Advanced Modern Macroeconomics

Trade in Labor and Goods Markets

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Chapter 4: Trade in Labor and Goods Markets

Chapter Summary

- Two representative agents, same utility, different labor productivity: "heterogenous agents".
- Each agent A and agent B solve own consumer and firm problems, with autarky or free trade.
- Trade: world market clearing condition, one world equilibrium wage.
- Less productive agent supplies labor to more productive agent, receives goods.
- Utility for each agent increases: measure of gains from trade.
- Trade with comparative advantage:
  - less productive agent better at supplying labor, works more;
  - more productive agent better at producing goods, produces more.
- Trade triangles: excess demand for goods and supply of labor.
- Applications: education, immigration restrictions, taxes;
- trade laws, treaties, immigration waves, and globalization.

Building on the Last Chapters

- Agent A and B Economies already seen previously
  - Example 2.5 and 3.5 calibration for agent A;
  - double productivity of Example 2.6 for agent B.
- Wage market clearing condition similar to Chapter 2, 3.
- Now 2 supply, 2 demand functions of each agent added together.
- Tax application in Example 4.6 extension to 2 agents of Example 3.5.

Learning Objective

- Aim representative agent model to two agents who trade.
- Wage under autarky
  - lower for less productive agent,
  - higher for more productive agent,
- but equal for both under trade,
  - with wage rising for less productive
  - and wage falling for more productive.
- Equilibrium excess goods, labor: market clearing trade flows
  - different from non-market clearing fixed price equilibrium, Chapter 3.
- Production function and utility curves not tangent to each other.
  - Production function tangent to budget line in one place;
  - utility level curve is tangent to the budget line in another place,
  - difference in tangencies gives trade flows.
- Breadth for explaining trends: education, globalization, immigration.

Who Made it Happen: Trade Theory

- Heckscher 1919, Ohlin 1933, trade with comparative advantage.
- Key: difference in factor prices in autarky,
  - factor price equalization 'an inescapable consequence of trade'
  - when similar technologies.
- Extended to general equilibrium, Stolper and Samuelson 1941.
- Samuelson (1948, 1949) proved "factor price equalization theorem".
- Heckscher-Ohlin: trade from different resource endowments.
  - Here, greater productivity given technology: bigger goods endowment
- Stolper-Samuelson 1941 consider protectionist trade policy.
  - Here, immigration restrictions analysed.

Two Agent Model, Example 4.1: Autarky vs. Trade

- $\alpha = 1$, $\gamma = 0.5$, $T = 24$.

$$\begin{aligned} u_A &= \ln c_A^d + \ln\left(24 - l_A^s\right), \; u_A = \ln c_B^d + \ln\left(24 - l_B^s\right); \\ c_A^s &= \sqrt{l_A^d}, \; c_B^s = 2\sqrt{l_B^d}. \end{aligned}$$

- Equilibria: $l_A = 8$, $c_A = \sqrt{8}$, $w_A = 0.177$; $l_B = 8$, $c_B = 2\sqrt{8}$, $w_B = 0.354$.
- Decentralized market supply, demand needed for trade

$$\begin{aligned} l_A^d + l_B^d &= l_A^s + l_B^s, \\ c_A^d + c_B^d &= c_A^s + c_B^s. \end{aligned}$$

- Either to find equilibrium wage w.
- Demand, supply functions of w from consumer, firm problems.

Autarky Supply, Demand Functions of Wage

$$\begin{aligned}
\underset{l_A^s}{\text{Max}}\ u &= \ln\left(\Pi_A + w l_A^s\right) + \ln\left(24 - l_A^s\right). \\
\frac{\partial L}{\partial l_A^s} &= \frac{w}{\Pi_A + w l_A^s} + \frac{(-1)}{24 - l_A^s} = 0. \\
l_A^s &= 12 - \frac{\Pi_A}{2w}.\ c_A^d = \Pi_A + w l_A^s = 12w + \frac{\Pi_A}{2}.
\end{aligned}$$

$$\begin{aligned}
\underset{l_B^s}{\text{Max}}\ u &= \ln\left(\Pi_B + w l_B^s\right) + \ln\left(24 - l_B^s\right). \\
\frac{\partial L}{\partial l_B^s} &= \frac{w}{\Pi_B + w l_B^s} + \frac{-1}{24 - l_B^s} = 0. \\
l_B^s &= 12 - \frac{\Pi_B}{2w},\ c_B^d = \Pi_A + w l_A^s = 12w + \frac{\Pi_B}{2}.
\end{aligned}$$

Firm Problems

$$\begin{aligned}
\underset{I_A^d}{\text{Max}}\ \Pi_A &= \sqrt{I_A^d} - wI_A^d. \\
\frac{\partial \Pi_A}{\partial I_A^d} &= 0.5\left(I_A^d\right)^{-0.5} - w = 0. \\
I_A^d &= \frac{1}{4w^2}.\ c_A^s = \sqrt{I_A^d} = \sqrt{\frac{1}{4w^2}} = \frac{1}{2w}.\ \Pi_A = \frac{1}{4w}.
\end{aligned}$$

$$\begin{aligned}
\underset{I_B^d}{\text{Max}}\ \Pi_B &= 2\sqrt{I_B^d} - wI_B^d. \\
\frac{\partial \Pi_B}{\partial I_B^d} &= \left(I_B^d\right)^{-0.5} - w = 0. \\
I_B^d &= \frac{1}{w^2}.\ c_B^s = 2\sqrt{I_B^d} = 2\sqrt{\frac{1}{w^2}} = \frac{2}{w},\ \Pi_B = \frac{1}{w}
\end{aligned}$$

