Advanced Modern Macroeconomics

Analysis and Application

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Chapter 2: Labor, Leisure and Productivity

- Demand and supply for goods and for labor,
- by deriving the labor-leisure trade-off, or margin.
- First the centralized general equilibrium: agent maximizes utility subject to production.
- Then agent problem split into problems of consumer and firm, with markets explicit.
- The marginal rate of substitution between goods and leisure derived.
- Equals the real wage which equals marginal product of labor in decentralization
- A change in labor productivity postulated with substitution and income effects.
- Growth policy instead of stabilization policy.

Building on Last Chapter and Learning Objective

- macroeconomics with only a representative consumer, as in microfoundations
- Labor static dimension, with closed economy, centralized and decentralized;
- intratemporal margin developed, labor-leisure time trade-off.
- You develop marginal rate of substitution between goods and leisure.
- See equals the real wage rate in equilibrium.
- Understand how this margin leads to derivation of aggregate supply and demand of both goods and labor.

Who Made It Happen

- Alfred Marshall eight editions of *Principles of Economics* from 1890 to 1920
- debate on "natural laws" of wages
  - John Bates Clark first on marginal productivity theory of wages,
  - 1899 *The Distribution of Wealth: A theory of wages, interest and profits*
  - 1901 article "Wages and Interest as Determined by Marginal Productivity", *JPE*
- Henry Ludwell Moore tested the theory in 1911 *Laws of Wages*
- J. R. Hicks expanded in context of unions, wage regulation and unemployment; 1932 *The Theory of Wages*.
- Gary S Becker : all time valued "at the margin" as is work time;
  - 1965 "The Allocation of Time" *Economic Journal;*
  - time spent in school, on-the-job training, fertility, child raising;
  - *Human Capital*, *The Economic Approach to Human Behavior*, and *A Treatise on the Family*.
  - Worldwide distribution of income, *American Economic Review* in 2005.

Representative Agent Goods-Leisure Choice

- A utility function, time allocation endowment, and production function: Robinson Crusoe.
- Utility u function of goods c and leisure x :

$$u = u\left(c, x\right).$$

$\frac{\partial u(c,x)}{\partial c} > 0$, $\frac{\partial u(c,x)}{\partial x} > 0$; diminishing marginal utility of increased consumption of goods or leisure.

- Log Utility

$$u\left(c, x\right) = \ln c + \alpha \ln x, \tag{1}$$

- α: degree to which the agent likes leisure.

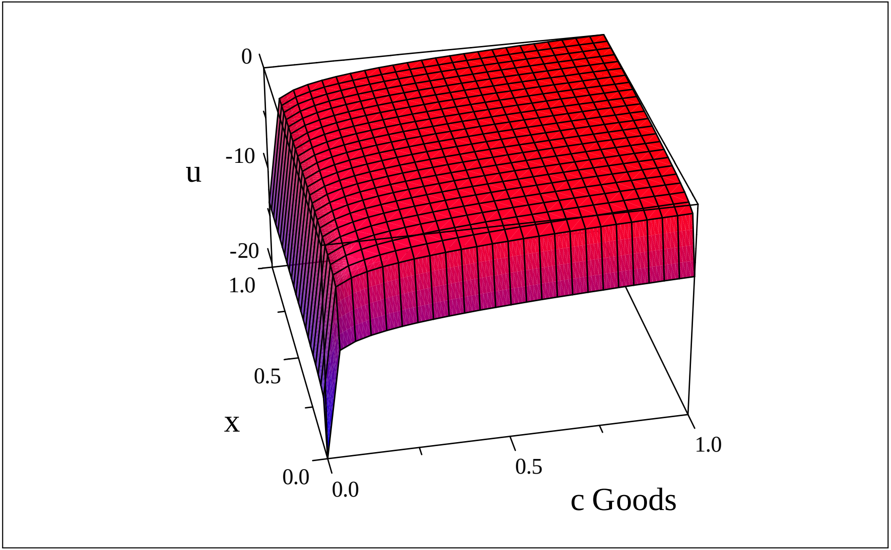

Figure 2.1 Log-Utility of Goods and Leisure

Cobb-Douglas Production

- Output y and $\gamma \in (0, 1)$, with

$$y = f(l, k) = A_G l^\gamma k^{1-\gamma}.$$

- Figure 2.2 graphs with $A_G = 1$ and $\gamma = \frac{1}{3}$.
- As we are focusing on the labor-leisure trade-off in this chapter, let $k = 1$ so that

$$f(l) = A_G l^\gamma. \tag{2}$$

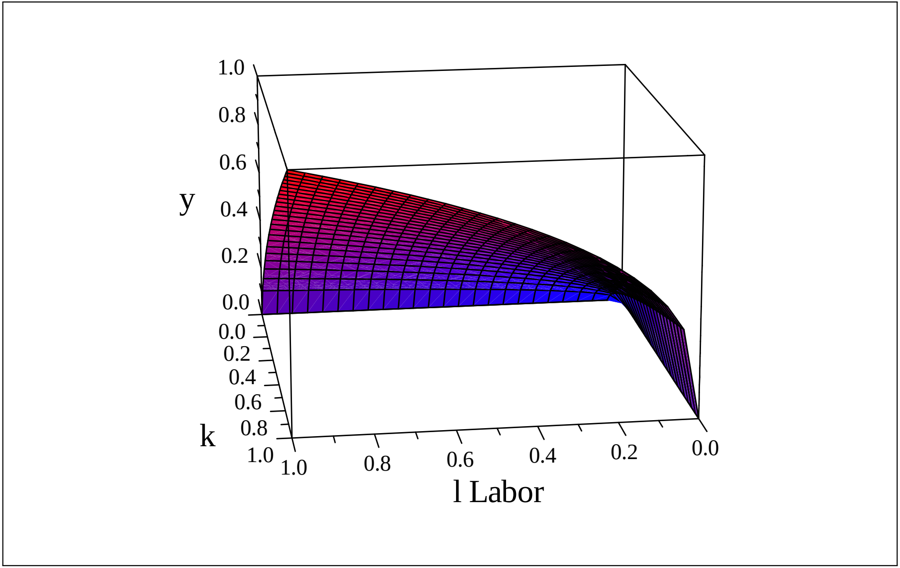

Figure 2.2. Cobb-Douglas Production of Goods Output y with Labor l and Capital k.

