Advanced Modern Macroeconomics

Employment Cycles and Taxes

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- Adds 2nd comparative static change: in time endowment.
- Expansion with employment rising and consumption increasing
  - explained with equal percent increase in both goods & time endowment.
  - A contraction with employment, goods falling as endowments decrease.
  - All equilibrium movements of labor and goods.
- Also, during recession, real wage is assumed to be ad hoc fixed.
- Gives deep recession or depression.
- Taxes on goods and labor introduced, with similar effects.
  - Tax revenue transferred back to consumer as income,
  - similar to unemployment compensation policy.
  - Tax causes labor supply and goods demand to shift back.
  - Wage rises, but wage "net of taxes" falls.
- The theory of economic regulation as implicit taxes.
- Applications to unemployment policy, moral hazard, insurance; industrial regulation.

Building on the Last Chapter and Learning Objective

- Extends Chapter 2 by adding a change in time endowment, to explain business cycles.
- Uses similar supply and demand as developed in last chapter.
- Markets now affected by taxes.
- Distortion seen in the marginal rate of substitution between goods and leisure.
- Aim to understand how equilibrium business cycle might occur
  - using simple comparative static analysis of a baseline model.
  - then challenge is to see also a non-equilibrium theory of unemployment
  - with concept of "surplus labor", an excess supply of labor at the fixed wage rate.
- Understand how tax and transfer policies comprise part of unemployment policy.

Who Made it Happen

- Samuelson: equilibrium suggested by Keynes (1936), Hicks's (1937)
  - surplus labor from rigid wages during 1930s international depression.
  - Samuelson: during crisis periods markets not clearing relevant;
  - Government should actively end such crises.
- Samuelson (1947) included analysis taxes; Harberger (1974) showed distortions
- David Ricardo1817 all major taxes increase cost of goods and labor.
  - "Taxes on wages will raise wages,
  - ...tax on necessities will raise their prices, followed by rise of wages."
- Old idea: Henry George 1926 "Progress and Poverty" wrote (p. 408):
  - "All such taxes have a tendency to reduce the production of wealth,
  - find the least distortive tax, tax on land, to extract "rents" of land.
- Rigid wages, prices as regulations upon markets, an implicit tax.
- Implicit taxes similar to explicit taxes: Becker 1957 "Economics of Discrimination".
- Stigler (1971),Peltzman (1976), Posner (1973): "rents" used up.

Business Cycles and Employment

- downturn: factor productivity falls, as in decrease in A_G.
- But in our economy employment rate does not fall: a dilemma.
- Employment too "smooth" relative to data.
- Complex utility, production functions possible but not attractive solution.
- Model's output, real wage rise & fall as productivity rises & falls: "procyclic".
- Evidence mixed but accepted that real wage rates probably procyclic.
- So just need labor employment to change also.

Internal versus External Labor Margins

- 8 hour day or 40 hour week consistent empirically yet labor employment cycles.
- Seeming paradox is central in explaining business cycles in standard model.
- "Internal" margin versus "external" margin: decision to work or not over business cycle.
- External labor margin as "labor force participation rate", instead of non-market sector.
- Explain change in the labor market participation rate to explain business cycle.
- Decrease in time endowment: more time devoted to non-market activity, human capital.

Time Endowment Increase

- Change in T abstracts from uses of time such as education,
- so time left for labor and leisure changes.
- Years of education in primary, secondary, and then tertiary schools:
- Education uses up time leftover for labor and leisure.
- More time spent in education or child-rearing decreases exogenous time endowment.
- Exogenous time endowment later made endogenous through "human capital".

Example 3.1

- As in Example 2.1, $\gamma = \frac{1}{3}$, $A_G = 1$ and $\alpha = 0.5$; but T rise 5% to 1.05.
- Centralized Problem:

$$\begin{aligned} \underset{l}{\text{Max}}\, u &= \ln\left(l^{\gamma}\right) + \alpha \ln\left(1.05 - l\right) \\ &= \ln\left(l^{\frac{1}{3}}\right) + 0.5 \ln\left(1.05 - l\right), \end{aligned}$$

$$\frac{(l)^{-\frac{2}{3}}}{3l^{\frac{1}{3}}} - 0.5\left(\frac{1}{1.05 - l}\right) = 0, \qquad (1)$$

$$l = 0.42. \qquad (2)$$

- Labor employed rises 5% to 0.42 from 0.40.

$$\begin{aligned} c &= l^{\gamma} = 0.42^{\frac{1}{3}} = 0.748\,89, \\ x &= 1.05 - 0.42 = 0.63\,, \end{aligned}$$

$$u\left(c, x\right) = \ln\left(l^{\frac{1}{3}}\right) + 0.5 \ln\left(1.05 - l\right) = \ln 0.7489 + 0.5 \ln 0.63 = -0.52. \qquad (3)$$

General Equilibrium Graph in Output Space

- The production curve in $(c : x)$ space

$$c = (1.05 - x)^{\frac{1}{3}}, \tag{4}$$

instead of

$$c = (1 - x)^{\frac{1}{3}}. \tag{5}$$

- Utility level curve shifts upwards to

$$c = \frac{e^{-0.520\,1}}{x^{0.5}}, \tag{6}$$

instead of

$$c = \frac{e^{-0.560\,58}}{x^{0.5}}. \tag{7}$$

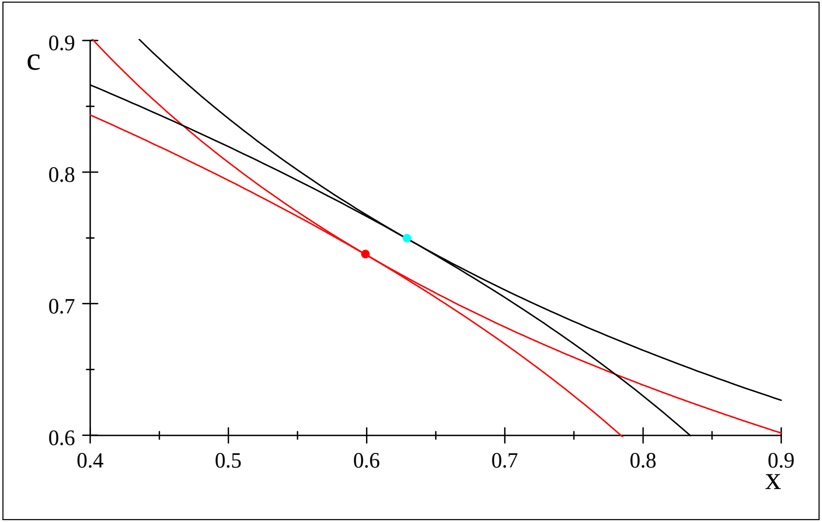

Figure 3.1. A 5% Increase in Time Endowment in Example 3.1 in Black Compared to Example 2.1 in Red.

