(w_t, h_t, k_t, g) = y_t^d - y_t^s = 0$,

$$0 = \frac{w_t h_t \left(1 - \frac{g+\delta_H}{A_H}\right) + \rho (1+g) k_t}{\left(1 + \frac{\alpha}{(1-\tau_l)}\right)} + k_t (g + \delta_k)$$
$$-A_G \left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t.$$

- Dividing through by $w_t h_t$ to get as function of g, $\frac{k_t}{w_t h_t}$:

$$0 = \frac{\left(1 - \frac{g+\delta_H}{A_H}\right) + \rho (1+g) \frac{k_t}{w_t h_t}}{\left(1 + \frac{\alpha}{(1-\tau_l)}\right)} + \frac{k_t}{w_t h_t} (g + \delta_k)$$
$$-A_G \left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} \frac{k_t}{w_t h_t}.$$

- Solve for $\frac{k_t}{w_t h_t}$ and w_t in terms of g, to get one equation in only g.

Solve for Effective Labor, Wage in Terms of g

$$
\begin{aligned}
l_t &= \frac{(1+g)(1-\beta)}{A_H \beta}, \; r_t = (1-\gamma) A_G \left(\frac{k_t}{h_t l_t}\right)^{-\gamma}, \\
\frac{k_t}{h_t l_t} &= \left(\frac{(1-\gamma) A_G}{r_t}\right)^{\frac{1}{\gamma}} = \left(\frac{(1-\gamma) A_G}{(1+g)(1+\rho) - 1 + \delta_k}\right)^{\frac{1}{\gamma}}, \\
w_t &= \gamma A_G \left(\frac{k_t}{h_t l_t}\right)^{1-\gamma} = \gamma A_G \left(\frac{(1-\gamma) A_G}{(1+g)(1+\rho) - 1 + \delta_k}\right)^{\frac{1-\gamma}{\gamma}}; \\
\frac{k_t}{w_t h_t} &= \left(\frac{k_t}{h_t l_t}\right) \frac{l_t}{w_t} = \frac{k_t}{w_t h_t} = \frac{(1-\gamma)(1+g)(1-\beta)}{\gamma A_H \beta \left[(1+g)(1+\rho) - 1 + \delta_k\right]}.
\end{aligned}
$$

Write Excess Demand as Function of g, Simplify

$$\frac{(1-\gamma)(1+g)(1-\beta)\left[(1-\tau_l)\rho(1+g)+(g+\delta_k)(1+\alpha-\tau_l)\right]}{\left[(1+g)(1+\rho)-1+\delta_k\right]\gamma A_H\beta(1+\alpha-\tau_l)}$$

$$+\quad \frac{(1-\tau_l)\left(1-\frac{g+\delta_H}{A_H}\right)}{1+\alpha-\tau_l}-\frac{(1+g)(1-\beta)}{\gamma A_H\beta}=0;$$

$$\begin{aligned}&\beta\gamma(1-\tau_l)(A_H-g-\delta_H)\left[(1+g)+\beta(\delta_k-1)\right]\\&+\beta(1-\gamma)(1+g)(1-\beta)\left[(1-\tau_l)\rho(1+g)+(g+\delta_k)(1+\alpha-\tau_l)\right]\\&-(1+g)(1-\beta)\left[(1+g)+\beta(\delta_k-1)\right](1+\alpha-\tau_l)=0.\end{aligned}$$

Example 19.2

- Same parameters as Example 13.2 : $\gamma = \frac{1}{3}$, $\alpha = 1$, $A_h = 0.20$, $\delta_k = 0.05$, $\delta_h = 0.015$, $\beta = \frac{1}{1+\rho} = 0.95$, $\rho = \frac{1}{0.95} - 1 = 0.0526$, $A_G = 0.282\,24$; plus $\tau_l = 0.144$.
  - Implies $g = 0.0120$.

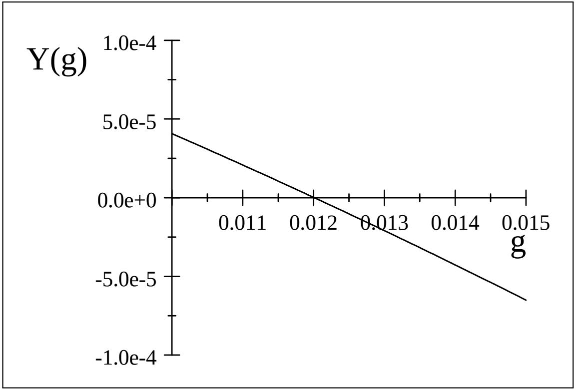


Figure 19.6. Normalized Excess Output Demand with Labor Income Tax of 14.4% in Example 19.2.