Supply and Demand in Both Markets

$$Goods_A \quad : \quad \frac{1}{w} = 2c_A^s, \ \frac{1}{w} = 4c^d - 4\sqrt{\left(c_A^d\right)^2 - 6} \tag{1}$$

$$Goods_B \quad : \quad \frac{1}{w} = \frac{c_B^s}{2}; \ \frac{1}{w} = c_B^d - \sqrt{\left(c_B^d\right)^2 - 24} \tag{2}$$

$$Labor_A \quad : \quad w = \frac{1}{2}\sqrt{\frac{1}{l_A^d}}, \ w = \sqrt{\frac{1}{8\left(12 - l_A^s\right)}}. \tag{3}$$

$$Labor_B \quad : \quad w = \sqrt{\frac{1}{l_B^d}}. \ w = \sqrt{\frac{1}{2\left(12 - l_B^s\right)}}. \tag{4}$$

Graphical Equilbrium

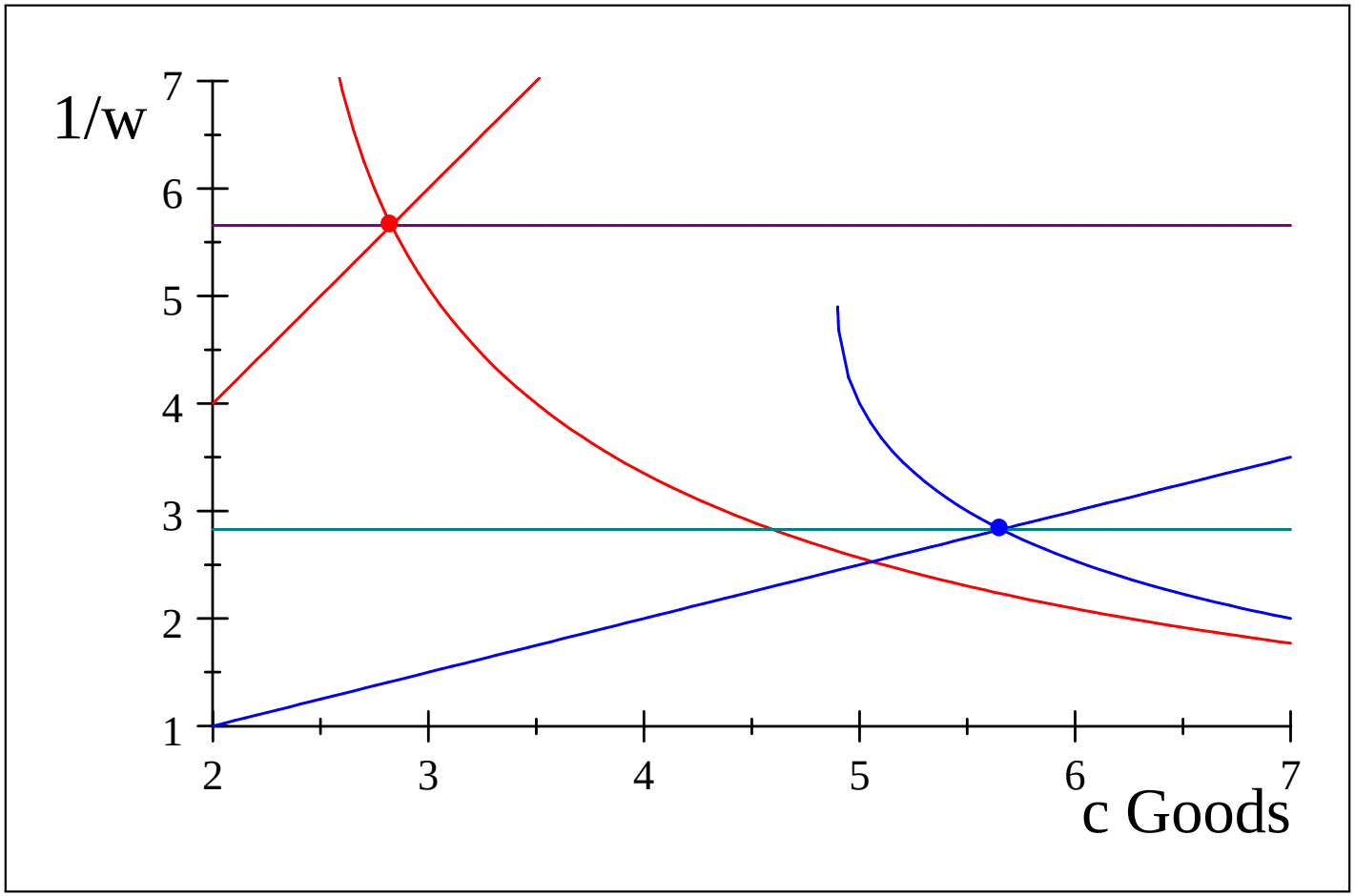

Figure 4.1. The Goods Market under Autarky in Example 4.1 for A in Red and B in Blue.