Decentralized Problem: Consumer

$$\underset{l^s}{\text{Max}}\ L = \ln\left(wl^s + \Pi\right) + 0.5\ln\left(1.05 - l^s\right), \tag{8}$$

$$\frac{w}{wl^s + \Pi} - \frac{0.5}{1.05 - l^s} = 0.$$

$$\begin{aligned} \frac{1.05}{0.5} - \frac{\Pi}{w} &= l^s\left(1 + \frac{1}{0.5}\right), \\ l^s &= \frac{2.10}{3} - \frac{\Pi}{3w} = 0.7 - \frac{\Pi}{3w}. \end{aligned}$$

$$c^d = wl^s + \Pi = w\left(\frac{2.10}{3} - \frac{\Pi}{3w}\right) + \Pi = 0.7w + \frac{2}{3}\Pi.$$

$$x = 1.05 - l^s = 1.05 - \frac{2.10}{3} + \frac{\Pi}{3w} = 0.35 + \frac{\Pi}{3w}.$$

Decentralized Problem: Firm

$$y = A_G \left(l^d\right)^{\gamma} = \left(l^d\right)^{\frac{1}{3}},$$

$$\underset{l^d}{\text{Max}}\ \Pi = \left(l^d\right)^{\frac{1}{3}} - wl^d. \tag{9}$$

$$\frac{1}{3}\left(l^d\right)^{-\frac{2}{3}} = w,\ l^d = \left(\frac{1}{3w}\right)^{1.5},\ w = \frac{1}{3\left(l^d\right)^{\frac{2}{3}}}. \tag{10}$$

$$\begin{aligned} c^s &= \left(\frac{1}{3w}\right)^{\frac{1.5}{3}} = \left(\frac{1}{3w}\right)^{0.5}, \qquad (11)\\ \Pi &= c^s - wl^d, \\ &= \left(\frac{1}{3w}\right)^{0.5} - w\left(\frac{1}{3w}\right)^{1.5} = \left(\frac{1}{3w}\right)^{0.5}\left(1-\frac{1}{3}\right) = \frac{2}{3\sqrt{3w}} = \frac{0.385}{\sqrt{w}}. \end{aligned}$$

Labor Market Supply and Demand

$$\Pi = \frac{2}{3\sqrt{3w}},\ c^d = 0.7w + \frac{2}{3}\Pi = 0.7w + \frac{4}{9\sqrt{3w}}.$$

$$l^s = 0.7 - \frac{\Pi}{3w} = 0.7 - \frac{2}{9\sqrt{3}w^{1.5}}. \tag{12}$$

$$w = \left(\frac{2}{(0.7 - l^s)\, 9\sqrt{3}}\right)^{\frac{2}{3}}. \tag{13}$$

$$w = \frac{1}{3\left(l^d\right)^{\frac{2}{3}}}. \tag{14}$$

$T = 1:$

$$w = \left(\frac{2}{\left(\frac{2}{3} - l^s\right) 9\sqrt{3}}\right)^{\frac{2}{3}},$$

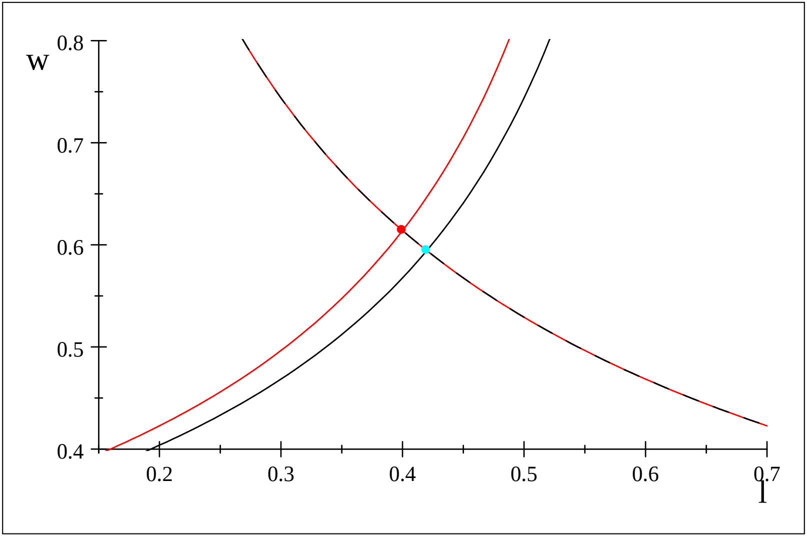

Figure 3.2. The Labor Market with a 5% Time Endowment Increase in Example 3.1.

Equilibrium Wage

$$l^s = 0.7 - \frac{2}{9\sqrt{3}w^{1.5}} = \left(\frac{1}{3}\right)^{1.5}\frac{1}{w^{1.5}} = l^d; \tag{15}$$

$$w = \left(\frac{\left(\frac{1}{3}\right)^{1.5} + \frac{2}{9\sqrt{3}}}{0.7}\right)^{\frac{2}{3}} = \left(\frac{\left(\frac{1}{3}\right)^{1.5} + 0.1283}{0.7}\right)^{\frac{2}{3}} = 0.594; \tag{16}$$

$$l^s = 0.7 - \frac{2}{9\,(0.594)^{1.5}}\frac{1}{\sqrt{3}} = 0.420 = \left(\frac{1}{3}\right)^{1.5}\frac{1}{(0.594)^{1.5}} = l^d. \tag{17}$$

$T = 1:$

$$\left(\frac{2}{\left(\frac{2}{3} - l^s\right) 9\sqrt{3}}\right)^{\frac{2}{3}} = \frac{1}{3\,(l^d)^{\frac{2}{3}}},\ l = \frac{\frac{2}{3}9\,(3)^{0.5}}{9\,(3)^{0.5} + 3^{1.5}2} = 0.4$$

$$w = \frac{1}{3\,(0.4)^{\frac{2}{3}}} = 0.614.$$

Economic Expansion: Example 3.2

- $\gamma = \frac{1}{3}$, $A_G = 1$, and $\alpha = 0.5$ and 5% increases in T, A_G : $T = 1.05$, and $A_G = 1.05$.