Example 19.2 Analytic Solution for Growth Rate

$Ag^2 + Bg + C = 0$:

$$
\begin{aligned}
A &\equiv -\beta\gamma\left(1-\tau_l\right)+\beta\left(1-\beta\right)\left(1-\gamma\right)\left[1+\alpha-\tau_l+\rho\left(1-\tau_l\right)\right] \\
&-\left(1+\alpha-\tau_l\right)\left(1-\beta\right), \\
B &\equiv -\left(1+\alpha-\tau_l\right)\left[2+\beta\left(\delta_k-1\right)\right]+\left(\beta-\beta^2\right)\left(\rho-\rho\gamma\right)\left(1-\tau_l\right) \\
&-\beta\gamma\left(1-\tau_l\right)\left(1+\beta\delta_k-\beta-A_H+\delta_H\right) \\
&+\left(1+\alpha-\tau_l\right)\left[\beta 2+\beta^2\left(\delta_k-1\right)\right] \\
&+\beta\left(1-\beta\right)\left(1-\gamma\right)\left\{\delta_k\left(1+\alpha-\tau_l\right)+\left[1+\alpha-\tau_l+\rho\left(1-\tau_l\right)\right]\right\}, \\
C &\equiv \beta\gamma\left(1-\tau_l\right)\left(A_H-\delta_H\right)\left[1+\beta\left(\delta_k-1\right)\right] \\
&+\beta\left(1-\beta\right)\left(1-\gamma\right)\left[\rho\left(1-\tau_l\right)+\delta_k\left(1+\alpha-\tau_l\right)\right] \\
&-\left(1+\alpha-\tau_l\right)\left(1-\beta\right)\left[1+\beta\left(\delta_k-1\right)\right].
\end{aligned}
$$

$$A = -0.3037,\ B = -0.0136,\ C = 0.0002067;\ g = 0.0120.$$

Example 19.2 Government Revenue, Equilibrium Values

- 30% labor tax yields about 10% of output; 14.4% tax: 5%

$$\frac{G_t}{y_t} = \frac{\tau_l w_t l_t h_t}{y_t} = \tau_l \gamma = \frac{0.144}{3} = 0.048.$$

- Growth rate falls from 3.33% with no tax to 1.21% with 0.144% tax.
- Other variables

$$l_t = \frac{(1+g)(1-\beta)}{A_H \beta} = \frac{(1+0.0120)(1-0.95)}{(0.20)\,0.95} = 0.26632;$$

$$\frac{k_t}{h_t l_t} = \left(\frac{\left(\frac{2}{3}\right) 0.282\,24}{(1+0.0120)(1+0.052632) - 1 + 0.05} \right)^3 = 4.350\,2;$$

$$\frac{k_t}{h_t} = \left(\frac{k_t}{h_t l_t} \right) l_t = (4.350\,2)(0.26632) = 1.1585.$$

- Rise in capital ratio as substitute from human to physical capital,
  - shifts out output supply, demand per unit of h.

Example 19.2 Calibrated AS-AD with Labor Tax

$$\frac{1}{w_t} = \frac{(1 - 0.144)\left(1 - \frac{0.027}{0.2}\right)}{y_t^d (2 - 0.144) - (1.1585)\left[(1.856)(0.0526)(1.012) + (0.062)\, 1.856\right]},$$

$$\frac{1}{w_t} = \frac{3}{0.282\,24}\left(\frac{1}{(0.28224)(1.1585)}\right)^2 \left(y_t^s\right)^2 .$$