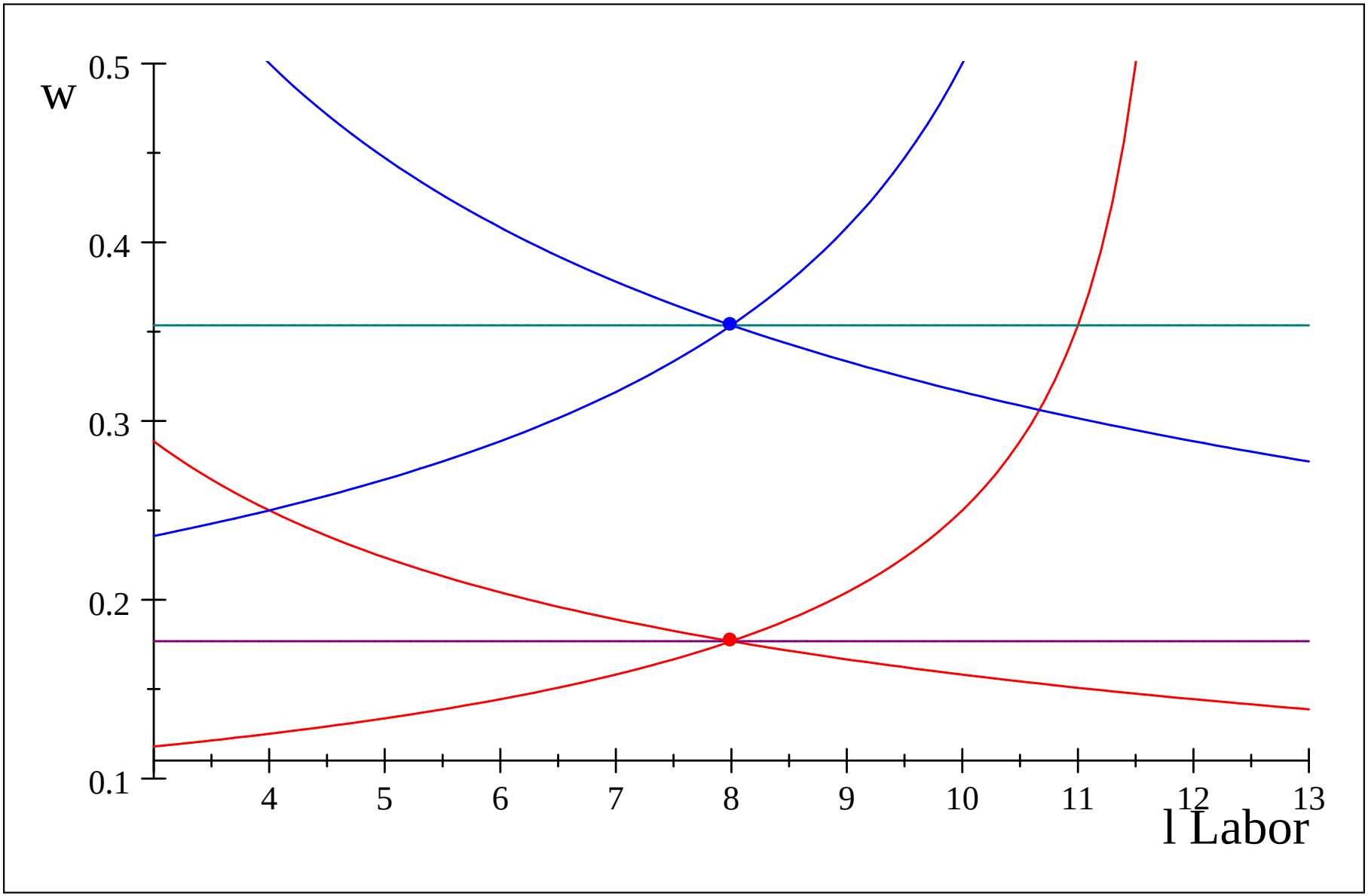

Figure 4.2. The Labor Market under Autarky in Example 4.1 for A in Red and for B in Blue.

Trade Solution for Wage

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$$
\begin{aligned}
l^d &= l_A^d + l_B^d = \frac{1}{4w^2} + \frac{1}{w^2} = 12 - \frac{1}{8w^2} + 12 - \frac{1}{2w^2} = l_A^s + l_B^s = l^s, \\
24 &= \frac{1}{4w^2} + \frac{1}{8w^2} + \frac{1}{w^2} + \frac{1}{2w^2} = \frac{1}{w^2}\left(\frac{1}{4} + \frac{1}{8} + 1 + \frac{1}{2}\right) = \frac{15}{8}\frac{1}{w^2}, \\
w &= \sqrt{\frac{15}{8}\frac{1}{24}} = 0.28.
\end{aligned}
$$

$$
\begin{aligned}
c_A^d + c_B^d &= 12w + \frac{1}{8w} + 12w + \frac{1}{2w} = \frac{1}{2w} + \frac{2}{w} = c_A^s + c_B^s, \\
24w &= \frac{1}{w}\left(2.5 - \frac{5}{8}\right) = \frac{1.875}{w}, \\
w &= \sqrt{\frac{1.875}{24}} = 0.28.
\end{aligned}
$$

Trade Solution for All Quantities

Table 4.1 Equilibrium with Free Trade; $w = 0.28$

EQUILIBRIUM	Agent A	Agent B
Labor Demand	$l_A^d = \frac{1}{4w^2} = 3.2$	$l_B^d = \frac{1}{w^2} = 12.8$
Labor Supply	$l_A^s = 12 - \frac{1}{8w^2} = 10.4$	$l_B^s = 12 - \frac{1}{2w^2} = 5.6$
Profit	$\Pi_A = \frac{1}{4w} = 0.89$	$\Pi_B = \frac{1}{w} = 3.57$
Goods Supply	$c_A^s = \frac{1}{2w} = 1.80$	$c_B^s = \frac{2}{w} = 7.14$
Goods Demand	$c_A^d = 12w + \frac{1}{8w} = 3.80$	$c_B^d = 12w + \frac{1}{2w} = 5.14$
Utility	$\ln c_A^d + \ln\left(24 - l_A^s\right) = 3.95$	$\ln c_B^d + \ln\left(24 - l_B^s\right) = 4.55$