Log utility as Cobb-Douglas

- Transform utility function

$$\begin{aligned} u(c,x) &= \ln c + \alpha \ln x; \\ u(c,x) &= \ln c + \ln x^{\alpha} = \ln\left(cx^{\alpha}\right); \quad (3) \\ e^{u} &= cx^{\alpha}. \quad (4) \end{aligned}$$

- More generally with

$$\alpha_1 + \alpha_2 = 1,$$

$$\begin{aligned} u(c,x) &= \alpha_1 \ln c + \alpha_2 \ln x; \\ u(c,x) &= \ln c^{\alpha_1} + \ln x^{\alpha_2} = \ln\left(c^{\alpha_1} x^{\alpha_2}\right); \\ e^{u} &= c^{\alpha_1} x^{\alpha_2}. \quad (5) \end{aligned}$$

- "homothetic" functions

Time and Goods Constraints

- Time Allocation Constraint: labor l and leisure x equal T :

$$l + x = T.$$

-
  - T can add up to 100% of the time, or 1, disregarding education time for now:
  $$l + x = 1. \tag{6}$$
  - labor and leisure are fractions of time endowment.
  - Or $T = 24$, as in 24 hours a day, for example.
- Goods Endowment Constraint
  - capital stock is assumed fixed at $k = 1$.
  - implies consumption equals output, with no investment, and

$$c = y = f(l, k) = A_G l^{\gamma} k^{1-\gamma} = A_G l^{\gamma}.$$

Equilibrium with Capital Fixed

- Substitute in $x = 1 - l$, and $c = y$, and $y = f(l)$: both x and c in terms of l
- $$\underset{l}{\text{Max}}\ L = u\left[f(l), 1 - l\right].$$
- "Chain rule" of calculus,

$$\frac{\partial u}{\partial c}\frac{\partial f(l)}{\partial l} + \frac{\partial u(c,x)}{\partial x}\frac{\partial(1-l)}{\partial l} = 0.$$

$$\frac{\partial f(l)}{\partial l} = \frac{\frac{\partial u(c,x)}{\partial x}}{\frac{\partial u(c,x)}{\partial c}}.$$

- Marginal product of labor (MP_n) equals marginal rate of substitution $(MRS_{c,x})$

$$MP_l \equiv \frac{\partial f(l)}{\partial l} = \frac{\frac{\partial u(c,x)}{\partial x}}{\frac{\partial u(c,x)}{\partial c}} \equiv MRS_{c,x}. \quad (7)$$

Log Utility Solution

$$\underset{l}{\text{Max}}\; u\left[f\left(l\right), 1-l\right] = \ln\left(A_G l^{\gamma}\right) + \alpha \ln\left(1-l\right). \tag{8}$$

$$\frac{\partial u\left[f\left(l\right), 1-l\right]}{\partial l} = \frac{\partial\left[\ln\left(A_G l^{\gamma}\right) + \alpha\ln\left(1-l\right)\right]}{\partial l} = 0; \tag{9}$$

$$0 = \frac{A_G \gamma \left(l\right)^{\gamma-1}}{A_G l^{\gamma}} - \alpha\left(\frac{1}{1-l}\right). \tag{10}$$

Note $\dfrac{\partial \ln z\left(n\right)}{\partial n} = \dfrac{\frac{\partial z(n)}{\partial n}}{z(n)}$.

$$MP_l \equiv \frac{\partial f\left(l\right)}{\partial l} = A_G\gamma\left(l\right)^{\gamma-1} = \frac{\frac{\alpha}{1-l}}{\frac{1}{A_G l^{\gamma}}} = \frac{\frac{\alpha}{x}}{\frac{1}{c}} = \frac{\frac{\partial u(c,x)}{\partial x}}{\frac{\partial u(c,x)}{\partial c}} \equiv MRS_{c,x}. \tag{11}$$

Solution:

$$l = \frac{\gamma}{\alpha+\gamma}; c = y = A_G l^{\gamma} = A_G\left(\frac{\gamma}{\alpha+\gamma}\right)^{\gamma}, x = 1 - l = 1 - \frac{\gamma}{\alpha+\gamma}.$$

Example 2.1. Baseline Model

- Calibration: $A_G = 1$, $\gamma = \frac{1}{3}$ and $\alpha = 0.5$.

$$\begin{aligned}
\left(\frac{1}{3}\right) l^{\left(\frac{1}{3}-1\right)} &= \frac{\frac{1.5}{x}}{\frac{1}{c}} = \frac{\frac{1.5}{1-l}}{\frac{1}{l^{\frac{1}{3}}}}. \\
l &= \frac{\gamma}{\alpha+\gamma} = \frac{\frac{1}{3}}{0.5+\frac{1}{3}} = 0.4; \\
x &= 1 - 0.4 = 0.6, \\
c &= l^{\frac{1}{3}} = 0.40^{\frac{1}{3}} = 0.737. \\
\ln(c) + \alpha \ln(x) &= \ln(0.737) + 0.5 \ln(0.6) = -0.560\,58.
\end{aligned}$$

$$c = \frac{e^u}{x^{0.5}} = \frac{e^{-0.560\,58}}{x^{0.5}};\ c = (1-x)^{\frac{1}{3}}; \tag{12}$$

$$c = \frac{e^u}{(1-l)^{0.5}} = \frac{e^{-0.560\,58}}{(1-l)^{0.5}};\ c = (l)^{\frac{1}{3}}. \tag{13}$$