Output Supply, Demand Per h Shift Out With Labor Tax

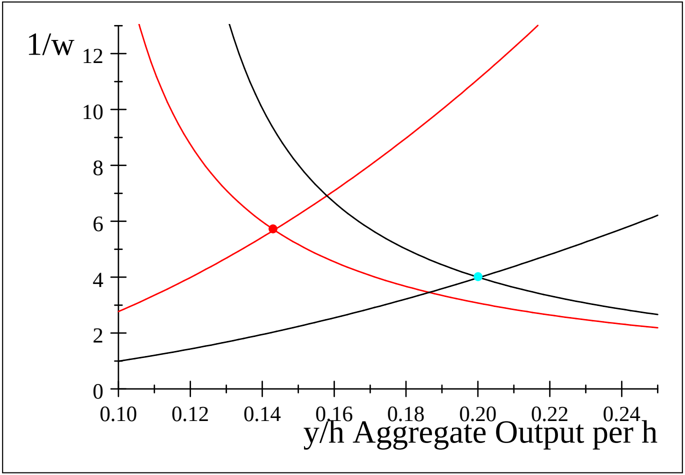


Figure 19.7. $AS - AD$ with a 14.4% Labor Income Tax in Example 19.2 (black) and a Zero Tax in Example 13.2 (red).

Wage Rate with Labor Tax

- $w_t = 0.250\,71$, with $\frac{1}{w_t} = \frac{1}{0.250\,71} = 3.\,988\,7$:

$$w_t = \gamma A_G \left(\frac{k_t}{h_t l_t} \right)^{1-\gamma} = \frac{1}{3} (0.282\,24) (4.350\,2)^{\frac{2}{3}} = 0.250\,71.$$

- Increase from 0.1757 with no tax in Example 13.2.
- After tax wage rate also higher than the baseline:

$$w_t (1 - \tau_l) = 0.25071 (1 - 0.144) = 0.214\,61.$$

Consumption-Output Ratio Rises with Labor Tax

$$\frac{c_t^d}{h_t} = = \frac{(1-0.144)}{(1+1-0.144)} 0.250\,71 \left(1 - \frac{0.0120 + 0.015}{0.20}\right) + \frac{(1-0.144)}{1+1-0.144} (1.1585) (0.05\,2632\,(1+0.0120)) = 0.128\,48.$$

$$\frac{y_t^d}{h_t} = 0.20031 = \left(\frac{1-0.144}{1+1-0.144}\right) 0.250\,71 \left(1 - \frac{0.0120 + 0.015}{0.20}\right) + \frac{(1.1585)}{2-0.144} \left[(1-0.144)\,0.0526\,(1.012) + (0.062)\,(2-0.144)\right]$$

$$\frac{c_t^d}{y_t^d} = \frac{0.128\,48}{0.200\,31} = 0.64.$$

- Above 0.5966 in Example 13.2, with no tax.

Government Revenue, Interest Rate

- Interest rate r_t falls, as g falls:

$$\begin{aligned} r_t &= (1+g)(1+\rho) - 1 + \delta_k, \\ r_t &= (1+0.0120)(1+0.052632) - 1 + 0.05 = 0.115\,26. \end{aligned}$$

- Present value of government revenue, with h_t normalized to 1 :

$$\begin{aligned} \frac{\frac{G_t}{y_t} y_t}{\rho(1+g)} &= \frac{\frac{\tau_l w_t l_t h_t}{y_t} y_t}{\rho(1+g)} = \frac{\tau_l \gamma y_t}{\rho(1+g)} \\ &= \frac{(0.144)(0.200\,31)}{0.052632(1+0.0120)3} = 0.18052. \end{aligned}$$

- 14.4% labor income tax about half of 30% capital income tax,
  - gives about half revenue that 30% capital income tax yields,
  - same growth rate decrease.

Labor Market with Labor Tax

$$
\begin{aligned}
l_t^s &= 1 - x_t - l_{Ht}.\ x_t = \frac{c_t^d \alpha}{w_t h_t \left(1 - \tau_l\right)},\ l_t^s = 1 - \frac{c_t^d \alpha}{w_t h_t \left(1 - \tau_l\right)} - l_{Ht}, \\
c_t^d &= \frac{\left(1 - \tau_l\right) w_t h_t \left(1 - \frac{g + \delta_H}{A_H}\right) + \left(1 - \tau_l\right) \rho \left(1 + g\right) k_t}{1 + \alpha - \tau_l},\ l_{Ht} - \frac{g + \delta_H}{A_H}, \\
l_t^s &= \frac{\left(1 - \tau_l\right)\left(1 - \frac{g + \delta_H}{A_H}\right) - \frac{\alpha \rho (1 + g) k_t}{w_t h_t}}{1 + \alpha - \tau_l}; \\
w_t &= \frac{\alpha \rho \left(1 + g\right) \frac{k_t}{h_t}}{\left(1 - \tau_l\right)\left(1 - \frac{g + \delta_H}{A_H}\right) - l_t^s \left(1 + \alpha - \tau_l\right)}.
\end{aligned}
$$