Consumer

$$c^d = wl^s + \Pi = 0.7w + \frac{2}{3}\Pi.$$

$$l^s = 0.7 - \frac{\Pi}{3w}.$$

$$y = A_G \left(l^d\right)^{\gamma} = 1.05 \left(l^d\right)^{\frac{1}{3}},$$

$$\underset{l^d}{\text{Max}}\, \Pi = 1.05 \left(l^d\right)^{\frac{1}{3}} - wl^d. \tag{18}$$

$$\frac{1}{3}(1.05)\left(l^d\right)^{-\frac{2}{3}} = w,\ l^d = \left(\frac{1.05}{3w}\right)^{1.5}. \tag{19}$$

$$w = \frac{1.05}{3\left(l^d\right)^{\frac{2}{3}}}. \tag{20}$$

Firm and Consumer

$$c^s = y = A_G\left(l^d\right)^{\gamma} = 1.05\left(\frac{1.05}{3w}\right)^{\frac{1.5}{3}} = \frac{(1.05)^{1.5}}{\sqrt{3w}},$$

$$\Pi = c^s - wl^d = \frac{(1.05)^{1.5}}{\sqrt{3w}} - w\left(\frac{1.05}{3w}\right)^{1.5} = \frac{0.414\,13}{\sqrt{w}}. \tag{21}$$

$$c^d = 0.7w + \frac{2}{3}\Pi = 0.7w + \frac{2}{3}\frac{2\,(1.05)^{1.5}}{3^{1.5}\sqrt{w}} = 0.7w + \frac{0.276\,08}{\sqrt{w}}$$

$$l^s = 0.7 - \frac{\Pi}{3w} = 0.7 - \left(\frac{1}{3w}\right)\frac{2\,(1.05)^{1.5}}{3^{1.5}\sqrt{w}} = 0.7 - \frac{0.138\,04}{w^{1.5}}. \tag{22}$$

$$w = \left(\frac{0.138\,04}{0.7 - l^s}\right)^{\frac{2}{3}}. \tag{23}$$

Equilibrium Wage

$$l^s = 0.7 - \frac{0.138\,04}{w^{1.5}} = \left(\frac{1.05}{3w}\right)^{1.5} = l^d; \tag{24}$$

$$0.7 = \frac{1}{w^{1.5}}\left(\frac{1.05^{1.5}}{3^{1.5}} + 0.138\,04\right), \tag{25}$$

$$w = \left(\frac{\frac{1.05^{1.5}}{3^{1.5}} + 0.138\,04}{0.7}\right)^{\frac{2}{3}} = 0.624\,07. \tag{26}$$

Fractional increase of $\frac{0.624\,07 - 0.614}{0.614} = 0.0164$, or 1.64%; employment at $w = 0.62407$ as in Example 3.1

$$l^s = 0.7 - \frac{0.138\,04}{w^{1.5}} = 0.7 - \frac{0.138\,04}{(0.62407)^{1.5}} = 0.42$$

$$0.42 = \left(\frac{1.05}{3\,(0.62407)}\right)^{1.5} = \left(\frac{1.05}{3w}\right)^{1.5} = l^d.$$

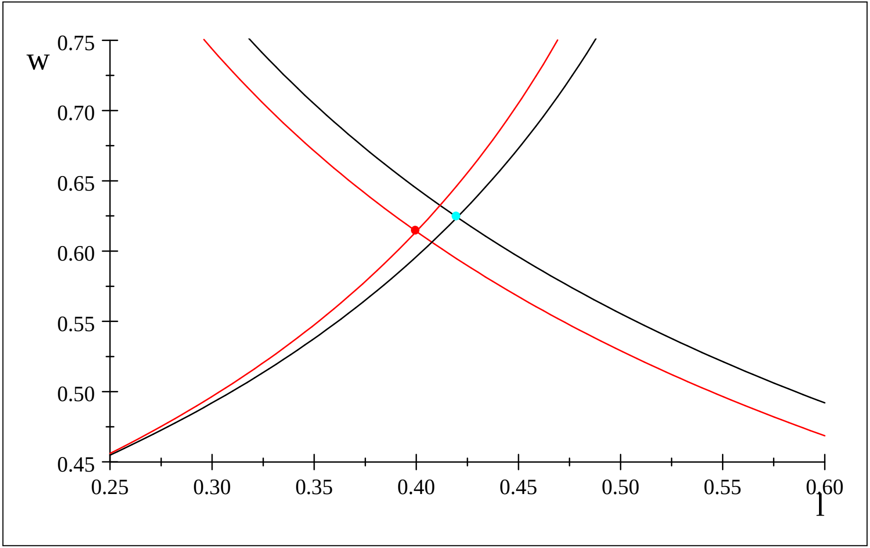

Figure 3.3. Expansion with a 5% Increase in Time Endowment T and in Goods Productivity A_G in Example 3.2.

Example 3.3 Contraction

With $A_G = T = 1$, decrease these by 5% to $A_G = T = 0.95$.

$$\underset{l^s}{\text{Max}}\ u = \ln\left(wl^s + \Pi\right) + 0.5\ln\left(0.95 - l^s\right), \tag{27}$$

$$\frac{w}{wl^s + \Pi} - \frac{0.5}{0.95 - l^s} = 0.\ \frac{0.95}{0.5} - \frac{\Pi}{w} = l^s\left(1 + \frac{1}{0.5}\right),$$

$$l^s = \frac{1.9}{3} - \frac{\Pi}{3w} = 0.63333 - \frac{\Pi}{3w}.$$

$$c^d = wl^s + \Pi = w\left(0.63333 - \frac{\Pi}{3w}\right) + \Pi = (0.63333)\,w + \frac{2}{3}\Pi.$$

Firm problem Example 3.3

$$\underset{l^d}{\text{Max}}\ \Pi = 0.95\left(l^d\right)^{\frac{1}{3}} - wl^d, \tag{28}$$

$$l^d = \left(\frac{0.95}{3w}\right)^{1.5}, \tag{29}$$

$$c^s = (0.95)\left(\frac{0.95}{3w}\right)^{\frac{1.5}{3}}, \tag{30}$$

$$\Pi = c^s - wl^d = \frac{2\,(0.95)^{1.5}}{3^{1.5}\sqrt{w}} = \frac{0.356\,40}{\sqrt{w}}. \tag{31}$$

$$c^d = (0.63333)\,w + \left(\frac{2}{3}\right)\frac{0.356\,40}{\sqrt{w}}. \tag{32}$$

Equilibrium in Example 3.3

$$l^s = 0.63333 - \frac{\Pi}{3w} = 0.633\,33 - \frac{0.118\,8}{w^{1.5}}.$$

$$l^s = 0.633\,33 - \frac{0.118\,8}{w^{1.5}} = \left(\frac{0.95}{3w}\right)^{1.5} = l^d,$$

$$w = \left(\frac{\frac{0.95^{1.5}}{3^{1.5}} + 0.118\,8}{0.633\,33}\right)^{\frac{2}{3}} = 0.603\,6.$$

$$l^s = 0.633\,33 - \frac{0.118\,8}{(0.603\,6)^{1.5}} = 0.38 = \left(\frac{0.95}{3\,(0.603\,6)}\right)^{1.5} = l^d.$$