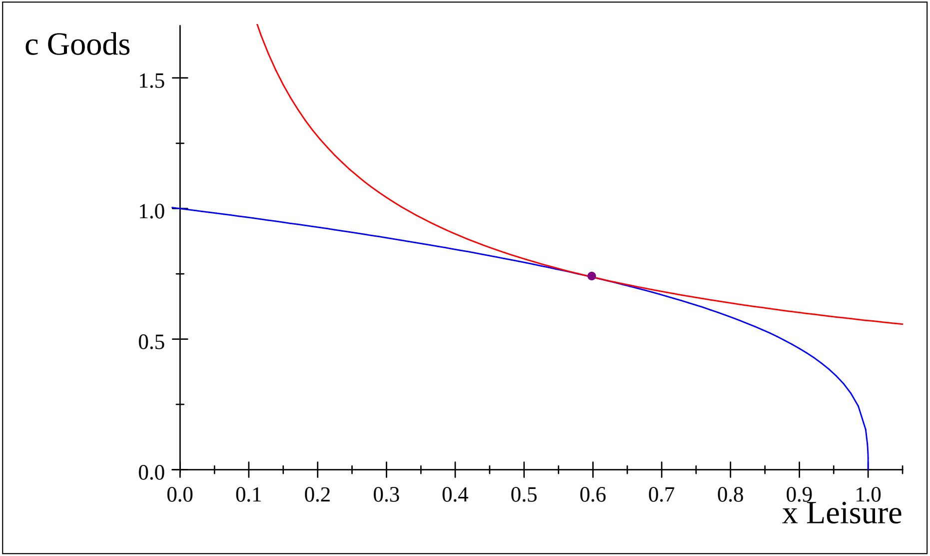

Figure 2.3. Consumption and Leisure Equilibrium at Tangency Point of Example 2.1.

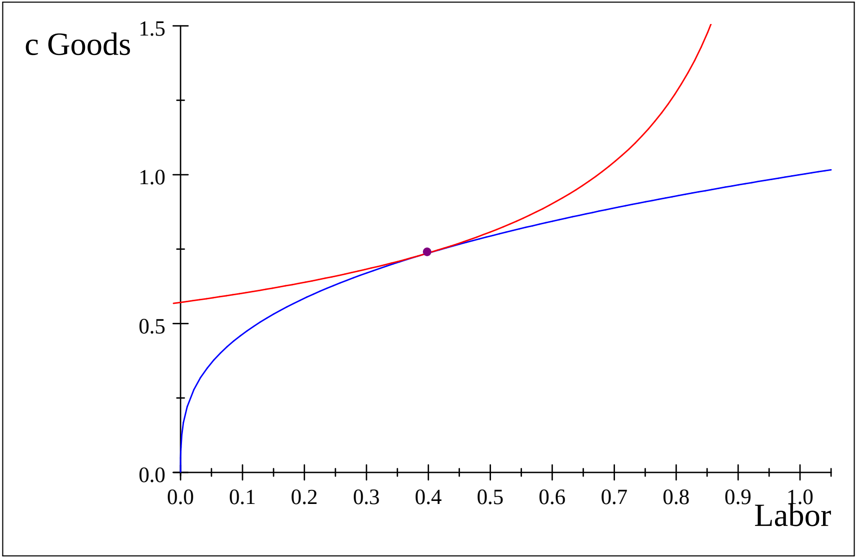

Figure 2.4. Consumption and Labor Equilibrium at Tangency Point of Example 2.1.

Smoothing Consumption

- balance between leisure and goods consumption.
- If instead "corner" solution, such as zero leisure, and only work; $l = 1$, $c = 1$.
- smoothing is across different "economic goods", here goods and leisure
- The ratio of marginal utilities $\frac{\partial u(c,x)}{\partial x} / \frac{\partial u(c,x)}{\partial c}$ gets higher as agent works more.
- Same as equalizing marginal utility of expenditure across different utility events,

$$\frac{\frac{\partial u(c,l)}{\partial c}}{1} = \frac{\frac{\partial u(c,l)}{\partial x}}{\frac{\partial f(l)}{\partial l}}.$$

Goods Productivity Increase

- Example 2.2: $\gamma = \frac{1}{3}$, $\alpha = 0.5$, A_G doubles from 1 to 2, relative to 2.1.
- A_G does not enter l solution. So $l = 0.60$ and $x = 0.40$. c :

$$
\begin{aligned}
c &= 2l^{\frac{1}{3}} = 2\left(0.40\right)^{\frac{1}{3}} = 1.474. && (14)\\
\ln\left(c\right) + \alpha \ln\left(x\right) &= \ln\left(1.474\right) + 0.5 \ln\left(0.6\right) = 0.132\,57; && (15)\\
c &= \frac{e^u}{x^{0.5}} = \frac{e^{0.132\,57}}{x^{0.5}}. && (16)
\end{aligned}
$$