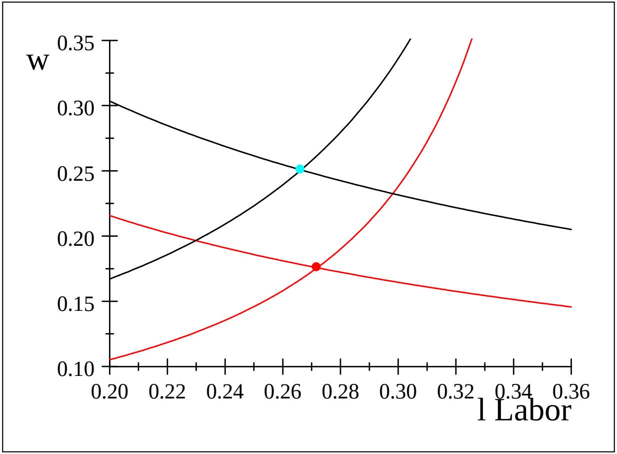
$$
l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} \frac{k_t}{h_t},\ w_t = \gamma A_G \left(\frac{k_t}{h_t l_t^d}\right)^{1-\gamma}.
$$

Calibrated Labor Supply, Demand with Labor Tax

$$w_t = \frac{1\left(0.052632\right)\left(1+0.0120\right)\left(1.1585\right)}{\left(1-0.144\right)\left(1-\frac{\left(0.0120+0.015\right)}{0.20}\right)-l_t^s\left(1+1-0.144\right)};$$

$$w_t = \frac{\left(0.282\,24\right)}{3}\frac{\left(1.1585\right)^{\frac{2}{3}}}{\left(l_t^d\right)^{\frac{2}{3}}}.$$

Supply Shifts Back, Demand Shifts Out, Employment Falls



Fitgure 19.8. Labor Market with 14.4% Labor Income Tax (in black) and Zero Tax (in red)

Isocost, Isoquant, Factor Input Ratio with Labor Tax

Isocost

$$0.200\,31 = y_t = w_t l_t h_t + r_t k_t = (0.250\,71)\, l_t + (0.115\,26)\, \frac{k_t}{h_t},$$

$$\frac{k_t}{h_t} = \frac{0.200\,31}{0.115\,26 h_t} - \frac{(0.250\,71)\, l_t}{0.115\,26}.$$

Isoquant

$$0.200\,31 = y_t^s = A_G \left(l_t^d h_t\right)^{\gamma} (k_t)^{1-\gamma} = (0.28224) \left(l_t^d h_t\right)^{\frac{1}{3}} (k_t)^{\frac{2}{3}};$$

$$\frac{k_t}{h_t} = \left(\frac{0.200\,31}{(0.28224)\, h_t \left(l_t^d\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}} = \frac{\left(\frac{0.200\,31}{(0.28224) h_t}\right)^{\frac{3}{2}}}{\left(l_t^d\right)^{\frac{1}{2}}}.$$

Factor input ratio

$$\frac{k_t}{h_t l_t} = \frac{1.1585}{0.26632} = 4.35.$$

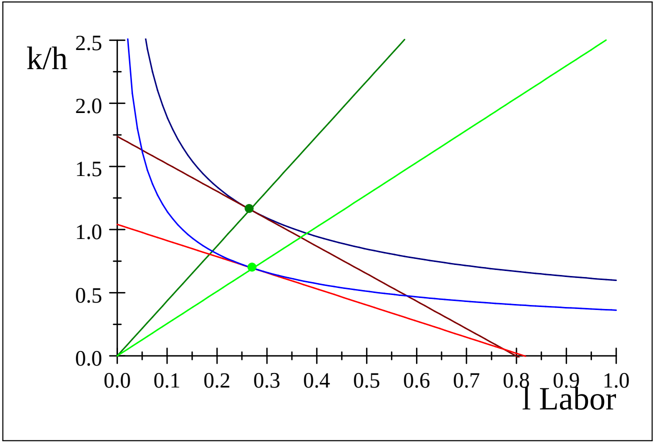


Figure 19.9. Factor Market Equilibrium with a Labor Income Tax of $\tau_l = 0.144$ in Example 19.2 Compared to the Zero Tax Example 13.2.