Wage rate falls by $\frac{0.614-0.6036}{0.614} = 0.01694$, or 1.69%; employment falls changes by $\frac{0.02}{0.40} = 0.05$, or 5%.

$$w = \left(\frac{0.118\,8}{0.633\,33 - l^s}\right)^{\frac{2}{3}}, \quad w = \frac{0.95}{3\left(l^d\right)^{\frac{2}{3}}}. \tag{33}$$

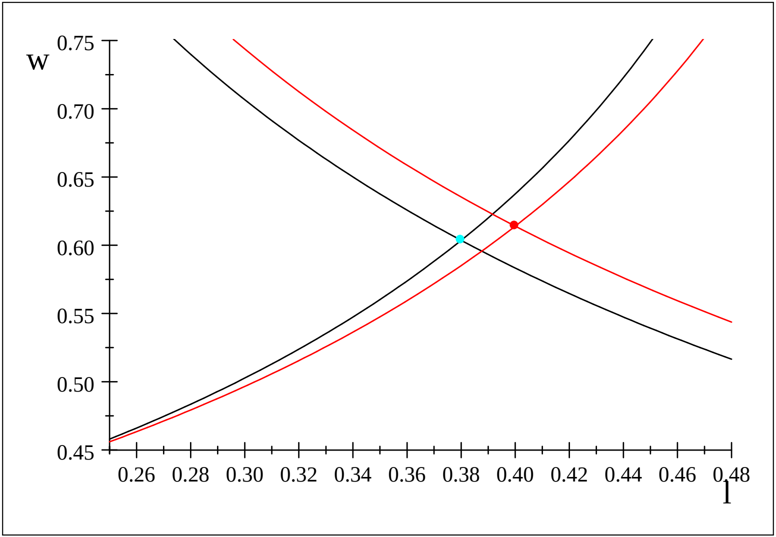

Figure 3.4. Contraction with a 5% Decrease in Time Endowment T and in Goods Productivity A_G in Example 3.3.

Unemployed Labor

- Labor movements over typical business cycle in "flexible price"
- Some recessions more severe, with decreases in employment not normal.
- Type of crisis unemployment, as in 1930s, and even in 2007-2009.
- Keynes's *General Theory:* interpreted as labor market not clearing;
- wage rate cannot adjust downwards sufficiently.
- Modeled by assuming fixed or inflexible wages and prices.
- Can assume wage rate fixed: ends up acting like an implicit tax.

Example 3.4: A Fixed Wage during a Contraction

- $w = \bar{w}$, and 5% decrease in productivity and time endowment.
- $A_G = T = 0.95$; $\gamma = \frac{1}{3}$, $\alpha = 0.5$, $w = \bar{w} = 0.614$ (when $T = 1$; $A_G = 1$)

$$
\begin{aligned}
l^s &= 0.633\,33 - \frac{0.118\,8}{(\bar{w})^{1.5}} = 0.633\,33 - \frac{0.118\,8}{(0.614)^{1.5}} = 0.386\,41, \\
l^d &= \left(\frac{0.95}{3\bar{w}}\right)^{1.5} = \left(\frac{0.95}{3\,(0.614)}\right)^{1.5} = 0.37038.
\end{aligned}
$$

$$
\textit{Excess Supply}: l^s - l^d = 0.386\,41 - 0.37038 = 0.016\,03,
$$

- $\frac{0.016\,03}{0.37038} = 0.04328$, or 4.3% of actual employment rate. "Surplus labor"
- 0.37 is equilibrium employment; instead of 0.40, decrease of $\frac{0.4-0.37}{0.4} = 0.075$,
- 50% bigger decrease of 7.5% compared to flexible wage equilibrium $l = 0.38$

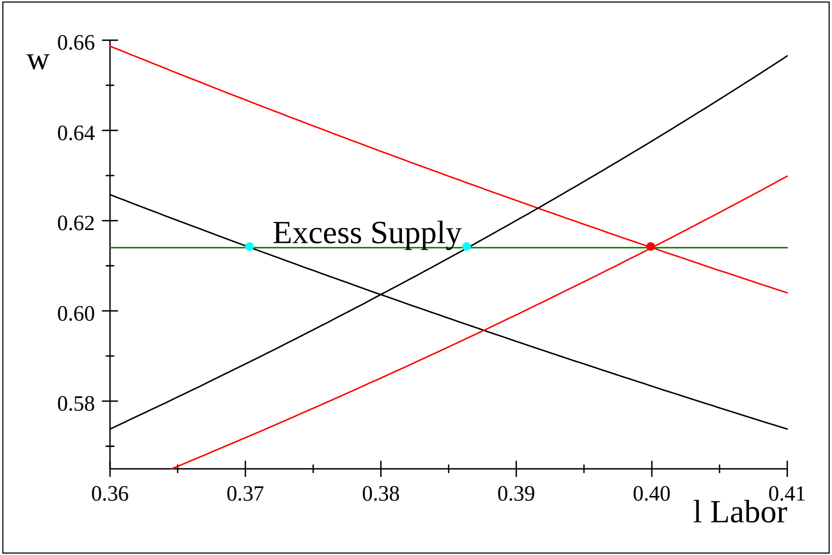

Figure 3.5. Excess Labor Supply with a Fixed Wage and Deeper Contraction in Example 3.4.

Goods Market Excess Demand

- excess supply of labor implies "rationing" of jobs, "queuing" in employment centers.
- Rationing also occurs in goods markets

$$
\begin{aligned}
c^s &= (0.95)\left(\frac{0.95}{3\bar{w}}\right)^{\frac{1.5}{3}} = (0.95)\left(\frac{0.95}{3\,(0.614)}\right)^{\frac{1.5}{3}} = 0.682\,25,\\
c^d &= (0.633)\,\bar{w} + \frac{2\,(0.356\,4)}{3\sqrt{\bar{w}}} = (0.633)\,(0.614) + \frac{2\,(0.356\,4)}{3\sqrt{0.614}} = 0.692\,09
\end{aligned}
$$

$$
Excess\ Demand : c^d - c^s = 0.692\,09 - 0.682\,25 = 0.009\,84,
$$

Fraction of actual consumption equal to $\frac{0.009\,84}{0.682\,25} = 0.0144$, or 1.44%.

- Rationing: queuing to buy goods: evident in 1930s? 2009? 1970s?
- 1970s oil rationing was only in one market.