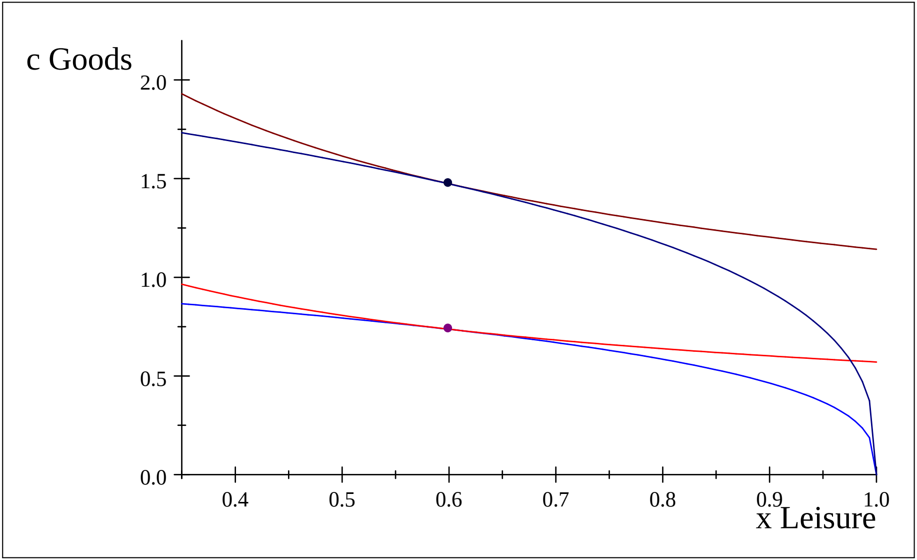

Figure 2.5. Productivity Doubling of Example 2.2 (darker blue and darker red).

Substitution and Income Effects

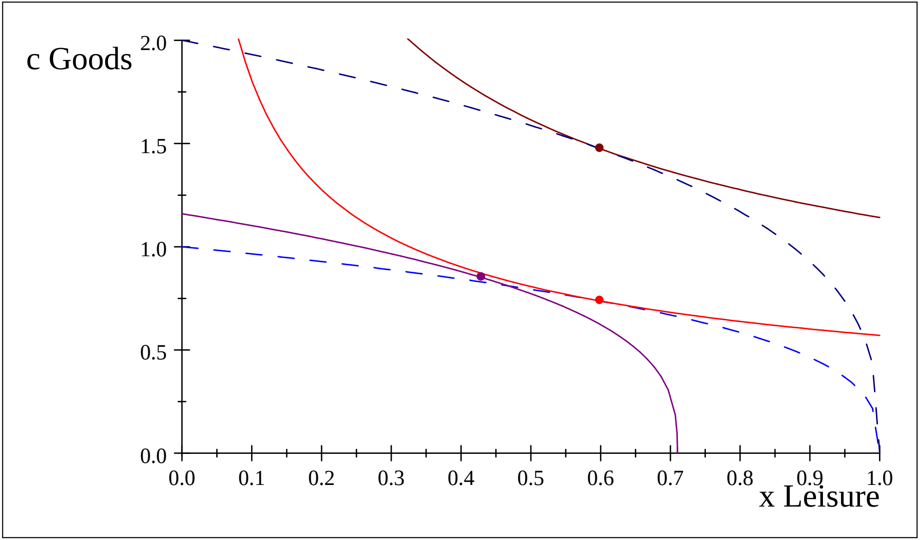

Figure 2.6. Substitution and Income Effects from a Productivity Doubling.

Example 2.3: An Eight Hour Day

- $T = 24$, $\alpha = 1$, $A_G = 1$, and let $\gamma = 0.5$, $u = \ln c + \ln x$, $c = \sqrt{l}$.

$$\underset{l}{\text{Max}}\ u = \ln\left(\sqrt{l}\right) + \ln\left(1 - l\right), \tag{17}$$

$$MP_l \equiv \frac{\partial f}{\partial l} = 0.5l^{-0.5} = \frac{\frac{1}{x}}{\frac{1}{c}} = MRS_{x,c}. \tag{18}$$

$$0.5l^{-0.5} = \frac{\frac{1}{24-l}}{\frac{1}{l^{0.5}}} = \frac{l^{0.5}}{24-l},\ 0.5\left(24 - l\right) = l,\ 12 = 1.5l, \tag{19}$$

$$l = 8,\ x = 16,\ c = \sqrt{8} = 2.83,\ u = \ln 2.83 + \ln 16 = 3.8 \tag{20}$$

- Doubling goods productivity $c = y = 2\sqrt{l}$, $l = 8$, $c = 2\sqrt{8} = 5.66$.

Example 2.4. Linear Indifference Curves

$$u = c + x,$$
$$c = \sqrt{l},\ T = 1,\ 1 = l + x.$$
$$\max_{l} u = (1 - l) + \sqrt{l},$$
$$\frac{\partial u}{\partial l} = -1 + 0.5 l^{-0.5} = 0.$$

$$l = 0.25,\ x = 1 - 0.25 = 0.75,\ c = \sqrt{0.25} = 0.5,\ u = \sqrt{0.25} + 0.75 = 1.25.$$

- Let labor productivity double $c = 2\sqrt{l}$, $\max_{l} u = (1 - l) + 2\sqrt{l}$,

$$\frac{\partial u}{\partial l} = -1 + l^{-0.5} = 0.$$
$$l = 1,\ x = 0,\ c = 2,\ u = 2 = c + x.$$

$$c = 1.25 - x,\ c = 2 - x. \tag{21}$$
$$c = \sqrt{1 - x},\ c = 2\sqrt{1 - x}. \tag{22}$$

