Advanced Modern Macroeconomics

Dynamic Analysis and AS-AD

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Chapter 8:Dynamic Analysis and AS-AD

Chapter Summary

- Labor-leisure, 2-period intertemporal trade-offs combined
- Using standard dynamic model with infinite horizon.
- Yet in "recursive dynamic" form: with only 2 time periods.
  - Recursive model: given "state" variable, capital k_t.
  - Makes utility a function of the state variable
  - Given k_t, choose investment through choice of k_{t+1}.
- Similar margins as before, but now fully dynamic.
- Dynamic update of aggregate supply (AS), demand (AD)
  - AS and AD depend upon state variable k_t.
  - Need stationary state equilibrium k_t to get $AS - AD$.
  - First derive consumer demand as function of w_t, k_t.
  - Add investment to get AD; get AS from firm given w_t, k_t.
  - Solving capital stock k_t requires all equilibrium conditions:
  - most challenging part, a focus of Chapter 10.
- Comparative static shifts in $AS - AD$,
  - plus general equilibrium graphs of output, inputs.

Building on the Last Chapters

- Simultaneous combination of previous margins
- Part 2 analyzed labor-leisure margin: same here.
- Part 3 two-period intertemporal consumption margin
  - Now infinite time instead of 2 periods.
  - Initial capital $k_0 = 0$ assumed in Part 3,
  - now equilibrium k_t given at time t.
  - Depreciation rate $\delta_k \in (0, 1)$, not 1 as before.
- AD depends upon wage rate: Part 2; interest rate: Part 3,
  - on both rates here.
  - Assuming exogenous growth implies interest rate.
  - This allows $AS - AD$ to depend only on w_t, k_t.
  - Endogenous growth: Part 5 with modified $AS - AD$.

Learning Objective

- $AS - AD$ dynamic formulation, comparative statics.
- Must see role of capital stock as state variable.
- AD : Consumption demand, permanent income concept
- AS : as in static model way
- Changes in any exogenous parameter: new k_t, $AS - AD$.

Who Made It Happen

- Frank Ramsey 1928 did all dynamic analysis here.
- Ramsey attributed this to his mentor Keynes;
- Dynamic theory here more than 80 years old,
- and main foundation of modern macroeconomics.
- Ramsey did both centralized, decentralized problems
- Ramsey analysis: also gives economic growth rate.
  - as determined by interest rate, time preference.
  - Ramsey no-growth equilibrium seen in this chapter
- Recursive framework: Stokey, Lucas and Prescott (1989).
- Permanent income: Friedman (1957).

The Recursive Problem

- "Recursive utility $V(k_t)$" : equilibrium over two periods t, $t+1$;
- Given state k_t.

$$V(k_t) = \underset{c_t, x_t, k_{t+1}}{Max:} \; u(c_t, x_t) + \beta V(k_{t+1}). \qquad (1)$$

- "Trick": reducing infinite horizon problem 2 periods.

$$V(k_t) = u(c_t, x_t) + \beta V(k_{t+1}) = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s, x_s); \qquad (2)$$

Recursive Utility as Infinite Sequence

$$
\begin{aligned}
V(k_t) &= u(c_t, x_t) + \beta V(k_{t+1}); \\
V(k_{t+1}) &= u(c_{t+1}, x_{t+1}) + \beta V(k_{t+2}). \\
V(k_t) &= u(c_t, x_t) + \beta \left[u(c_{t+1}, x_{t+1}) + \beta V(k_{t+2}) \right]; \\
V(k_t) &= u(c_t, x_t) + \beta u(c_{t+1}, x_{t+1}) + \beta^2 V(k_{t+2}). \\
V(k_{t+2}) &= u(c_{t+2}, x_{t+2}) + \beta V(k_{t+3}). \\
V(k_t) &= u(c_t, x_t) + \beta u(c_{t+1}, x_{t+1}) + \beta^2 V(k_{t+2}); \\
V(k_t) &= u(c_t, x_t) + \beta u(c_{t+1}, x_{t+1}) + \beta^2 \left[u(c_{t+2}, x_{t+2}) + \beta V(k_{t+3}) \right]; \\
V(k_t) &= u(c_t, x_t) + \beta u(c_{t+1}, x_{t+1}) + \beta^2 u(c_{t+2}, x_{t+2}) + \beta^3 V(k_{t+3}).
\end{aligned}
$$

Plus "transversality condition" $\lim_{t \to \infty} \left[\beta^t V(k_t) \right] = 0.$

General Equilibrium Representative Agent Problem

$$u\left(c_t, x_t\right) = \ln c_t + \alpha \ln x_t. \tag{3}$$

$$y_t = A l_t^{\gamma} k_t^{1-\gamma}, \tag{4}$$

$$y_t = c_t + i_t. \tag{5}$$

$$k_{t+1} = i_t + \left(1 - \delta_k\right) k_t,$$

$$i_t = k_{t+1} - k_t \left(1 - \delta_k\right). \tag{6}$$

$$T = x_t + l_t. \tag{7}$$

$$V\left(k_t\right) = \underset{c_t, x_t, l_t, k_{t+1}}{Max} : u\left(c_t, x_t\right) + \beta V\left(k_{t+1}\right), \tag{8}$$

$$V\left(k_t\right) = \underset{l_t, k_{t+1}}{Max} : u\left(A l_t^{\gamma} k_t^{1-\gamma} - k_{t+1} + k_t\left(1 - \delta_k\right), T - l_t\right) + \beta V\left(k_{t+1}\right). \tag{9}$$

Equilibrium and Envelope Conditions

Equilibrium

$$0 = \frac{\partial u\left(c_t, x_t\right)}{\partial c_t}\left(\gamma A l_t^{\gamma-1} k_t^{1-\gamma}\right) + \frac{\partial u\left(c_t, x_t\right)}{\partial x_t}(-1);$$
$$0 = \frac{\partial u\left(c_t, x_t\right)}{\partial c_t}(-1) + \beta \frac{\partial V\left(k_{t+1}\right)}{\partial k_{t+1}}.$$