Production, Utility Level, Budget Line with Labor Tax

$$c_t^d = y_t^s - i_t = A_G \left(l_t^d h_t \right)^{\gamma} (k_t)^{1-\gamma} - (g + \delta_k)\, k_t,$$

$$\frac{c_t^d}{h_t} = (0.28224) \left(l_t^d \right)^{\frac{1}{3}} (1.1585)^{\frac{2}{3}} - (0.0120 + 0.05)(1.1585).$$

$$u = \ln c_t + \alpha \ln x_t = \ln c_t + \alpha \ln (1 - l_{Ht} - l_t),$$

$$-2.565 = \ln 0.128\,48 + 1 \ln (1 - (0.26632 + 0.135)),$$

$$-2.565 = \ln c_t + \ln (T_t - l_t),$$

$$c_t = \frac{e^{-2.565}}{(1 - 0.135 - l_t)}.$$

$$c_t^d = w_t l_t^s h_t (1 - \tau_l) + k_t [r_t - (g + \delta_k)] + G_t, \;\; G_t = 0.0096147,$$

$$\frac{c_t^d}{h_t} = (0.250\,71)\, l_t^s (1 - 0.144) + (1.1585)((0.115) - 0.0620) + 0.0096$$

Budget Line Crosses Production Function with Labor Tax

Utility Level Tangent to Budget Line

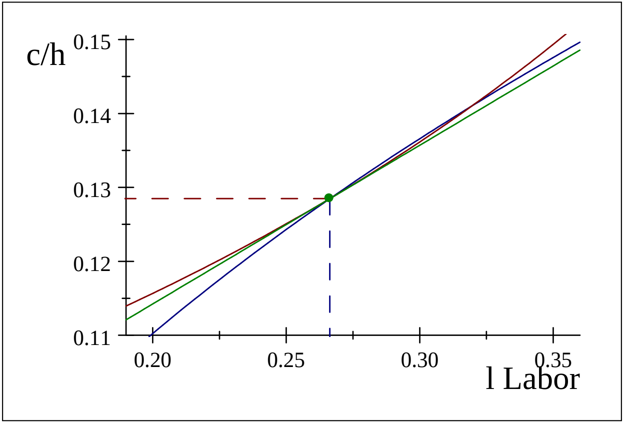


Figure 19.10. General Equilibrium Goods and Labor with a Labor Income Tax of $\tau_k = 0.144$ in Example 19.2.

Tax Wedge from Labor Tax

- Budget line intersects the production function in 2 places
  - rather than tangent to it: tax wedge graphical result.
  - Utility tangent to budget line at lower intersection point.
- Upper intersection point is if a tax subsidy to labor income.
- Wedge not apparent seen for capital tax in $\left(\frac{c}{h}, l\right)$ dimensions
  - because capital tax does not cause wedge intratemporally,
  - only intertemporally a wedge from capital tax.
- Similar labor tax wedge in Chapter 9, with exogenous growth.

Consumption VAT Tax

- Proportional goods sales tax: value-added tax (VAT) denoted by τ_c.
- Budget constraints

$$G_t = \tau_c c_t;$$

$$\begin{aligned} c_t^d (1+\tau_c) &= w_t h_t l_t^s + r_t k_t + G_t - k_{t+1} + k_t (1-\delta_k); \\ c_t^d &= \frac{w_t h_t l_t^s + r_t k_t + G_t - k_{t+1} + k_t (1-\delta_k)}{(1+\tau_c)}. \end{aligned}$$

Consumer problem

$$\begin{aligned} V(k_t, h_t) &= \underset{k_{t+1}, l_t, l_{Ht}}{Max} \\ & \ln \left[\frac{w_t h_t l_t + r_t k_t + G_t - k_{t+1} + k_t (1-\delta_k)}{(1+\tau_c)} \right] \\ & + \alpha \ln (1 - l_{Ht} - l_t) + \beta V \left[k_{t+1}, h_t (1-\delta_h) + A_H l_{Ht} h_t \right]. \end{aligned}$$