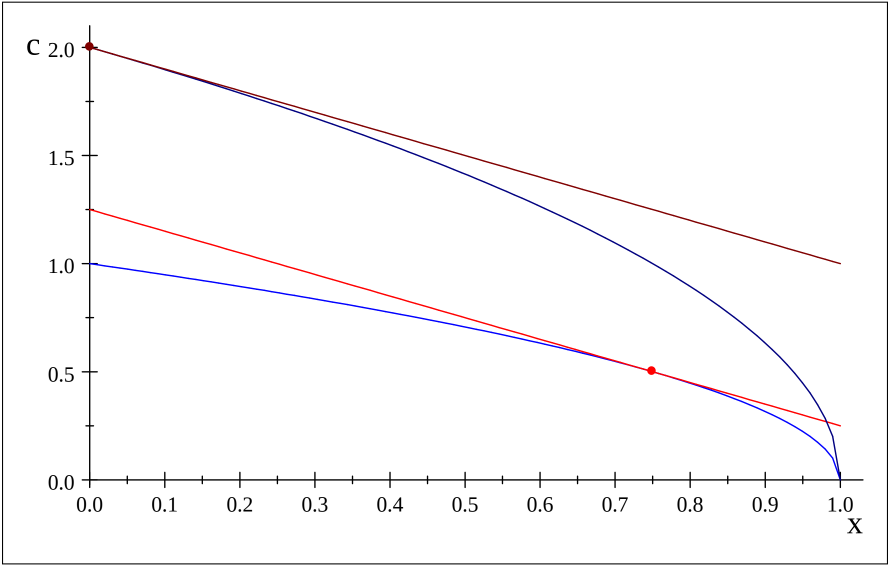

Figure 2.7. Linear Utility with a Doubling of Goods Productivity in Example 2.4.

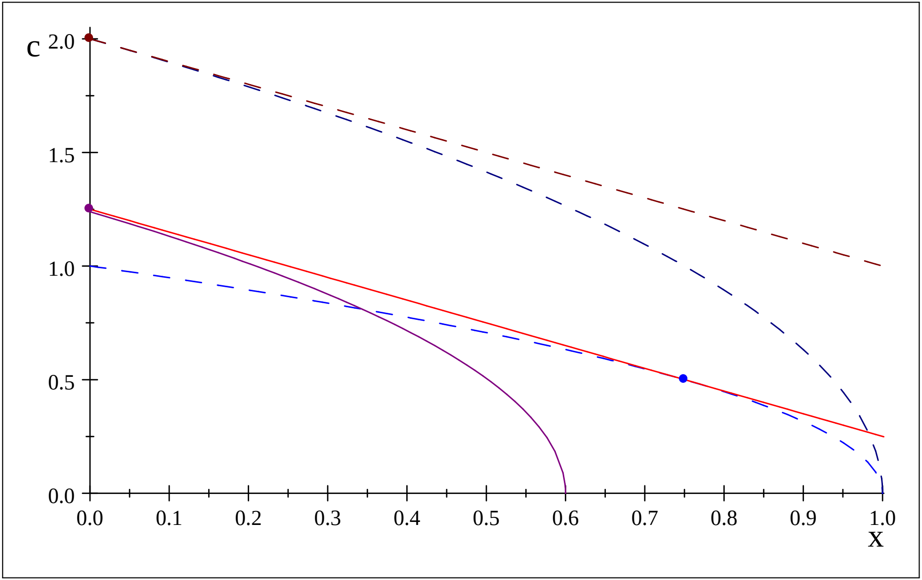

Figure 2.8. Linear Utility with a Decomposition into Substitution and Income Effects in Example 2.4.

Decentralization: Consumer and Firm Problems

$$MRS_{c,x} = w = MP_l. \tag{23}$$

- Consumer: Demand for Goods, Supply of Labor

$$c^d = wl^s + \Pi.$$

$$x + l^s = 24 = T.$$

$$\underset{l^s}{\text{Max}}\; u\left(wl^s + \Pi, 24 - l^s\right). \tag{24}$$

$$\frac{\partial u}{\partial c}\frac{\partial\left(wl^s + \Pi\right)}{\partial l^s} + \frac{\partial u}{\partial x}\frac{\partial\left(24 - l^s\right)}{\partial l^s} = 0.$$

$$\frac{\partial u}{\partial c}w + \frac{\partial u}{\partial x}\left(-1\right) = 0,$$

$$w = \frac{\frac{\partial u(c,l^s)}{\partial x}}{\frac{\partial u(c,l^s)}{\partial c}} = MRS_{c,x}.$$

Firm: Supply of Goods, Demand for Labor

$$\Pi = c^s - wl^d.$$

$$y = f\left(l\right).$$

$$c^s = y.$$

$$\underset{l^d}{\text{Max}}\,\Pi = f\left(l^d\right) - wl^d. \tag{25}$$

$$MP_l = \frac{\partial f\left(l^d\right)}{\partial l^d} = w. \tag{26}$$

Aggregate Demand and Supply: Example 2.5.

- $T = 24$, $\alpha = 1$, $\gamma = 0.5$, and $A_G = 1$. Consumer:

$$\underset{l^s}{\text{Max}}\ u = \ln\left(wl^s + \Pi\right) + \ln\left(24 - l^s\right). \tag{27}$$

$$\begin{aligned} \frac{\partial L}{\partial l^s} &= \frac{\partial u}{\partial c^d}(w) + \frac{\partial u}{\partial x}(-1) \\ &= \frac{w}{wl^s + \Pi} - \frac{1}{24 - l^s} = 0. \end{aligned}$$

$$l^s = 12 - \frac{\Pi}{2w};\ \frac{\partial l^s}{\partial w} = \frac{\Pi}{2w^2} > 0 \tag{28}$$

$$c^d = wl^s + \Pi = w\left(12 - \frac{\Pi}{2w}\right) + \Pi = 12w + \frac{\Pi}{2},$$

$$\begin{aligned} x &= 24 - l^s = 12 + \frac{\Pi}{2w} \\ l^s &= 12 - \frac{\Pi}{2w}. \end{aligned}$$