A Goods Tax

- how tax rates affect long run employment, output, economic growth.
- government spending and taxation to cover the spending
- Has to do with efficacity of government policy in broad sense.
- Spending partly in form of benefits called "income transfers", or unemployment benefits.
- With perfect unemployment system, no "moral hazard", no distortion from benefit transfer
- Moral hazard: decrease in incentive to work from getting unemployment benefits.
- Iincome benefits as a permanent benefit is distortionary: shifts back labor supply.
- Many work without reporting work, but work less: "dole" in UK, Australia, New Zealand.

Goods Tax and Transfer

- Consumer
$$(1+\tau_c)\,c^d = wl^s + \Pi + G. \tag{34}$$

- Government
$$G = \tau_c c^d. \tag{35}$$

- In equilibrium, sub in for $G : c^d = \Pi + wl^s$: consumer's net income unchanged.
- Distortion on margin, less work:
$$MRS_{c,x} = \frac{\frac{\alpha}{x}}{\frac{1}{c^d}} = \frac{w}{1+\tau_c},$$

Example 3.5: Consumer.

- $A_G = 1,\ T = 24,\ \gamma = 0.5,\ \alpha = 1,\ \tau_c = 0.20$

$$c^d = \frac{wl^s + \Pi + G}{1 + \tau_c}, \tag{36}$$

$$\underset{l^s}{\text{Max}}\ u = \ln\left(\frac{wl^s + \Pi + G}{1 + \tau_c}\right) + \ln(24 - l^s),$$

$$\frac{\frac{w}{1+\tau_c}}{\frac{\Pi + wl^s + G}{1+\tau_c}} - \frac{1}{24 - l^s} = 0;\ \frac{w}{1+\tau_c} = \frac{\frac{1}{24 - l^s}}{\frac{1}{\frac{\Pi + wl^s + G}{1+\tau_c}}} = \frac{\frac{1}{x}}{\frac{1}{c}} = \frac{\frac{\partial u(c,x)}{\partial x}}{\frac{\partial u(c,x)}{\partial c^d}}.$$

$$l^s = 12 - \left(\frac{\Pi + G}{2w}\right). \tag{37}$$

$$c^d = \frac{0.5\,(24w + \Pi + G)}{1 + \tau_c}. \tag{38}$$

Example 3.5: Firm

$$c^s = \left(l^d\right)^{0.5}, \tag{39}$$

$$\begin{aligned}\Pi &= (1+\tau_c)\,c^s - wl^d - \tau_c c^s \\ &= c^s - wl^d.\end{aligned}$$

$$\underset{l^d}{\text{Max}}\Pi = \left(l^d\right)^{0.5} - wl^d, \tag{40}$$

$$0.5\left(l^d\right)^{-0.5} - w = 0,$$

$$l^d = \frac{1}{4w^2},\; c^s = \frac{0.5}{w},\; \Pi = \frac{1}{4w}.$$

Example 3.5: Equilibrium

$$c^d = \frac{0.5\left(24w + \frac{1}{4w} + G\right)}{1+\tau_c}, \tag{41}$$

$$l^s = 12 - \left(\frac{\frac{1}{4w} + G}{2w}\right). \tag{42}$$

$$c^d = \frac{w12 + \dfrac{1}{8w}}{1 + \frac{\tau_c}{2}}. \tag{43}$$

$$\frac{1}{w} = \frac{c^s}{0.5}; \left(\frac{1}{w}\right)^2 - \left[\left(1 + \frac{\tau_c}{2}\right) 8c^d\right]\left(\frac{1}{w}\right) + 96 = 0. \tag{44}$$

$$\frac{1}{w} = \frac{-B - \sqrt{B^2 - 4AC}}{2a}; A = 1, B = -\left(1 + \frac{\tau_c}{2}\right) 8c^d, C = 96,$$

$$\frac{1}{w} = 0.5\left[\left(1 + \frac{\tau_c}{2}\right) 8\left(c^d\right) - \sqrt{\left(\left(1 + \frac{\tau_c}{2}\right) 8\right)^2 \left(c^d\right)^2 - 4\left(96\right)}\right]. \tag{45}$$

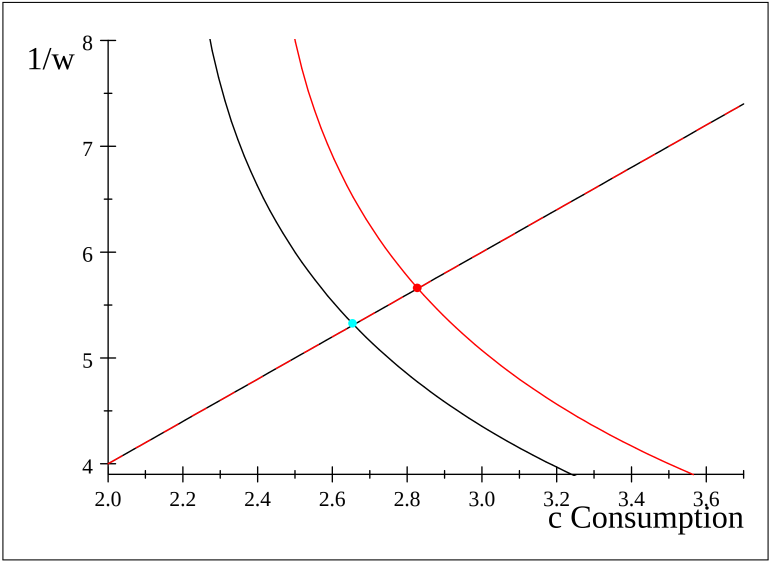

Figure 3.6. A 20% Tax on Goods with Income Transfer in Example 3.5.

$$l^s = 12 - \frac{1}{8w^2} - \frac{\tau_c c^d}{2w};\ c^s = c^d;\ c^s = \frac{1}{2w} \tag{46}$$

$$l^s = 12 - \frac{1}{8w^2} - \frac{\tau_c}{4w^2}. \tag{47}$$

$$w = \sqrt{\frac{1.40}{8\left(12 - l^s\right)}};\ w = \frac{1}{2\sqrt{l^d}}. \tag{48}$$

$$l^s = 12 - \frac{1}{8w^2} - \frac{\tau_c c^d}{2w} = \frac{1}{4w^2} = l^d,$$

$$w = \sqrt{\frac{3 + 2\tau_c}{96}} = \sqrt{\frac{3 + 2\left(0.2\right)}{96}} = 0.18819. \tag{49}$$

- $\tau_c = 0.2$, $w = 0.188$, compared to $\tau_c = 0$ and $w = 0.176\,78$.
- $l = \dfrac{1}{4\left(0.188\,19\right)^2} = 7.0591$; from $\dfrac{1}{4\left(0.17678\right)^2} = 8.0$
- decrease of $\frac{8-7.0591}{8} = 0.117\,61$, or 11.8%.