Equilibrium, Envelope Conditions with VAT

$$k_{t+1} \quad : \quad \frac{1}{c_t}\left(\frac{-1}{1+\tau_c}\right) + \beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial k_{t+1}} = 0,$$

$$l_t \quad : \quad \frac{1}{c_t}\left(\frac{w_t h_t}{1+\tau_c}\right) + \frac{\alpha}{x_t}\left(-1\right) = 0,$$

$$l_{Ht} \quad : \quad \frac{\alpha}{x_t}\left(-1\right) + \beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial h_{t+1}}\left(A_H h_t\right) = 0;$$

$$k_t \quad : \quad \frac{\partial V\left(k_t, h_t\right)}{\partial k_t} = \frac{1}{c_t}\frac{\left(1 + r_t - \delta_k\right)}{1+\tau_c},$$

$$h_t \quad : \quad \frac{\partial V\left(k_t, h_t\right)}{\partial h_t} = \frac{1}{c_t}\frac{\left(w_t l_t\right)}{1+\tau_c} + \beta \frac{\partial V\left(k_{t+1}, h_{t+1}\right)}{\partial h_{t+1}}\left(1 + A_H l_{Ht} - \delta_H\right).$$

Comparison of Goods and Labor Taxes

- Again intertemporal margins for capital unaffected by tax.
- Intratemporal goods-leisure margin affected as with labor tax:

$$x_t = \frac{\alpha c_t (1 + \tau_c)}{w_t h_t}.$$

- Labor income tax: $x_t = \frac{\alpha c_t}{w_t h_t (1 - \tau_l)}$. Same analysis if τ_c defined as

$$1 + \tau_c = \frac{1}{1 - \tau_l}.$$

- Aggregate Demand

$$c_t^d = \frac{w_t h_t \left(1 - \frac{g + \delta_H}{A_H}\right) + \rho (1 + g) k_t}{1 + \alpha (1 + \tau_c)}.$$

$$y_t^d = \frac{w_t h_t \left(1 - \frac{g + \delta_H}{A_H}\right) + \rho (1 + g) k_t}{1 + \alpha (1 + \tau_c)} + k_t (g + \delta_k).$$

- However goods tax raises more revenue.

Example 19.3: VAT Tax

- Assume same calibration as Example 19.2,
  - including $\tau_l = 0.144$ in effect by assuming that
  - $\tau_c = \frac{1}{1-\tau_l} - 1 = \frac{1}{1-0.144} - 1 = 0.16822.$
  - Gives same equilibrium growth rate, goods, labor market equilibria.
- But present value of revenue raised is higher:

$$\frac{G_t}{\rho\left(1+g\right)} = \frac{\tau_c c_t}{\rho\left(1+g\right)} = \frac{\left(0.16822\right)0.124}{0.052632\left(1.012\right)} = 0.393,$$

  - as compared to 0.181 with 14.4% labor tax.
  - Twice the revenue as from labor income tax.
- Equivalence of goods, labor taxes, but revenue differences,

Value Added Taxes

- Value-added taxes (VAT) differ internationally in 2009.
  - US has no federal VAT, but almost all 50 states have sales tax.
  - Italy, France, Germany have highest VAT near to 20%.
  - Spain, United Kingdom near 15% level,
  - Australia, New Zealand in $10 - 12.5\%$ range,
  - United States, Canada at bottom of range near 5%.
- Graph shows marked segmentation by region.
  - Europe has highest VAT rates,
  - Australasia middle rates,
  - North America at low end.
  - Nations may need comparable VAT because of tax evasion.
- May be why European countries lag in growth rates
  - relative to North American countries.
  - If higher VAT, but similar labor, capital tax rates,
  - bigger negative growth effects in high VAT regions.