Envelope:

$$\frac{\partial V\left(k_t\right)}{\partial k_t} = \frac{\partial u\left(c_t, x_t\right)}{\partial c_t}\left[(1-\gamma) A l_t^{\gamma} k_t^{-\gamma} + \left(1-\delta_k\right)\right].$$

Combining to get standard "Euler Equation" (Intertemporal Margin):
First bring forward time index in Envelope,
then substitute in 2nd Equilibrium condition (for k_{t+1}) :

$$\frac{\partial V\left(k_{t+1}\right)}{\partial k_{t+1}} = \frac{\partial u\left(c_{t+1}, x_{t+1}\right)}{\partial c_{t+1}}\left[(1-\gamma) A l_{t+1}^{\gamma} k_{t+1}^{-\gamma} + \left(1-\delta_k\right)\right].$$
$$\frac{\partial u\left(c_t, x_t\right)}{\partial c_t} = \beta \frac{\partial u\left(c_{t+1}, x_{t+1}\right)}{\partial c_{t+1}}\left[(1-\gamma) A l_{t+1}^{\gamma} k_{t+1}^{-\gamma} + \left(1-\delta_k\right)\right].$$

Two Standard Margins: Intertemporal, Intratemporal

$$
\begin{aligned}
MRS_{c_t,c_{t+1}} &= \frac{\frac{\partial u(c_t,x_t)}{\partial c_t}}{\beta \frac{\partial u(c_{t+1},x_{t+1})}{\partial c_{t+1}}} = 1 + (1-\gamma)\, A l_{t+1}^{\gamma} k_{t+1}^{-\gamma} - \delta_k \\
&= 1 + MP_{k_{t+1}} - \delta_k .
\end{aligned}
$$

Log Utility:

$$
\frac{c_{t+1}}{c_t} = \frac{1 + \left[(1-\gamma)\, A_G\, l_{t+1}^{\gamma} k_{t+1}^{-\gamma} \right] - \delta_k}{1+\rho}.
$$

$$
MP_{l_t} = \gamma A_G\, l_t^{\gamma-1} k_t^{1-\gamma} = \frac{\frac{\partial u(c_t,x_t)}{\partial x_t}}{\frac{\partial u(c_t,x_t)}{\partial c_t}} = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t}} = MRS_{x,c}.
$$

Decentralized:

$$
\frac{c_{t+1}}{c_t} = \frac{1 + r_{t+1} - \delta_k}{1+\rho};\; w_t = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t}}
$$

Comparison to Part 3

- Part 3, Chapt. 5, 6, 7: y_0 given, $y_1 = Ak_1^{1-\gamma}$, $y_0 = c_0 + k_1$.
  - Savings/investment decision: k_1.
- With Part 4 capital accumlation, $k_0 = 0$ (or $\delta_k = 1$),
  - get that $i_0 = k_1 - k_0 (1 - \delta_k) = k_1$,
  - as in Part 3: $i_0 = k_1$ in Chapt 5-7.
- Part 3: no investment in period 1: $i_1 = 0$, $c_1 = y_1$;
  - in Part 4: positive investment in period $t + 1$, and on;
  - in Part 4: $y_t = c_t + i_t$ for all t,
  - with $i_t > 0$ if at least 0 growth or positive growth.

Decentralized Consumer Problems

- Savings s_t :

$$s_t = k^s_{t+1} - k^s_t \left(1 - \delta_k\right). \tag{10}$$

- Income: wages $w_t l^s_t$, rental income $r_t k^s_t$, profit Π_t.
- Consumption equals income minus savings:

$$c^d_t = w_t l^s_t + r_t k^s_t + \Pi_t - k^s_{t+1} + k^s_t \left(1 - \delta_k\right).$$

$$V\left(k^s_t\right) = \underset{c^d_t, x_t, l^s_t, k^s_{t+1}}{Max} : u\left(c^d_t, x_t\right) + \beta V\left(k^s_{t+1}\right),$$

$$V\left(k^s_t\right) = \underset{l^s_t, k^s_{t+1}}{Max} : u\left[w_t l^s_t + r_t k^s_t + \Pi_t - k^s_{t+1} + k^s_t \left(1 - \delta_k\right), T - l_t\right] + \beta V\left(k^s_{t+1}\right).$$

Consumer Equilibrium

$$
\begin{aligned}
0 &= \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t} w_t - \frac{\partial u\left(c_t^d, x_t\right)}{\partial x_t}; \\
0 &= \frac{-\partial u\left(c_t^d, x_t\right)}{\partial c_t^d} + \beta \frac{\partial V\left(k_{t+1}^s\right)}{\partial k_{t+1}^s}. \\
Envelope &: \frac{\partial V\left(k_t^s\right)}{\partial k_t^s} = \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t^d}\left(1 + r_t - \delta_k\right). \\
&\Rightarrow \frac{c_{t+1}^d}{c_t^d} = \frac{1 + r_{t+1} - \delta_k}{1 + \rho}, \; w_t = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t^d}}.
\end{aligned}
$$

Goods Producer

$$\begin{aligned}
TECHNOLOGY &: y_t = A_G \left(l_t^d\right)^{\gamma} \left(k_t^d\right)^{1-\gamma}, \\
\underset{y_t, l_t^d, k_t^d}{Max}\ \Pi_t &= y_t - w_t l_t^d - r_t k_t^d; \\
\underset{l_t^d, k_t^d}{Max}\ \Pi_t &= A_G \left(l_t^d\right)^{\gamma} \left(k_t^d\right)^{1-\gamma} - w_t l_t^d - r_t k_t^d : \\
w_t &= \gamma A_G \left(l_t^d\right)^{\gamma-1} \left(k_t^d\right)^{1-\gamma}, \\
r_t &= (1-\gamma) A_G \left(l_t^d\right)^{\gamma} \left(k_t^d\right)^{-\gamma}.
\end{aligned}$$