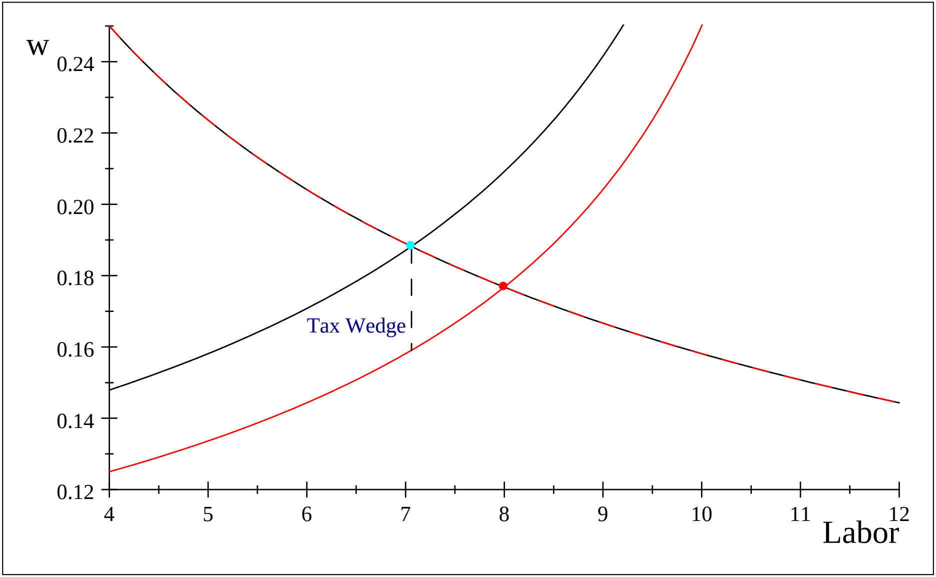

Figure 3.7. Labor Market with Goods Tax $\tau_c = 0.2$ and Income Transfer in Example 3.6

After Tax Wage Rate and Tax Wedge

- $w = \sqrt{\frac{3+2\tau_c}{96}}$ instead of $\sqrt{\frac{3}{96}}$. Labor hours from 8 hours to 7.06.
- The "tax wedge" is the difference between
  - 1) equilibrium wage rate when $\tau_c = 0.2$ and
  - 2) wage rate that induces 7.06 labor time
  - Wedge measured as vertical difference in Figure 3.7 at $l^s = 7.06$,
  - dashed vertical line.
- Computation of wedge

$$
\begin{aligned}
7.06 &= l^s = 12 - \frac{1}{8w^2}, \\
w &= \sqrt{\frac{1}{8\,(12 - 7.0591)}} = 0.159\,06.
\end{aligned}
$$

$$
\begin{aligned}
Tax\ Wedge &: 0.188 - 0.159 = 0.029; \\
\frac{0.029}{0.159} &= 0.18,\ 18\%.
\end{aligned}
$$

General Equilibrium Tax Wedge

- Utility Level Curve with Tax

$$w = 0.188; \tag{50}$$

$$c^d = \frac{(0.18819)\,12 + \frac{1}{8\,(0.18819)}}{1 + \frac{0.2}{2}} = 2.6568, \tag{51}$$

$$u = \ln c^d + \ln\left(24 - l^s\right) = \ln 2.\,6568 + \ln\left(24 - 7.0591\right) = 3.8069,$$

$$c^d = \frac{e^{3.8069}}{(24 - l)} = \frac{45.011}{24 - l^s}. \tag{52}$$

- Production Possibility Curve: $c^s = \left(l^d\right)^{0.5}$

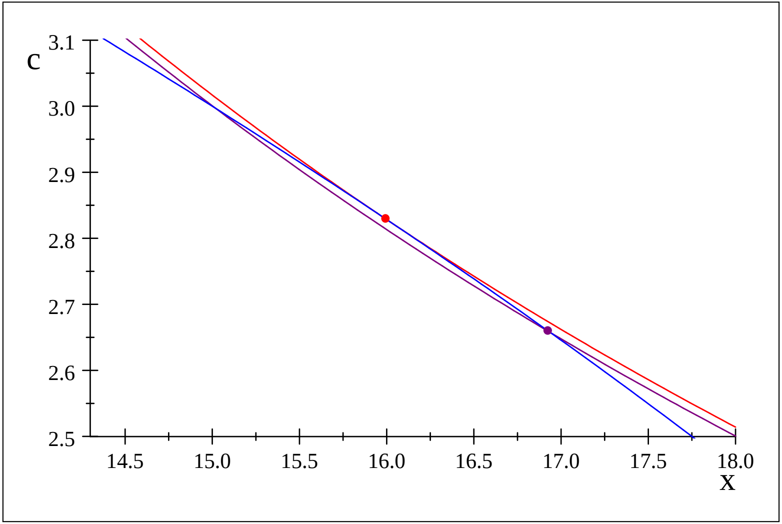

Figure 3.8. General Equilibrium with Tax Distortion in Example 3.5.

Zero Government Transfer

- $$c^d = wl^s + \Pi - \tau_c c^d + G. \quad (53)$$
  $$T \equiv \tau_c c^d,$$
  $$y = wl^s + \Pi,$$
  $$c^d = y - T + G. \quad (54)$$
  $$T = G$$
  $$c^d = y.$$
- No addition of government expenditure since cancel out with taxes.

- Suppose no transfer G: taxes "used up".

$$c^d = wl^s + \Pi - \tau_c c^d = y - T; \quad (55)$$

$$c^d + G = y - T + G, \; G = T \quad (56)$$

$$c^d + G = y. \quad (57)$$

$$c^d = \frac{0.5\left(24w + \frac{1}{4w}\right)}{1 + \tau_c}. \quad (58)$$

$$\left(\frac{1}{w}\right)^d = 0.5\left[(1+\tau_c)\,8\left(c^d\right) - \sqrt{((1+\tau_c)\,8)^2\left(c^d\right)^2 - 4\,(96)}\right], \quad (59)$$

$$\left(\frac{1}{w}\right)^s = 2c^s.\ \tau_c = 0.2;\ G = \tau_c c^d = (0.2)\,(2.51226) = 0.5025. \quad (60)$$

$$E\,(c) = \left(\frac{1}{w}\right)^d - \left(\frac{1}{w}\right)^s = 0.\ c = 2.5126,\ \frac{1}{w} = 5.0252.$$