Firm Problem and Consumer's Solution using Profit

$$\underset{l^d}{\text{Max}}\ \Pi = \sqrt{l^d} - wl^d. \tag{29}$$

$$0.5\left(l^d\right)^{-0.5} - w = 0. \tag{30}$$

$$l^d = \frac{1}{4w^2};\ \frac{\partial l^d}{\partial w} = -\frac{1}{2w^3} < 0. \tag{31}$$

$$c^s = \sqrt{l^d} = \frac{1}{2w},\ \frac{\partial c^s}{\partial\left(\frac{1}{w}\right)} = \frac{1}{2} > 0. \tag{32}$$

$$\Pi = \sqrt{l^d} - wl^d = \frac{1}{2w} - \frac{w}{4w^2} = \frac{1}{4w}. \tag{33}$$

$$c^d = 12w + \frac{\Pi}{2} = 12w + \frac{1}{8w}. \tag{34}$$

$$l^s = 12 - \frac{1}{8w^2}. \tag{35}$$

The Goods Markets: AS and AD

- $$AS : \frac{1}{w} = 2c^s, \tag{36}$$

$$c^d = 12w + \frac{1}{8w}, \; \frac{8\left(c^d\right)}{w} = \frac{8\left(12w\right)}{w} + \frac{\frac{1}{w}}{w}, \tag{37}$$

$$AD : \left(\frac{1}{w}\right)^2 - 8c^d\frac{1}{w} + 96 = 0; \; \frac{1}{w} = \frac{-B - \sqrt{B^2 - 4AC}}{2A}. \tag{38}$$

$$A\left(\frac{1}{w}\right)^2 + B\frac{1}{w} + C = 0; \; A = 1, B = -8c^d, C = 96 :$$

$$AD : \frac{1}{w} = \frac{8c^d - \sqrt{64\left(c^d\right)^2 - 4\left(96\right)}}{2} = 4c^d - 4\sqrt{\left(c^d\right)^2 - 6}; \tag{39}$$

$$AS = AD : \frac{1}{w} = 5.65; \; c^d = 12w + \frac{1}{8w} = \frac{12}{5.65} + \frac{1}{8}\left(5.65\right) = 2.83.$$

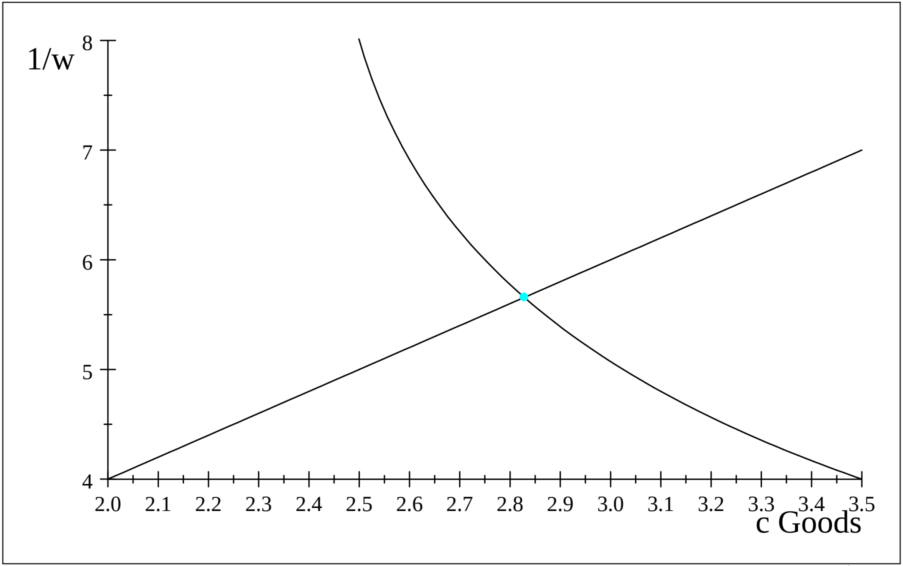

Figure 2.9. Aggregate Goods Demand and Supply as a Function of $\frac{1}{w}$ in Example 2.5.

Aggregate Labor Market

$$w = \frac{1}{2}\sqrt{\frac{1}{l^d}}, \quad (40)$$

$$w = \sqrt{\frac{1}{8\left(12 - l^s\right)}}; \quad (41)$$

$$w = 0.177; \quad (42)$$

$$l = 8.. \quad (43)$$

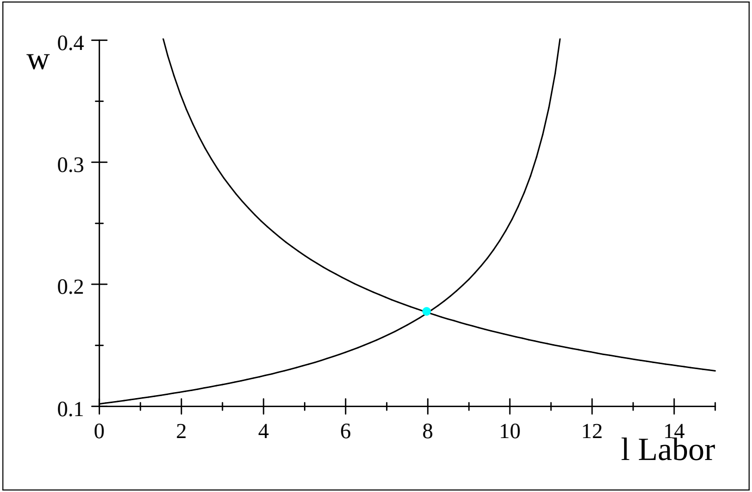

Figure 2.10. Aggregate Labor Demand and Supply as Function of w.