Note Zero Profit: $w_t l_t^d = \gamma y_t$, and $r_t k_t^d = (1-\gamma) y_t \Longrightarrow$

$$\Pi_t = y_t - w_t l_t^d - r_t k_t^d = y_t - \gamma y_t - (1-\gamma) y_t = 0.$$

Firm, Consumer: Same Budget/Profit Line

Firm:

$$\begin{aligned} y_t &= c_t^s + i_t^d. \\ i_t &= k_{t+1}^d - k_t^d\left(1-\delta_k\right), \\ y_t &= c_t^s + k_{t+1}^d - k_t^d\left(1-\delta_k\right). \end{aligned}$$

$$\begin{aligned} profit &: \quad y_t - w_t l_t^d - r_t k_t^d = \Pi_t, \\ w_t l_t^d + r_t k_t^d + \Pi_t &= c_t^s + k_{t+1}^d - k_t^d\left(1-\delta_k\right), \\ c_t^s &= w_t l_t^d + r_t k_t^d + \Pi_t - k_{t+1}^d + k_t^d\left(1-\delta_k\right). \end{aligned}$$

Consumer:

$$c_t^d = w_t l_t^s + r_t k_t^s + \Pi_t - k_{t+1}^s + k_t^s\left(1-\delta_k\right).$$

Given Market Clearing:

$$k_t^s = k_t^d, \; l_t^s = l_t^d.$$

Firm, Consumer: Market/Budget/Profit Line is Same

Aggregate Demand: AD

- Aggregate demand:
  - Sum of demand for goods for by consumer for consumption,
  - and demand for goods by firm as capital inputs in production.
- Consumer Demand:

$$x_t = \frac{\alpha c_t^d}{w_t}, \; x_t = T - l_t, \Longrightarrow l_t^s = T - \frac{\alpha c_t^d}{w_t}.$$

$$\begin{aligned}
c_t^d &= w_t\left(T - \frac{\alpha c_t^d}{w_t}\right) + r_t k_t + \Pi_t - k_{t+1} + k_t\left(1 - \delta_k\right). \\
c_t^d &= \frac{w_t T + r_t k_t + \Pi_t - k_{t+1} + k_t\left(1 - \delta_k\right)}{1+\alpha}, \\
c_t^d &= \frac{w_t T + \Pi_t + k_t\left(1 + r_t - \delta_k - \frac{k_{t+1}}{k_t}\right)}{1+\alpha}.
\end{aligned}$$

Zero Growth Implications for Consumer Demand

$$\frac{k_{t+1}}{k_t} = 1;\ k_{t+1} = k_t = k_{t-1} = ...$$

$$\Longrightarrow \quad c_t^d = \tag{11}$$

$$\frac{w_t T + \Pi_t + k_t \left(1 + r_t - \delta_k - \frac{k_{t+1}}{k_t}\right)}{1+\alpha} = \frac{\left[w_t T + \Pi_t + k_t \left(r_t - \delta_k\right)\right]}{1+\alpha} \tag{12}$$

$$\frac{c_{t+1}^d}{c_t^d} = 1;\ c_{t+1} = c_t = c_{t-1} = ...$$

$$\frac{c_{t+1}^d}{c_t^d} = \frac{1 + r_t - \delta_k}{1+\rho};$$

$$\Longrightarrow \quad r_t - \delta_k = \rho;$$

$$\Longrightarrow c_t^d = \frac{w_t T + \Pi_t + k_t \rho}{1+\alpha};\ \Pi_t = 0 \Longrightarrow c_t^d = \frac{w_t T + k_t \rho}{1+\alpha}. \tag{13}$$

Consumption, Permanent Income, Wealth

- Consumption a fraction of Permanentn Income:

$$c_t^d = \frac{w_t T + \rho k_t}{1+\alpha}, \tag{14}$$

$$y_{pt} \equiv w_t T + \rho k_t, \tag{15}$$

$$c_t^d = \left(\frac{1}{1+\alpha} \right) y_{pt}, \tag{16}$$

- Permanent income: Interest flow on Human and Physical Capital Wealth

$$W_t = \frac{y_{pt}}{\rho} = \frac{w_t T + \rho k_t}{\rho} = \frac{w_t T + \rho k_t}{\rho} = \frac{w_t T}{\rho} + k_t, \tag{17}$$

- Consumption a fraction of Wealth:

$$c_t^d = \left(\frac{1}{1+\alpha} \right) y_{pt} = \left(\frac{\rho}{1+\alpha} \right) W_t$$

Adding Capital Maintenance to Derive AD

$$
\begin{aligned}
i_t &= k_{t+1}^d - k_t^d\left(1-\delta_k\right), \\
k_{t+1}^d &= k_t^d, \\
i_t &= k_t^d - k_t^d\left(1-\delta_k\right) = \delta_k k_t^d. \\
k_t^s &= k_t^d = k_t,
\end{aligned}
$$

$$
AD \quad : \quad y_t^d = c_t^d + i_t = \left(\frac{1}{1+\alpha}\left[w_t T + \rho k_t\right]\right) + \delta_k k_t, \qquad (18)
$$

$$
y_t^d = \frac{w_t T + k_t\left[\rho + (1+\alpha)\,\delta_k\right]}{1+\alpha}. \qquad (19)
$$

$$
Inversely: \frac{1}{w_t} = \frac{T}{y_t^d\left(1+\alpha\right) - k_t\left[\rho + \left(1+\alpha\right)\delta_k\right]}. \qquad (20)
$$

Aggregate Supply: AS

- Substitute firm labor demand into production function:

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t.$$

$$y_t^s = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma}.$$

- AS as function of w_t, k_t :

$$AS : y_t^s = A_G (k_t)^{1-\gamma} \left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} (k_t)^{\gamma} = A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t.$$

$$Inversely : \frac{1}{w_t} = \frac{\left(y_t^s\right)^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{\gamma}} (k_t)^{\frac{1-\gamma}{\gamma}}}.$$

Marginal Cost of Output

$$
\begin{aligned}
Total\ Cost &\equiv TC_t = w_t l_t + r_t k_t; \\
l_t^d &= \left(\frac{y_t}{A_G}\right)^{\frac{1}{\gamma}} (k_t)^{\frac{\gamma-1}{\gamma}}; \\
TC_t &= w_t \left[\left(\frac{y_t}{A_G}\right)^{\frac{1}{\gamma}} (k_t)^{\frac{\gamma-1}{\gamma}}\right] + r_t k_t.
\end{aligned}
$$

$$
\begin{aligned}
MC_t &= \frac{\partial (TC_t)}{\partial y_t} = \frac{\partial \left[w_t \left(\frac{y_t}{A_G}\right)^{\frac{1}{\gamma}} (k_t)^{\frac{\gamma-1}{\gamma}} + (\rho + \delta_k) k_t\right]}{\partial y_t}, \\
MC_t &= \frac{w_t y_t^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{\gamma}} (k_t)^{\frac{1-\gamma}{\gamma}}},
\end{aligned}
$$

Goods Price Normalized to One

- Let P_t be price of output.
- Normalize this to one: $P_t = 1$.
- Microeconomic result that $P_t = MC_t$, implies $MC_t = 1$.
- The relative price of output is then $\frac{1}{w_t}$,
- giving the AS curve for graphing in terms of MC_t :

$$AS \quad : \quad \frac{MC_t}{w_t} \equiv \frac{1}{w_t} = \check{A} y_t^{\frac{1-\gamma}{\gamma}};$$

$$\check{A} \equiv \frac{1}{\gamma A_G^{\frac{1}{\gamma}} \left(k_t\right)^{\frac{1-\gamma}{\gamma}}}$$

- Curvature of AS graph in $\left(\frac{1}{w_t} : y_t\right)$ space depends on $\frac{1-\gamma}{\gamma}$.

Baseline Calibration: Example 8.1

- $\gamma = \frac{1}{3}$, $\alpha = 0.5$, $T = 1$, $\rho = 0.03$, $A_G = 0.15$, $\delta_k = 0.03$.
- $r = \rho + \delta_k \implies r = 0.03 + 0.03 = 0.06$.
- Equilibrium k_t value: $k_t = 2.3148$.
- $\gamma = \frac{1}{3}$ gives convex AS supply curve;
  - Convexity implies increasing marginal cost: realistic.
  - used in Mankiw, Romer and Weil (1992).
  - $\gamma = \frac{2}{3}$ gives concave AS supply curve.
-

$$AD \quad : \quad \frac{1}{w_t} = \frac{1}{y_t^d\left(1+0.5\right) - 2.3148\left[0.03 + \left(1.5\right)0.03\right]}, \tag{21}$$

$$AS \quad : \quad \frac{1}{w_t} = \frac{\left(y_t^s\right)^2}{\frac{1}{3}\left(0.15\right)^3\left(2.3148\right)^2}, \tag{22}$$

AD

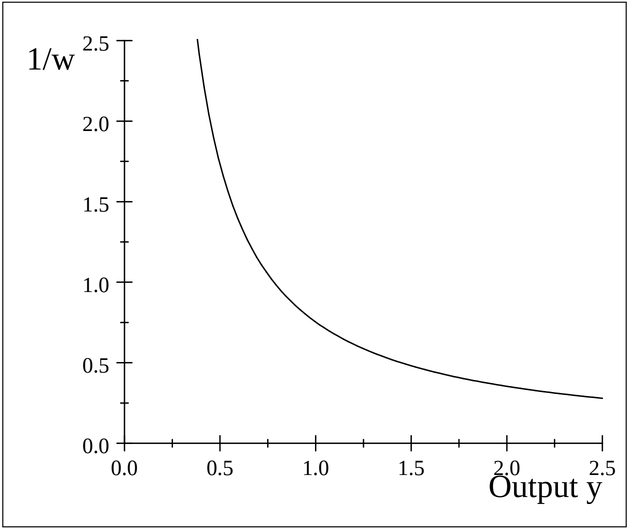


Figure 8.1. Baseline Dynamic Aggregate Output Demand *AD*

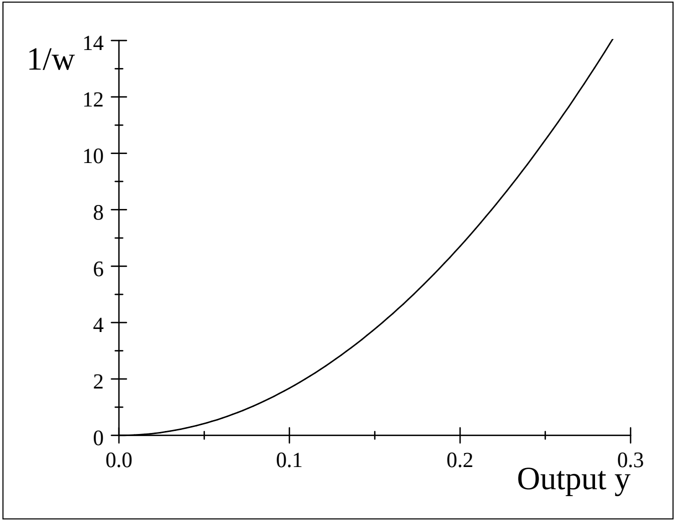


Figure 8.3. Baseline Dynamic Aggregate Output Supply *AS*

C+I=Y

Green: c^d; Black: $c^d + i$:

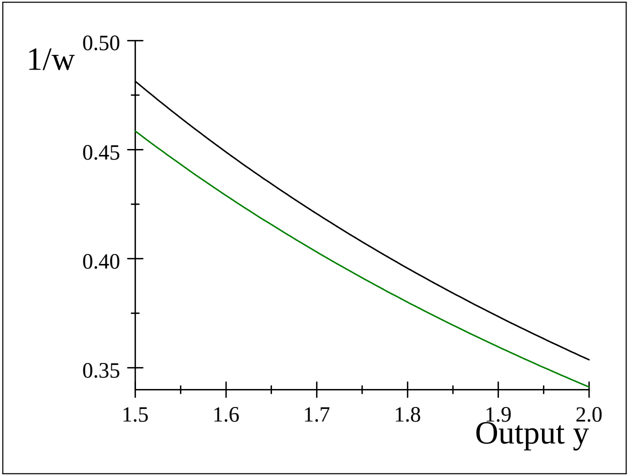


Figure 8.2 Aggregate Output Demand *AD* (black) and Consumption Demand (green)

Graphical AS-AD Equilibrium

$\frac{1}{w} = 7.2$; $w = \frac{1}{7.2} = 0.13889$.

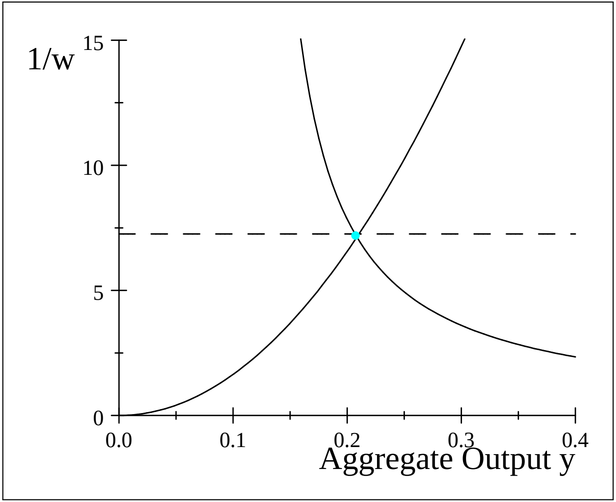


Figure 8.4. Baseline Dynamic $AS - AD$ Equilibrium

Solving the Equilibrium Wage Rate

$$y_t^d = \frac{w_t T + k_t \left[\rho + (1+\alpha)\,\delta_k\right]}{1+\alpha} = A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t = y_t^s;$$

$$y_t^d - y_t^s = \frac{w_t T + k_t \left[\rho + (1+\alpha)\,\delta_k\right]}{1+\alpha} - A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t.$$

$$\textit{Excess Demand} \; : \; Y(w_t; k_t) \equiv y_t^d - y_t^s = 0,$$

$$Y(w_t; k_t) = \frac{w_t T + k_t \left[\rho + (1+\alpha)\,\delta_k\right]}{1+\alpha} - A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t.$$

$$\textit{Given} \; : \; k_t = 2.3148 \Longrightarrow$$

$$0 = \frac{w_t + (2.3148)\left[0.03 + (1.5)\,0.03\right]}{1.5} - (0.15)^{1.5} \left(\frac{\frac{1}{3}}{w_t}\right)^{\frac{1}{2}} (2.3148).$$

$$\Longrightarrow w_t = 0.13889$$

Excess Demand Y(w) Equals Zero at Equilibrium Wage

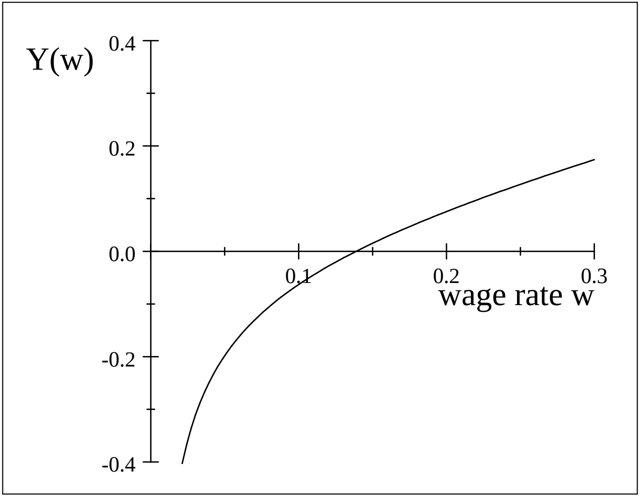


Figure 8.5. Excess Output Demand $Y(w_t)$ as a Function of the Wage Rate

Graphically Focus In on Exact Wage at Y(w)=0

$w = 0.13889$:

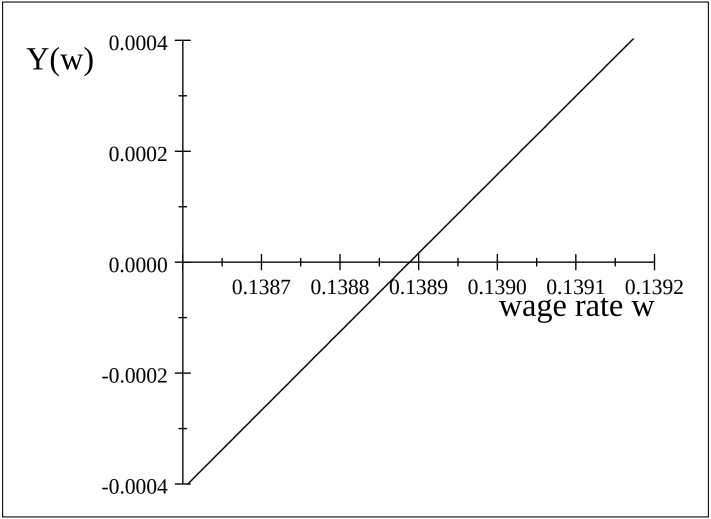


Figure 8.6. Excess Output Demand $Y(w_t)$ as a Function of the Wage Rate

Around Equilibrium Wage

- If $w < 0.13899$, relative price $\frac{1}{w_t}$ higher than equilibrium $\frac{1}{0.1389}$.
  - at high price, less demand for output than supply.
  - Excess demand is negative (positive excess supply).
- If $w > 0.13889$, relative price $\frac{1}{w}$ too low,
  - at low price, more demand for output than supply.
  - Excess demand is positive
- $Y(w_t) = 0$ at equilibrium w
  - aggregate supply of y equals aggregate demand for y.