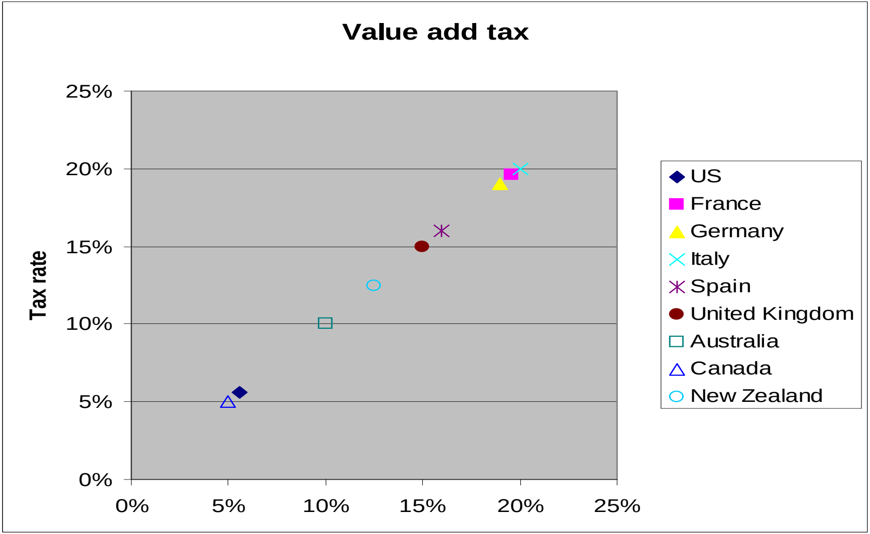


Figure: Figure 19.11.

Application: Trickle Down Economics

- US Economic Recovery Tax Act of 1981
  - reduced top marginal tax rates from 70% to 50%,
  - bottom tax bracket from 14% to 11%;
  - called "trickle down" economics that would not affect most people.
- C. Frenze, Joint Economic Committee (April 1996, JEC Report):
  - "The Reagan tax cuts, like similar measures enacted in the 1920s and 1960s, showed that reducing excessive tax rates stimulates growth."
- Chapter shows how lower taxes increase BGP growth rate g.
  - "Trickle down" might lead to worse distribution of income
  - Figure 19.12 shows distributional consequences of 1980s tax cuts,
  - as computed in April 1996, JEC Report.
  - Share of total income paid in taxes increased
  - for Top 1%, Top 5%, Top 10% of income,
  - decreased for Lowest 50% of income.
  - Income distribution appeared to become more equitable.

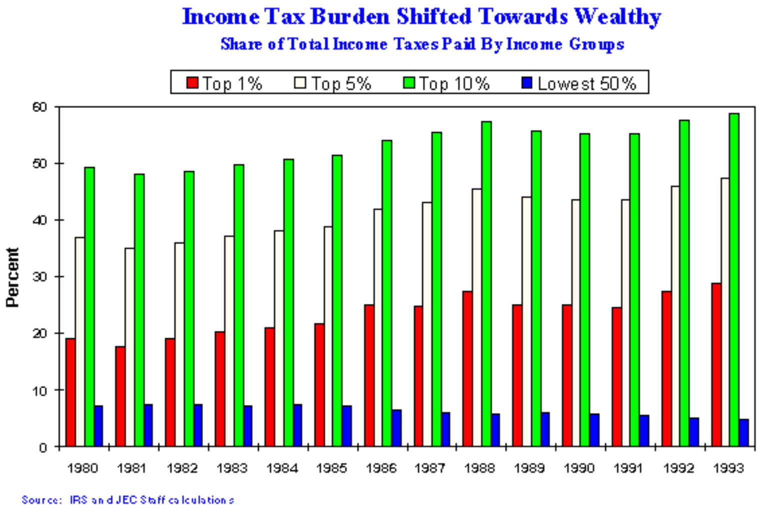


Figure: Figure 19.12.

Capital, Market Interest Rates, and Equity Value

- Chapter shows a capital tax can decrease growth rate.
- Effect of taxes on stock markets and equity premium
  - McGrattan, Prescott, 2003 "Average Debt, Equity Returns: Puzzling?"
  - Tax rates on corporate profit, like capital tax, were high but reduced.
  - Caused tax wedge that pushed up before-tax equity interest returns.
- Explain equity premium gap relative to government bond interest rates
  - as a result of tax wedges on capital
  - in US from 1880 to 2002.
  - Equity premium rose as average US marginal corporate tax rate rose
  - up through WWII; declined as average declined post WWII.
- McGrattan, Prescott explain value of corporations similarly,
  - 2005,"Taxes, Regulations, Value of U.S., U.K. Corporations".
  - Changes in market value of corporations in US, UK
  - from changes in corporate tax rate.