Market Equilibrium: the Real Wage Rate

$$c^s = \frac{1}{2w} = 12w + \frac{1}{8w} = c^d,$$
$$w = \sqrt{\frac{3}{96}} = 0.177.$$

$$\begin{aligned} 12 - \frac{1}{8w^2} &= \frac{1}{4w^2},\ w = \sqrt{\frac{3}{96}} = 0.177,\ \frac{1}{w} = \frac{1}{0.177} = 5.65 \\ l^s &= l^d = 8;\ x = 16,\ c = 2.83,\ u = \ln(2.83) + \ln(16) = 3.81 \end{aligned}$$

General Equilibrium Representation

$$\begin{aligned} c^d &= wl^s + \Pi = w(24 - x) + \Pi \\ c^d &= (0.177)(24 - x) + \frac{1}{4(0.177)} \end{aligned} \tag{44}$$

$$c = \frac{e^u}{x} = \frac{e^{3.81}}{x},\ c = \sqrt{l} = \sqrt{24 - x}. \tag{45}$$

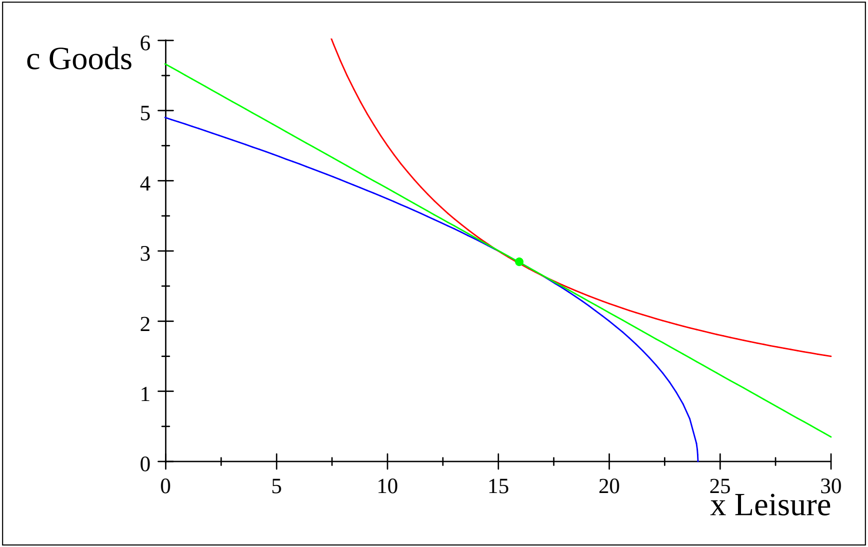

Figure 2.11. General Equilibrium Goods and Labor Market with Budget/Profit Line in Example 2.5.

Labor Productivity Increase: Example 2.6.

$T = 24$, $\alpha = 1$, $\gamma = 0.5$, $A_G = 2$ instead of $A_G = 1$.

$$\underset{l^d}{\text{Max}}\ \Pi = 2\sqrt{l^d} - wl^d. \tag{46}$$

$$\left(l^d\right)^{-0.5} - w = 0;\ \ l^d = \frac{1}{w^2}; \tag{47}$$

$$c^s = \frac{2}{w}; \tag{48}$$

$$\Pi = \frac{2}{w} - \frac{w}{w^2} = \frac{1}{w}. \tag{49}$$

$$c^d = 12w + \frac{\Pi}{2} = 12w + \frac{1}{2w}; \tag{50}$$

$$l^s = 12 - \frac{\Pi}{2w} = 12 - \frac{1}{2w^2}. \tag{51}$$

$$l^s = 12 - \frac{1}{2w^2} = \frac{1}{w^2} = l^d;\ \ w = \sqrt{\frac{1}{8}} = \frac{1}{4}\sqrt{2} = 0.354, \tag{52}$$

Goods Market

$$c^d = \frac{12}{\frac{1}{w}} + \frac{1}{2}\frac{1}{w},$$
$$0 = \frac{1}{2}\left(\frac{1}{w}\right)^2 - c^d\left(\frac{1}{w}\right) + 12.$$

$$A\left(\frac{1}{w}\right)^2 + B\frac{1}{w} + C = 0,\ A = \frac{1}{2}, B = -c^d, C = 12.\ \frac{1}{w} = \frac{-B - \sqrt{B^2 - 4A}}{2A}$$

$$\frac{1}{w} = \frac{c^d - \sqrt{\left(c^d\right)^2 - 4\left(\frac{1}{2}\right)(12)}}{2\left(\frac{1}{2}\right)}. \tag{53}$$

$$c^s = \frac{2}{w},$$
$$\frac{1}{w} = \frac{c^s}{2}. \tag{54}$$

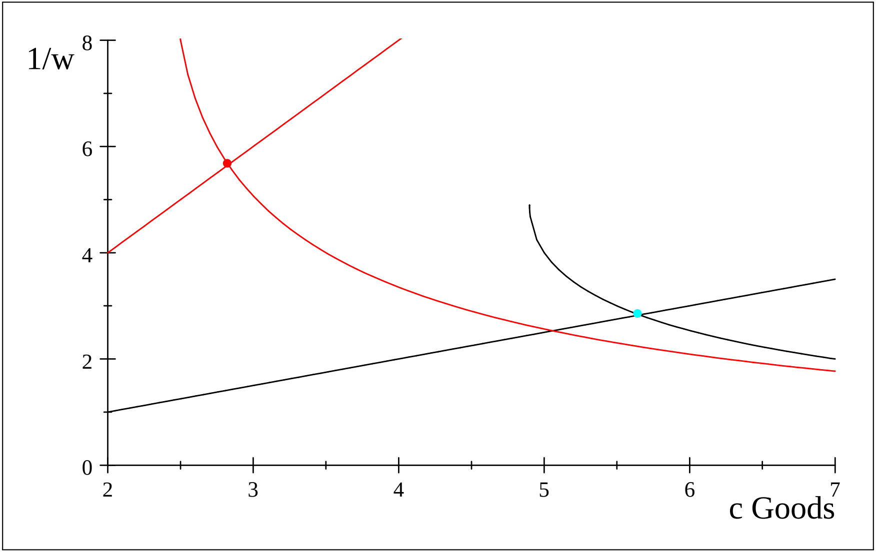

Figure 2.12. Productivity Increase in the Goods Market in Example 2.6.