Market for Goods

$$
\begin{aligned}
c^d &= c_A^d + c_B^d = 12w + \frac{1}{8w} + 12w + \frac{1}{2w} = 24w + \frac{5}{8w}, \\
\frac{1}{w} &= \frac{4}{5}c^d - \frac{4}{5}\sqrt{(c^d)^2 - 60}; \qquad (5) \\
c^s &= c_A^s + c_B^s = \frac{1}{2w} + \frac{2}{w} = \frac{2.5}{w}, \\
\frac{1}{w} &= \frac{c^s}{2.5}. \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
\textit{Excess Demand}_A &: c_A^d - c_A^s = 3.80 - 1.80 = 2.0, \qquad (7) \\
\textit{Excess Supply}_B &: c_B^s - c_B^d = 7.14 - 5.14 = 2.0. \qquad (8)
\end{aligned}
$$

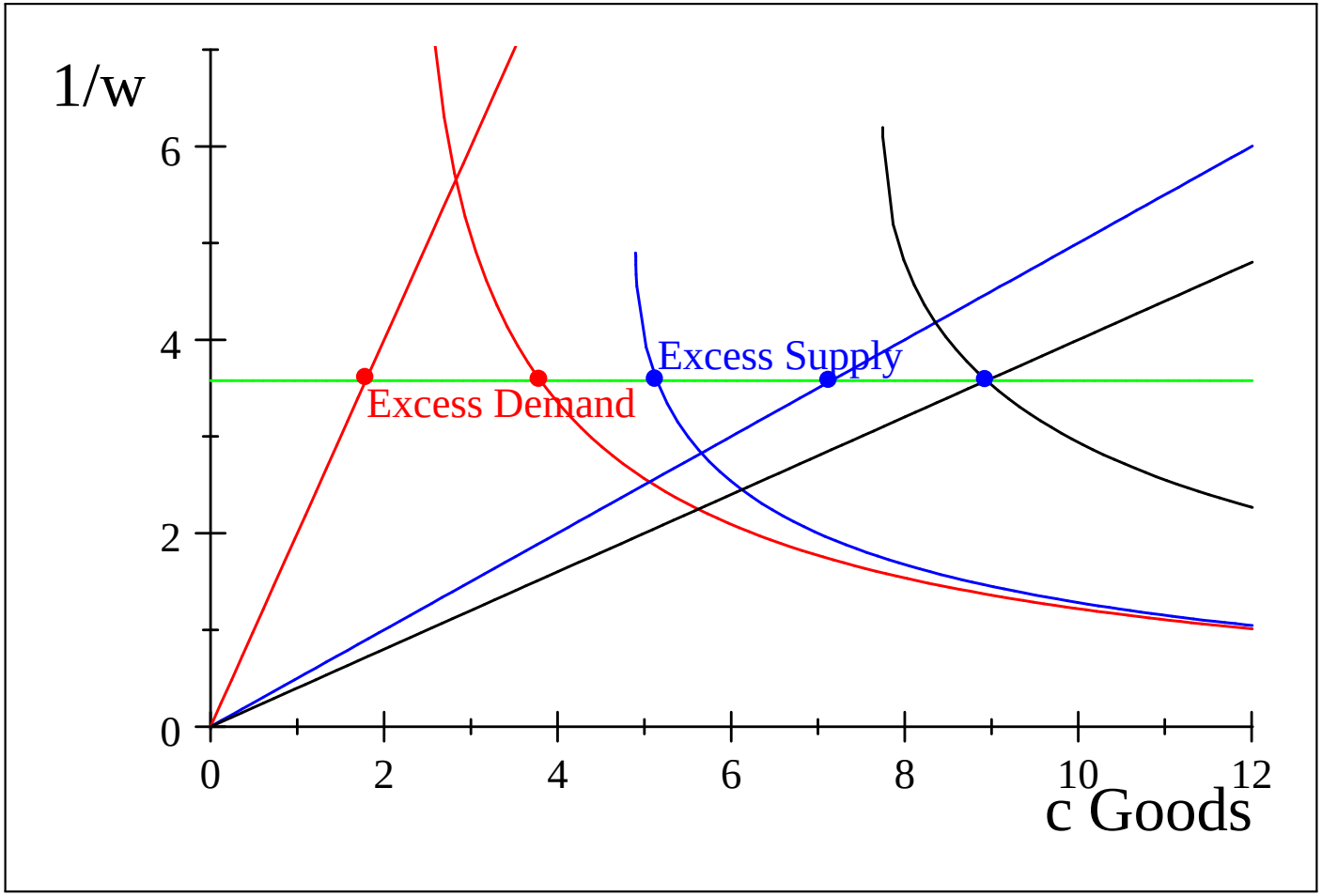

Figure 4.3. The Goods Market under Free Trade in Example 4.3.