E(c)=0 Implies Equilibrium c

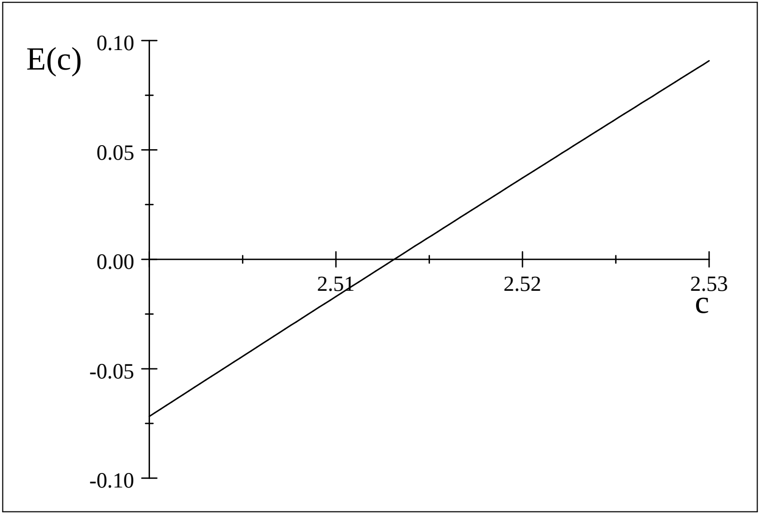

Figure 3.9. Zero Excess Relative Price at Equilibrium $c = 2.51$ in Example 3.6.

Goods Market

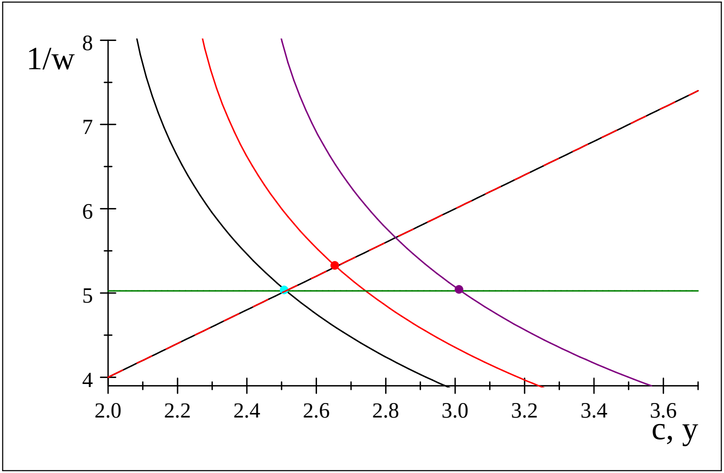

Figure 3.10. Goods Market with Tax on Goods with Zero Transfer in Example 3.6 in Black, with Transfer of Example 3.5 in Red, and Zero Transfer Output y in Purple.

Labor Market

- $$G = 0$$
$$\begin{aligned} l^s &= 12 - \frac{1}{8w^2}, \\ w &= \sqrt{\frac{1}{8\left(12 - l^s\right)}}; && (61) \\ w &= \frac{1}{2\sqrt{l^d}}. && (62) \end{aligned}$$
- original labor supply when no tax and income transfer; same demand

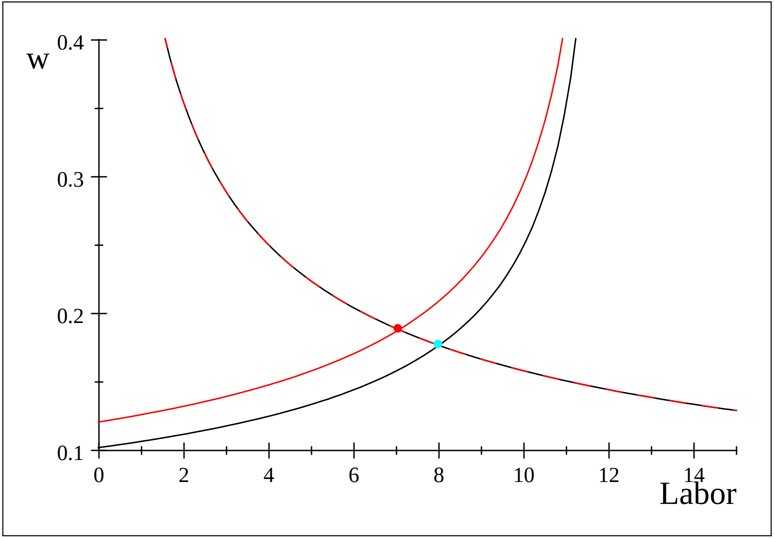

Figure 3.11. Labor Market with a Goods Tax and Zero Transfer in Example 3.6.

$$\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial c^d}} = \frac{w}{1+\tau_c};$$
$$\frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial c^d}} = \frac{w\left(1-\tau_l\right)}{1}.$$

- $T = 24$, $\alpha = 1$, $A_G = 1$ and $\gamma = 0.5$.

$$c^d = \left(1-\tau_l\right) wl^s + \Pi + G, \tag{63}$$

$$G = \tau_l wl^s.$$

$$\underset{l^s}{\text{Max}}\; u = \ln\left[\left(1-\tau_l\right) wl^s + \Pi + G\right] + \ln\left(24 - l^s\right). \tag{64}$$

$$\frac{\left(1-\tau_l\right) w}{\left(1-\tau_l\right) wl^s + \Pi + G} - \frac{1}{24 - l^s} = 0. \tag{65}$$

$$l^s = \frac{24 + \Pi + G}{2\left(1-\tau_l\right) w}. \tag{66}$$

Labor Tax Example 3.7: Firm and Equilibrium

$$\operatorname*{Max}_{l^d} \Pi = \sqrt{l^d} - wl^d. \tag{67}$$

$$0.5\left(l^d\right)^{-0.5} - w = 0; \tag{68}$$

$$l^d = \frac{1}{4w^2};\ \Pi = \frac{1}{4w}. \tag{69}$$

$$l^s = 24\left(\frac{1-\tau_l}{2-\tau_l}\right) - \frac{1}{4w^2\left(2-\tau_l\right)}. \tag{70}$$

$$w = \frac{1}{2\left[\sqrt{19.2 - 1.8\left(l^s\right)}\right]},\ w = \frac{1}{2\sqrt{l^d}} \tag{71}$$

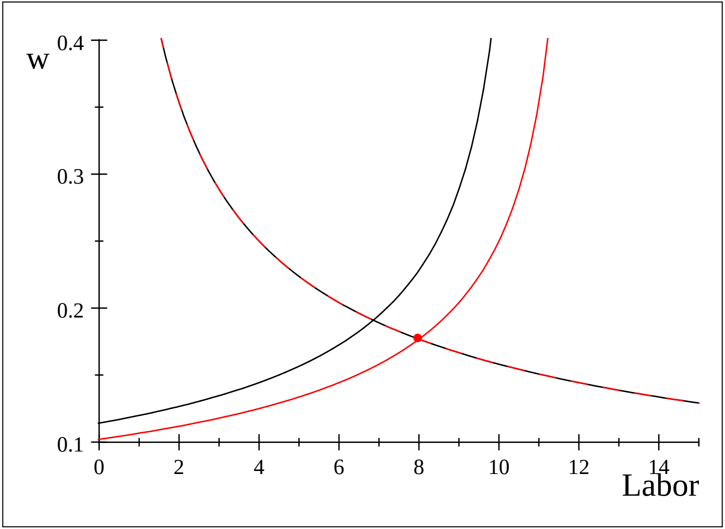

Figure 3.12. Labor Market with Labor Tax and Transfer in Example 3.7.