Consumption to Output Ratio

- $$\begin{aligned} y_t^s &= A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t = (0.15)^{1.5} \left(\frac{1}{3\,(0.13889)}\right)^{0.5} 2.3148 \\ &= 0.20832\,. \end{aligned}$$

$$\begin{aligned} c_t^d &= \frac{1}{1+\alpha}\left(w_t + \rho k_t\right) = \frac{1}{1.5}\left(0.13889 + 0.03\,(2.3148)\right) \\ &= 0.13889. \end{aligned}$$

- Note, in this Example:

$$c_t^d = w_t = 0.13889.$$

$$\frac{c^d}{y^s} = \frac{0.13889}{0.20832} = \frac{2}{3} = \frac{\rho + \delta_k \gamma}{\rho + \delta_k} = \frac{0.03 + (0.03)\,\frac{1}{3}}{0.03 + 0.03}.$$

- Two-thirds of output consumed and one third invested.
- Savings rate: $1 - \frac{c^d}{y^s}$ is $1/3$.

General Equilibrium: Input Market

Isocost line (red):

$$\begin{aligned} \textit{Zero profit} &\implies y_t = w_t l_t + r_t k_t. \\ 0.20832 &= (0.13889)\, l_t + (0.06)\, k_t; \\ k_t &= \frac{0.20832}{0.06} - \frac{(0.13889)\, l_t}{0.06}; \\ k_t &= 3.472 - 2.3148 l_t. \end{aligned}$$

Isoquant curve (blue): constant level of output, different inputs

$$\begin{aligned} 0.20832 &= y_t^s = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} = 0.15 \left(l_t^d\right)^{\frac{1}{3}} (k_t)^{\frac{2}{3}}; \\ k_t &= \left(\frac{0.20832}{0.15 \left(l_t^d\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}} = \frac{1.64}{\left(l_t^d\right)^{\frac{1}{2}}} \end{aligned}$$

Factor input ratio (green):

$$k_t / l_t = (2.3148) \, / \, (0.50) = 4.63. \tag{23}$$

Input Market Equilibrium

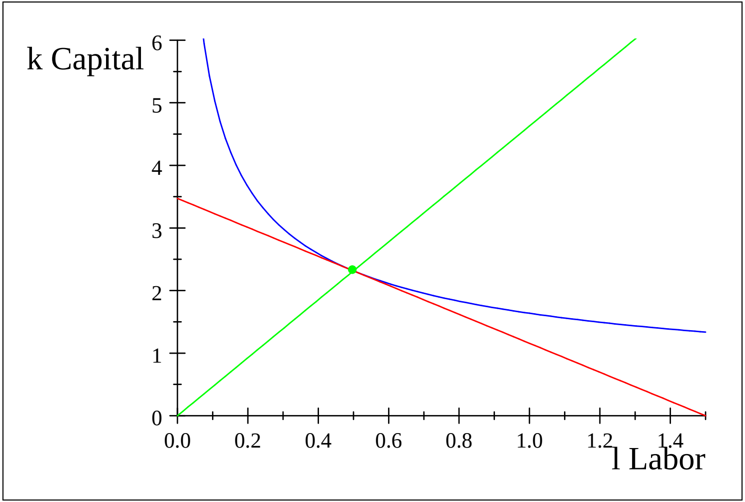


Figure 8.7. Factor Market Equilibrium in Example 8.1.

Production, Utility Level, Budget Line

$$c_t^d = y_t^s - i_t = A_G \left(l_t^d \right)^{\gamma} (k_t)^{1-\gamma} - \delta_k k_t = 0.262 \left(l_t^d \right)^{\frac{1}{3}} - 0.069;$$

$$\begin{aligned} u &= \ln c_t + \alpha \ln (1 - l_t) = \ln 0.13889 + 0.5 \ln 0.5 = -2.32, \\ c_t &= \frac{e^{-2.32}}{(1 - l_t)^{0.5}}. \end{aligned} \tag{24}$$

$$c_t^d = w_t l_t^s + \rho k_t^s = (0.13889)\, l_t^s + (0.03)\,(2.3148)\,.$$

Tangency Between Production, Utility, Budget Lines

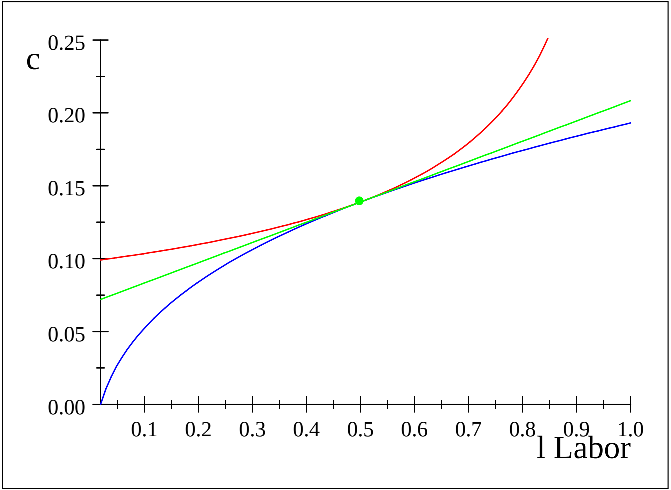


Figure 8.8. General Equilibrium Consumption and Utility Levels in Example 8.1.