Market for Labor

$$
\begin{aligned}
l^d &= l_A^d + l_B^d = \frac{1}{4w^2} + \frac{1}{w^2} = \frac{5}{4w^2}, \\
w &= \sqrt{\frac{5}{4l^d}}; \qquad (9) \\
l^s &= l_A^s + l_B^s = 12 - \frac{1}{8w^2} + 12 - \frac{1}{2w^2} = 24 - \frac{5}{8w^2}, \\
w &= \sqrt{\frac{5}{8\left(24 - l^s\right)}}. \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
\textit{Excess Supply}_A &: \quad l_A^s - l_A^d = 10.4 - 3.2 = 7.2, \qquad (11) \\
\textit{Excess Demand}_B &: \quad l_B^d - l_B^s = 12.8 - 5.6 = 7.2. \qquad (12)
\end{aligned}
$$

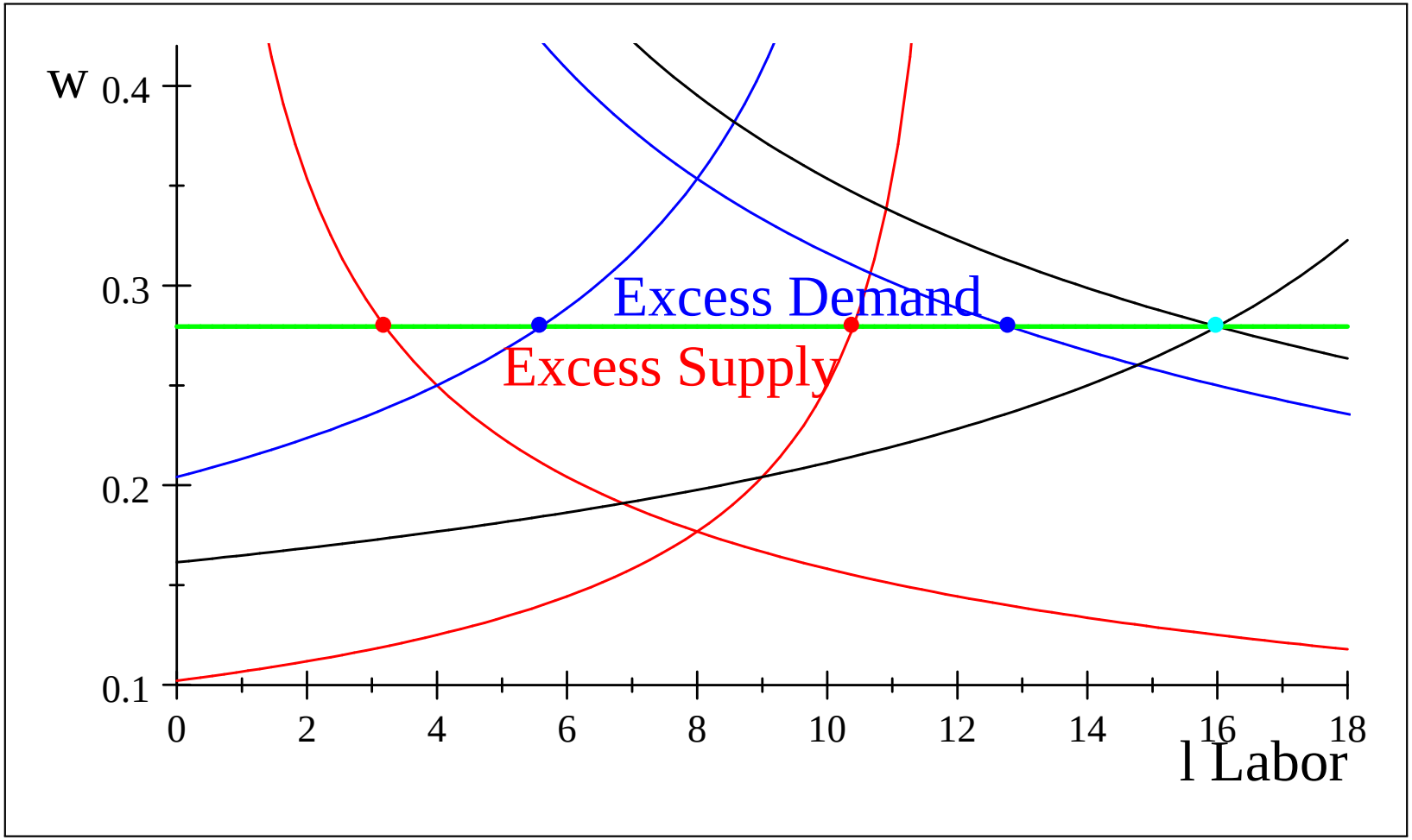

Figure 4.4. The Labor Market under Free Trade in Example 4.3.

Budget Lines, Production Functions, Level Curves

$$
\begin{aligned}
c_A^d &= wl_A^s + \Pi_A = wl_A^s + 0.894 = (0.28)(24 - x_A) + 0.894, \\
c_B^d &= wl_B^s + \Pi_B = wl_B^s + 3.578 = (0.28)(24 - x_B) + 3.578,
\end{aligned}
$$

$$
c_A^s = \sqrt{l_A^d} = \sqrt{(24 - x_A)},\ c_B^s = 2\sqrt{l_B^d} = 2\sqrt{(24 - x_B)}.
$$

$$
u_A = 3.9454,\ c_A^d = \frac{e^{3.9454}}{24 - l_A^s} = \frac{e^{3.9454}}{x_A}; \tag{13}
$$

$$
u_B = 4.55,\ c_B^d = \frac{e^{4.55}}{24 - l_B^s} = \frac{e^{4.55}}{x_B}. \tag{14}
$$