Labor Market

$$
\begin{aligned}
l^s &= 12 - \frac{1}{2w^2}, \\
w &= \sqrt{\frac{1}{2\left(12 - l^s\right)}}; \qquad (55) \\
l^d &= \frac{1}{w^2}, \\
w &= \frac{1}{\sqrt{l^d}}. \qquad (56)
\end{aligned}
$$

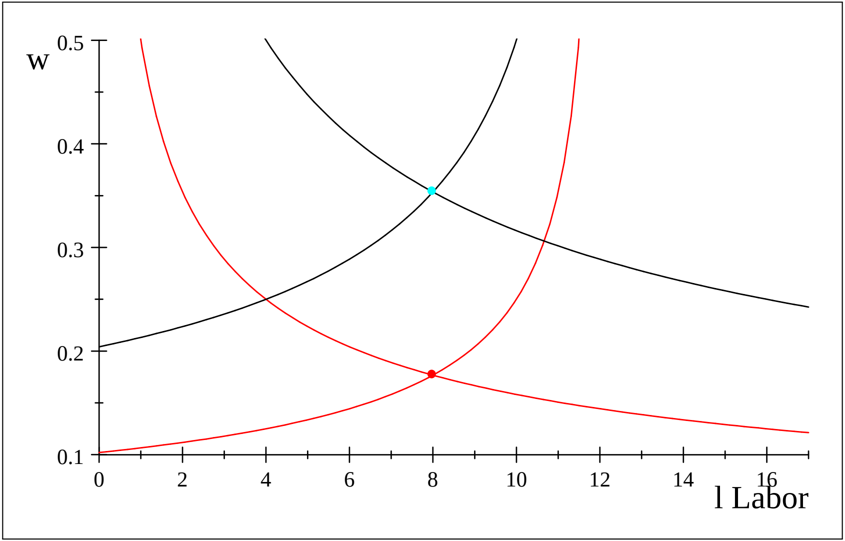

Figure 2.13. Productivity Increase in the Labor Market of Example 2.6.

General Equilibrium

$$
\begin{aligned}
c^d &= wl^s + \Pi = w(1-x)l^s + \Pi \\
c^d &= (0.3535)(24-x) + \frac{1}{0.3535},
\end{aligned}
$$

$$
u = \ln 16 + \ln 5.65 = 4.505;\ c^d = \frac{e^u}{x} = \frac{e^{4.505}}{x} = \frac{90.5}{x},
$$

$$
c^s = \frac{2}{w} = \frac{2}{0.3535} = 5.66.
$$

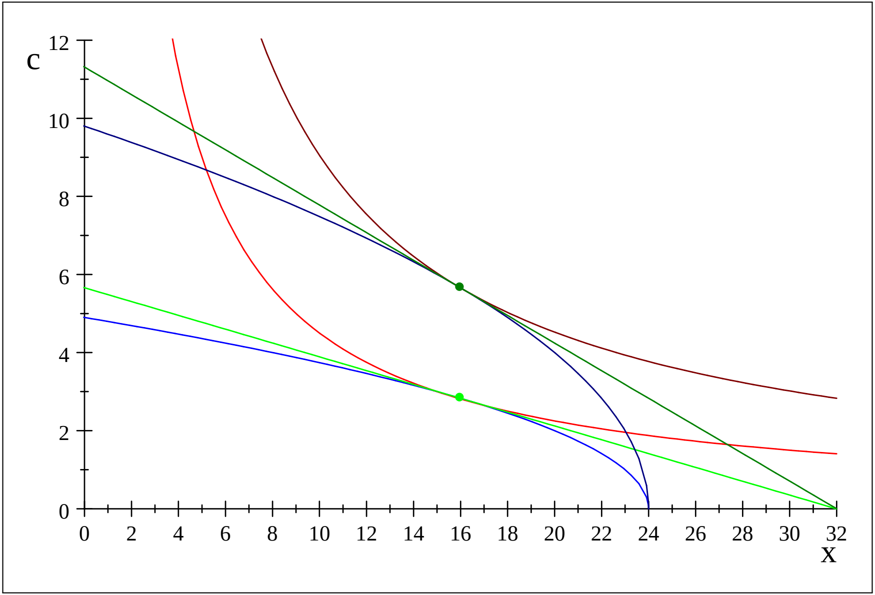

Figure 2.14. General Equilibrium Goods and Labor Market in Example 2.6 (black) and Example 2.5 (red).

Application: Productivity and Growth Policy

- US Legislation: Employment Act of 1946, still in force today in amended form.
- Section 2 "promote the maximum employment, production and purchasing power."
- Must “provide such volume of Federal investment and expenditure as may be needed ...to assure continuing full employment".
- "maximum" employment instead of "full" after Congressional debate.
- "full" employment entered Act upon amendment in 1978.
- Then also economic growth entered the legislation.
- Amended Act: entitled Full Employment and Balanced Growth Act.
- full employment, balanced economic growth, growth in productivity, and balanced government budget (Title I,1.).
- "Encourages the adoption of a fiscal policy that would reduce federal spending as a percent of GNP,"
- Set inflation targets of 3% by 1983, 0% by 1988.

Modern Economic Views on Stabilization Vs. Growth

- Robert E. Lucas, Jr. popularized cost of business cycles much less than gain from small economic growth increase.
- In 1987 book: *Models of Business Cycles*,
- shows amount of goods consumer needs to be compensated with in order to not be worried about the consumption variation
- is much less than consumption increase from small amount of economic growth.
- The implication: focus on growth not "stabilization".
- Many have used Lucas's; concept still in place that focusing on facilitating economic growth may be the most important policy a government might pursue.