Productivity Increase: Example 8.2

- A_G : 5% increase, 0.15 to 0.1575.
- Affects AS directly; $AS - AD$ through new k_t.
- $\gamma = \frac{1}{3}$, $\rho = 0.03$, $\delta_k = 0.03$, $T = 1$, $\alpha = 0.5$.
- k_t increases from 2.3148 to 2.6797, or 15.75%.

$$AD : \frac{1}{w_t} = \frac{1}{y_t\,(1.5) - 2.6797\,(0.03 + (1.5)\,0.03)}. \tag{25}$$

$$AS : \frac{1}{w_t} = \frac{\left(y_t^s\right)^2}{\left(\frac{1}{3}\right)(0.1575)^3\,(2.6797)^2}. \tag{26}$$

AD-AD Shift Back, Relative Price Rises

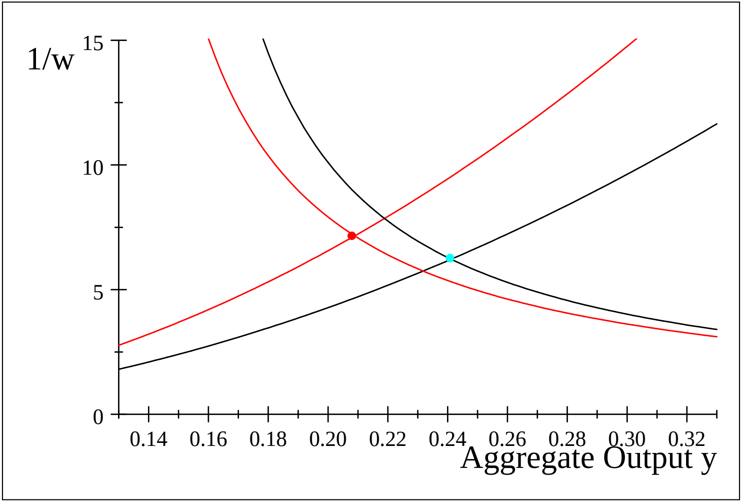


Figure 8.9. $AS - AD$ Equilibrium with Goods Productivity Increase (in black) in Example 8.2, compared to the Baseline (in red) of Example 8.1.

Equilibrium Wage

$$Y(w_t) = y_t^d - y_t^s = \frac{w_t + (2.6797)\left[0.03 + (1.5)\,0.03\right]}{1.5} - (0.1575)^{1.5}\left(\frac{\frac{1}{3}}{w_t}\right)^{\frac{1}{2}}(2.6797).$$

$$\Longrightarrow \quad w_t = 0.16078.$$

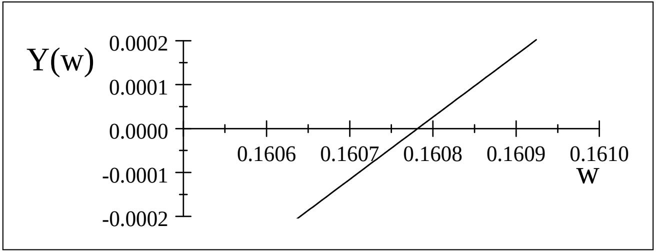


Figure 8.10. Excess Output Demand $Y(w_t)$ with a Goods Productivity Increase in Example 8.2.

Consumption and Output

$$c_t^d = \frac{1}{1.5}\left(0.16078 + 0.03\left(2.6797\right)\right) = 0.16078.$$

$$y_t^s = \left(0.1575\right)^{1.5}\left(\frac{1}{3\left(0.16078\right)}\right)^{0.5} 2.6797 = 0.24117\,.$$

$$\frac{c^d}{y^s} = \frac{0.16078}{0.24117} = 0.667.$$

Same consumption and savings rate.

Input Market with Productivity Increase

Isocost line (dark red):

$$\begin{aligned} y_t &= w_t l_t + r_t k_t, \\ 0.24117 &= (0.16078)\, l_t + (0.06)\, k_t, \\ k_t &= 4.0195 - (2.6797)\, l_t. \end{aligned} \tag{27}$$

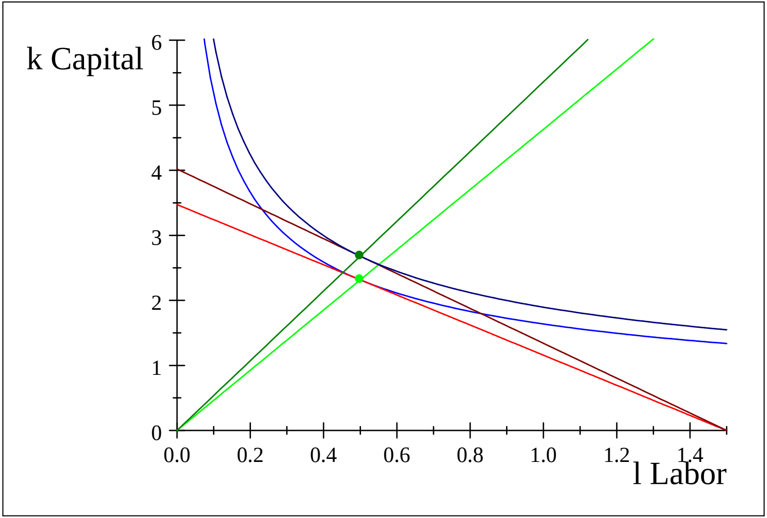
Isoquant curve (navy blue): constant level of output, different inputs

$$\begin{aligned} 0.24117 &= y_t^s = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} = 0.1575 \left(l_t^d\right)^{\frac{1}{3}\gamma} (k_t)^{\frac{2}{3}}; \\ k_t &= \left(\frac{(0.24117)}{0.1575 \left(l_t^d\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}} = k_t = \frac{1.8948}{\left(l_t^d\right)^{\frac{1}{2}}}. \end{aligned} \tag{28}$$

Factor input ratio (dark green):

$$\frac{k_t}{l_t} = \frac{2.6797}{0.50} = 5.3594. \tag{29}$$

Isoquant Shift Up, Isocost Pivots Up, Input Ratio Up



Example 8.11. Factor Market Equilibrium with Goods Productivity Increase of Example 8.2.