Equal Sized Trade Triangles

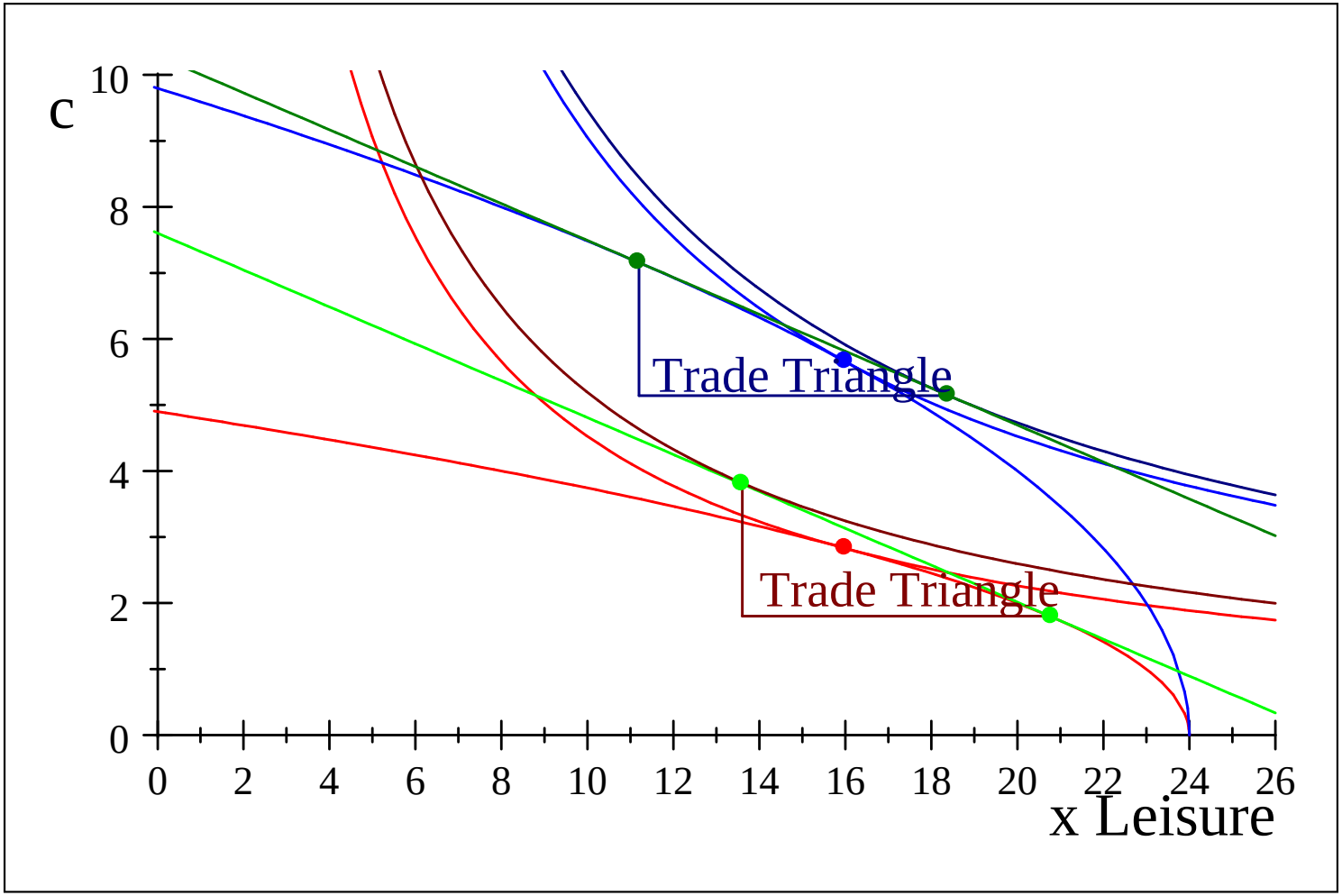

Figure 4.5. General Equilibrium Goods and Labor Markets Under Free Trade in Example 4.3.

Marginal Productivity Theory of Income Distribution

- More productive agent: higher consumption, leisure; higher income "class"
- Agent A works for agent B, engages in a bit of self-production,
- Agent B works only for self-production, employs agent A.
- Theory of income distribution: "labor", "capitalist" classes.
- Even with no capital yet introduced; more productive as capitalist.
- Veblen's "Theory of Leisure Class": higher income class, more leisure.
- Here: no negative connotation of higher income, more leisure.
- Only "normative" suggestion:
  - education important in equalizing income across classes,
  - as it comes with greater marginal productivity.

Multiple Heterogenous Agents. Example 4.4: More B than A

$$c^d = 400c_A^d + 600c_B^d = 400c_A^s + 600c_B^s = c^s; \quad (15)$$
$$l^d = 400l_A^d + 600l_B^d = 400l_A^s + 600l_B^s = l^s; \quad (16)$$

$$400\left(\frac{1}{4w^2}\right) + 600\left(\frac{1}{w^2}\right) = 400\left(12 - \frac{1}{8w^2}\right) + 600\left(12 - \frac{1}{2w^2}\right),$$
$$w = \sqrt{\frac{8400}{1000\,(12)\,8}} = 0.2958.$$

Quantities Solution with New Wage

Table 4.2 Equilibrium with Free Trade, Multiple Agents

$w = 0.296$	Agent A	Agent B
Labor Demand	$l_A^d = \frac{1}{4w^2} = 2.86$	$l_B^d = \frac{1}{w^2} = 11.43,$
Labor Supply	$l_A^s = 12 - \frac{1}{8w^2} = 10.57$	$l_B^s = 12 - \frac{1}{2w^2} = 6.3$
Profit	$\Pi_A = \frac{1}{4w} = 0.85$	$\Pi_B = \frac{1}{w} = 3.4,$
Goods Supply	$c_A^s = \sqrt{l_A^s} = 1.69$	$c_B^s = 2\sqrt{l_B^s} = 6.8,$
Goods Demand	$c_A^d = \Pi_A + wl_A^s = 3.97$	$c_B^d = \Pi_B + wl_B^s = 5.24.$
Utility Level	$\ln(3.97) + \ln(13.43) = 3.98$	$\ln(5.24) + \ln(17.7) = 4.5$