Labor Tax Equilibrium and Goods Market

- $$l^d = \frac{1}{4w^2} = 24\left(\frac{1-\tau_l}{2-\tau_l}\right) - \frac{1}{4w^2\left(2-\tau_l\right)} = l^s, \tag{72}$$

$$w = \sqrt{\frac{1}{96}\left(\frac{3-\tau_l}{1-\tau_l}\right)}.\ \tau_l = 0.2,\ w = 0.191;\ \tau_l = 0;\ w = 0.177 \tag{73}$$

$$c^d = \left(1-\tau_l\right) wl^s + \Pi + \tau_l wl^s = wl^s + \Pi.$$

$$c^d = w\left[24\left(\frac{1-\tau_l}{2-\tau_l}\right) - \frac{1}{4w^2\left(2-\tau_l\right)}\right] + \frac{1}{4w}.$$

- Tax τ_l shifts back labor supply, shifts back consumption goods demand
- goods supply unchanged. $\frac{1}{w}$ falls in goods market; w rises in labor market.

Comparison of Taxes

$$\frac{1-\tau_l}{1} = 1 - 0.177 = 0.833 = \frac{1}{1+\tau_c}.$$

- $\tau_l = 0.177$, same as goods tax rate of $\tau_c = 0.2$,
- $\tau_l = 0.177$; $w = 0.189$; close to wage rate when $\tau_c = 0.2$; $w = 0.188$.
- Marginal distortion quantitatively similar.

Regulation and Implicit Taxes

- Tax revenue from explicit tax collected by government,
- Tax revenue from a regulation, an implicit tax, called "rents", can go to anyone:
- those passing the legislation, members of government, or those working for legislation;
- often just "power" yielded to people involved in protective legislation.

Analytic Rents and Corruption: Consumer

- Π_{cr}, "consumer rents" transfered back; Π profit from firm
- implicit tax of regulation, τ, on goods.

$$(1+\tau)\, c^d = \Pi + wl^s + \Pi_{cr}.$$

$$\underset{l^s}{\text{Max}}\ u = \ln\left(\frac{\Pi + wl^s + \Pi_{cr}}{1+\tau}\right) + \ln(24 - l^s).$$

$$c^d = \frac{w12 + \frac{\Pi + \Pi_{cr}}{2}}{1+\tau},$$

$$l^s = 12 - \left(\frac{\Pi + \Pi_{cr}}{2w}\right). \qquad (74)$$

Analytic Rents and Corruption: Firm

- Π_{fr} firm rents that must be paid out to consumer
- equal to some fraction $\eta \in [0,1]$ of implicit tax revenue τc^s.

$$\Pi_{fr} = \eta \tau c^s. \tag{75}$$

- Competitive market for the rents, then $\eta = 1$ and $\Pi_{cr} = \Pi_{fr}$.
- If no rents paid out, then $\eta = 0$, and as when $G = 0$.

$$\underset{l^d}{\text{Max}}\, \Pi = (1+\tau)\, c^s - wl^d - \Pi_{fr}, \tag{76}$$

$$\begin{aligned} \underset{l^d}{\text{Max}}\, \Pi &= (1+\tau)\sqrt{l^d} - wl^d - \eta\tau\sqrt{l^d} && (77) \\ &= [1+\tau(1-\eta)]\sqrt{l^d} - wl^d. && (78) \end{aligned}$$

- As in tax and dole by government $\eta = 1$.
- Smaller is η, less is transfer to consumer, less is income effect shifting back labor supply.

- 1. Monopoly rents collected by firm: $\eta = 0$ and $\Pi_{fr} = \Pi_{cr} = 0$.
- 2. Efficient regulation: $\eta = 1$, and $\Pi_{fr} = \tau c^s = \Pi_{cr}$.
- 3. Inefficient regulation: $\eta = 1$, $\Pi_{fr} = \tau c^s$, but $\Pi_{cr} = 0$.
- More generally, some rent reaches consumer $\Pi_{cr} = \eta_c \Pi_{fr}$, where $\eta_c \leq 1$.

Applications: Moral Hazard

- Unemployment insurance systems can cover probabilistic employment loss.
- Private contract of employees or by government programs.
- Distorting incentives to work can cause long term structural unemployment.
- Lower probability of getting work from policy is called "moral hazard".
- Increases probability of the bad state to increase.
- Duration importanti: a permanent subsidy causes more long-term unemployment.
- Widely accepted as why "dole", or "welfare", cultures arise.
- Unemployment Benefits as Insurance: James Meade, 1937.
- Breakdown of insurance system likely if benefits for unlimited time "duration".
- perfect insurance: no moral hazard, no shift back in labor supply.

Tax Reduction as Policy

- Reducing tax rate on goods and labor income increases incentives to work.
- Only if level of government spending also falls.
- Spending must be financed somehow.
- Temporary reduction in tax rates must be offset by higher future taxes
- Temporary reductions not perceived as efficacious
- Long term reductions in tax rates good if government gradually more efficient
- Less spending as a percent of national income allows this.

Regulation of Airlines, Oil and Pollution

- Airline deregulation in 1970s,
  - airline prices were uniformly set and entry dramatically limited.
  - increased price of tickets to consumers.
  - some rents transferred back as luxurious meal service, little crowding.
  - With deregulation low cost carriers entered market, prices dramatically fell
  - all but eliminating implicit tax τ that airlines could levy onto price.
- Oil policy in the US
  - forced price of oil within US domestic markets to be below world price
  - ceiling on the sales price for oil in old oil wells in the US.
  - makes the tax τ equal to a subsidy for oil
  - results in large automobiles, led to US auto industry competitive disadvantage.
  - $\Pi_{cr} = \tau c^s$, where $\tau < 0$. interpretation of US oil crisis of 1970s.
- Market for trading excess industrial emissions can minimize cost of regulation.