Trade Flows

$$\textit{Excess Demand}_A \quad : \quad c_A^d - c_A^s = 3.97 - 1.69 = 2.28 \tag{17}$$

$$\textit{Excess Supply}_B \quad : \quad c_B^s - c_B^d = 6.77 - 5.25 = 1.52. \tag{18}$$

$$\begin{aligned} 400\left(c_A^d - c_A^s\right) &= 400\,(2.2819) = 912.8; \\ 600\left(c_B^s - c_B^d\right) &= 600\,(1.521\,4) = 912.8. \end{aligned}$$

$$\textit{Excess Supply}_A \quad : \quad I_A^s - I_A^d = 10.57 - 2.86 = 7.71, \tag{19}$$

$$\textit{Excess Demand}_B \quad : \quad I_B^d - I_B^s = 11.43 - 6.29 = 5.14. \tag{20}$$

$$\begin{aligned} 600\left(I_B^d - I_B^s\right) &= 600\,(5.1433) = 3086; \\ 400\left(I_A^s - I_A^d\right) &= 400\,(7.714\,2) = 3086. \end{aligned}$$

$$c_A^d = \frac{e^{3.98}}{24 - l_A^s} = \frac{e^{3.98}}{x_A}; \tag{21}$$

$$c_B^d = \frac{e^{4.53}}{24 - l_B^s} = \frac{e^{4.53}}{x_B}. \tag{22}$$

$$\begin{aligned} c_A^d &= w l_A^s + \Pi_A, \\ \Pi_A &= \frac{1}{4w} = \frac{1}{4\left(0.2958\right)} = 0.84517, \\ c_A^d &= w l_A^s + 0.84517 = \left(0.2958\right)\left(24 - x_A\right) + 0.84517. \end{aligned} \tag{23}$$

$$\begin{aligned} c_B^d &= w l_B^s + \Pi_B, \\ \Pi_B &= \frac{1}{w} = \frac{1}{\left(0.2958\right)} = 3.3807, \\ c_B^d &= w l_B^s + 3.3807 = \left(0.2958\right)\left(24 - x_B\right) + 3.3807. \end{aligned} \tag{24}$$

Trade Triangle Bigger For Each A Than for Each B

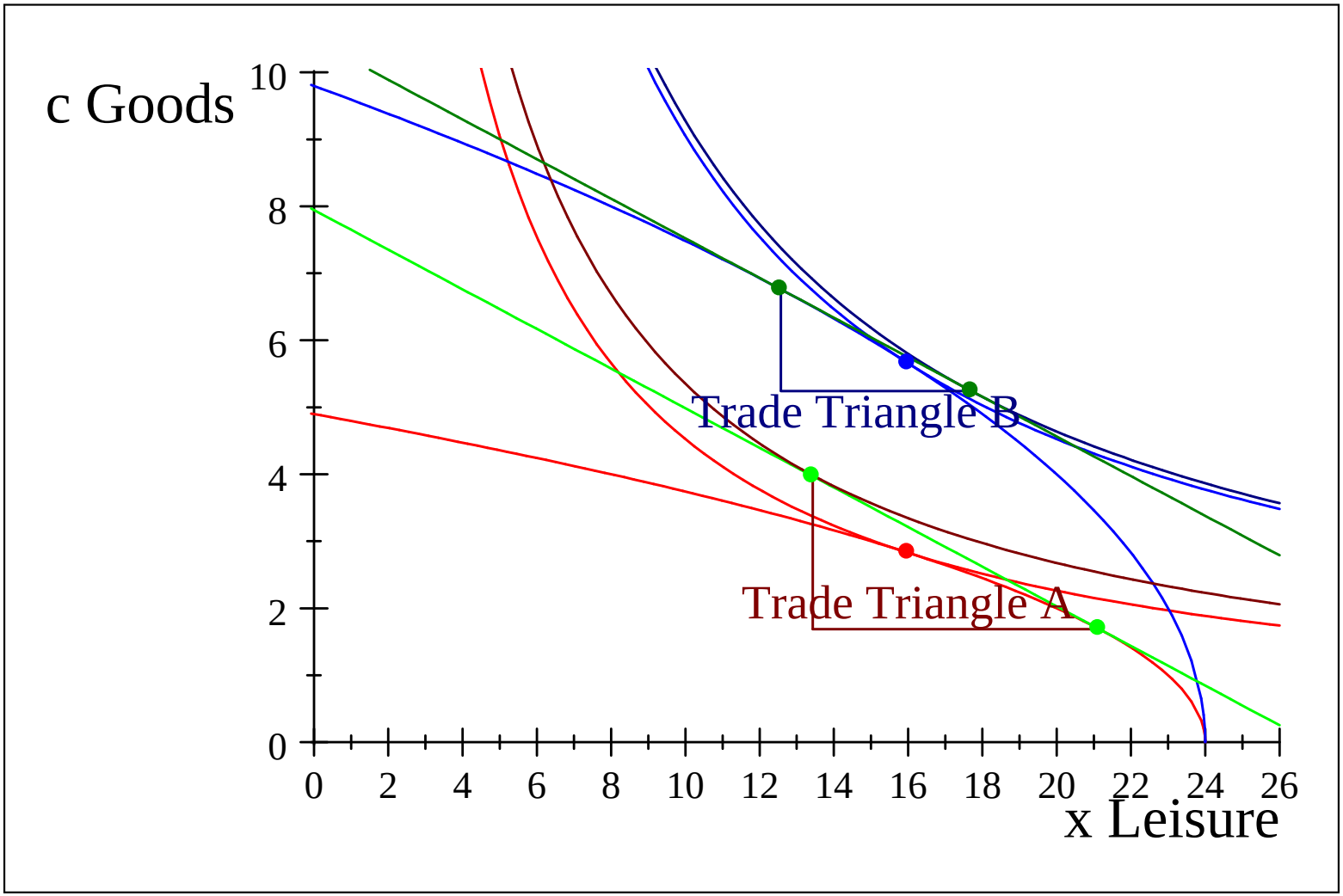

Figure 4.6. Multiple Agents Under Free Trade in Example 4.4.

Example 4.4: Education Increase

- Suppose that 100 of 400 A agents obtain education (at no cost)
- Now of 1000, 300 group A and 700 group B.

$$300\left(\frac{1}{4w^2}\right)+700\left(\frac{1}{2w^2}\right)=300\left(12-\frac{1}{8w^2}\right)+700\left(12-\frac{1}{2w^2}\right).$$

- $w=0.31$, higher than $w=0.296$ with the 400/600 mix.

$$c^d=300c_A^d+700c_B^d=4.12\,(300)+5.33\,(700)=4969$$

- Trade triangles shrinks for B and gets bigger for A.
- Utility level for each B decreases, consumption for B increases.
- Income gap between two groups narrows.
- B agents each worse off; more productive groups may not favor expanding education levels of less productive classes or countries.

Example 4.5: Immigration Restrictions

- 100 of A agents are precluded from entering country or deported.
- 300/600 mix in one country and 100 of agent A in other.

$$300\left(\frac{1}{4w^2}\right)+600\left(\frac{1}{w^2}\right)=300\left(12-\frac{1}{8w^2}\right)+600\left(12-\frac{1}{2w^2}\right)$$

- $w = 0.306$ for 300/600 mix, instead of $w = 0.296$.
- Consumption up by 2.8% and 1.5% for each A and B.
- May explain why unions support immigration restricts.
- World accounting: average $w = 900(0.3062) + 100(0.177) = 0.2932$, from $w = 0.2958$.
- Aggregate consumption: $100(2.83) + 300(4.08) + 600(5.31) = 4693$,
  - decline from 4732 in the integrated economy.
  - Such immigration restrictions reduce average:
  - wages, consumption, utility levels.
- Policy: country or domestic segretation of markets reduces utility.
  - example: "welfare" system & separate housing, shops, schools.

Taxes and Two Classes of Agents

- Example 4.6. Tax Both and Transfer to A

$$(1+\tau_c)\, c_A^d = \Pi_A + w l_A^s + G,\ (1+\tau_c)\, c_B^d = \Pi_B + w l_B^s,$$

$$400G = \tau_c 400 c_A^d + \tau_c 600 c_B^d.\ l_A^s = 12 - \frac{1}{8w^2} - \frac{G}{2w}.$$

$$\left(\frac{400}{4w^2}\right) + \left(\frac{600}{w^2}\right) = 400\left(12 - \frac{1}{8w^2} - \frac{G}{2w}\right) + 600\left(12 - \frac{1}{2w^2}\right),$$

$$G = \tau_c\left[\frac{1}{4w} + \frac{1.5}{w} + w\left(12 - \frac{1}{8w^2}\right) + (1.5)w\left(12 - \frac{1}{2w^2}\right)\right].$$

$$w = \sqrt{\left(\frac{7}{2.5}\right)\frac{(3+2\tau_c)}{96}}.$$

- Tax raises real wage; due to tax "wedge"; "after-tax" wage down
- Total consumption: $c^d = 400(4.27) + 600(4.73) = 4546$, from 4732.
- A agents gain 7.1% goods; B agents losing 10% goods.

As an Unemployment/Welfare System

- The income transfer here subsidizes unemployment,
- Less permanent transfer with more "insurance" features:
  - would subsidize unemployment less
  - act more to smooth income with less moral hazard
- Can create an underclass,
  - "depreciate" agents "human capital",
  - segregate classes of people within a society.
  - People on welfare work, but in underground, criminal, economy
  - while maintaining their benefits.
- Transfer can increase barriers, discriminate against those receiving it.
- Permanent, long term, transfer may have limited justification.

Applications

- USA, Canada, Mexico: North American Free Trade Agreement.
- European Union internal market, ASEAN internal market.
- UK's Magna Carta 1297; Australia's ASEAN FTA 2009; Globalization
- Magna Carta (Great Charter) of 1297: part of statutory law, England, Wales.
  - Clause 9: "The city of London and ... all other Cities, Boroughs, Towns, and the Barons of the Five Ports, and all other Ports, shall have all their liberties and free customs."
  - Clause 30:"All Merchants ... shall have their safe and sure Conduct to depart out of England, to come into England, to tarry in, and go through England, as well by Land as by Water, to buy and sell without any manner of evil Tolts, by the old and rightful Customs, except in Time of War."
  - Clause 1: freedom of the Church,
  - Clause 29: due process of law for individuals.
  - Establishment of internal market with free trade of goods, labor
  - Cited as basis for US Constitution.
- FTAs continue to be established: what is called "globalization".

Immigration Waves and Rural-Urban Migration

- East to West Germany: after 1989 Fall of the Wall
  - first large labor movement in Europe after demise of Soviet Union
  - Germany set free trade in labor, goods within internal market,
  - with years of subsidies aimed at rebuilding East Germany.
- Allowing Eastern European workers into UK, W. Europe, in 2004
  - new EU Accession Treaty
  - immigration from new EU countries.
- Historic immigration waves built UK, USA, Australia, New Zealand.
- China: modern mass migration from rural areas into cities.
  - education and productivity rises,
  - seek out higher wages in cities.
  - Part of common historical developments as countries opens up to trade.