Production, Utility Level, Budget Line

$$c_t^d = y_t^s - i_t = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} - \delta_k k_t = 0.3 \left(l_t^d\right)^{\frac{1}{3}} - 0.08,$$

$$\begin{aligned} u &= \ln c_t + \alpha \ln (1 - l_t) \\ &= \ln 0.16078 + 0.5 \ln 0.5 = -2.1743, \qquad (30) \\ c_t &= \frac{e^{-2.17}}{(1 - l_t)^{0.5}}. \qquad (31) \end{aligned}$$

$$c_t^d = w_t l_t^s + \rho k_t^s = (0.161)\, l_t^s + (0.03)\,(2.68)\,. \qquad (32)$$

Tangency Moves Upwards, Same Labor

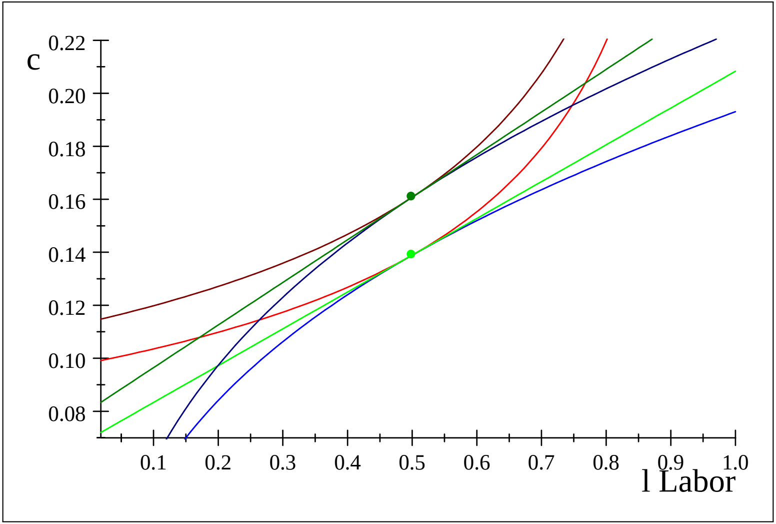


Figure 8.12. General Equilibrium with Goods Productivity Increase of Example 8.2.

Time Endowment Increase: Example 8.3

$T = 1 \Longrightarrow T = 1.05$. k_t : rises to 2.4306.

$$1.05 = x_t + l_t, \; l_t^s = 1.05 - \frac{\alpha c_t^d}{w_t}.$$

$$c_t^d = w_t \left(1.05 - \frac{\alpha c_t^d}{w_t} \right) + r_t k_t - k_{t+1} + k_t (1 - \delta_k);$$

$$c_t^d = \frac{1.05 w_t + r_t k_t - k_{t+1} + k_t (1 - \delta_k)}{1 + \alpha} = \frac{1}{1 + \alpha} (1.05 w_t + \rho k_t).$$

$$y_t^d = \frac{1}{1 + \alpha} (1.05 w_t + k_t [\rho + (1 + \alpha) \delta_k]).$$

$$AD : \frac{1}{w_t} = \frac{1.05}{y_t^d (1.5) - 2.43 (0.03 + (1.5) 0.03)}.$$

$$AS : \frac{1}{w_t} = \frac{(y_t^s)^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{\gamma}} (k_t)^{\frac{1-\gamma}{\gamma}}} = \frac{3 (y_t^s)^2}{(0.15)^3 (2.43)^2}.$$

AS-AD Shift Out, Wage the Same

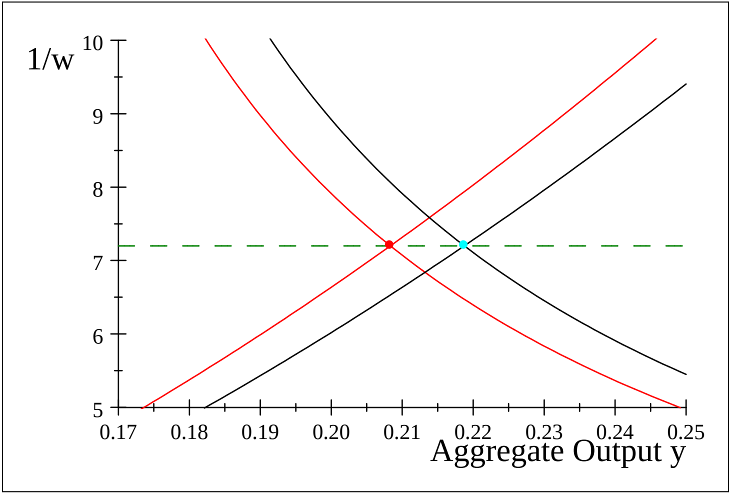


Figure 8.13. Shift in $AS - AD$ with Time Endowment Increase

$$
\begin{aligned}
Y\left(w_t\right) &= \frac{1.05w_t + (2.43)\left(0.03 + (1.5)\,0.03\right)}{1.5} - (0.15)^{1.5}\left(\frac{\frac{1}{3}}{w_t}\right)^{\frac{1}{2}} 2.43 \\
\implies & \quad w = 0.13889.
\end{aligned}
$$

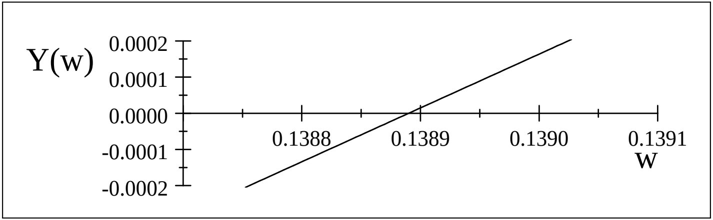


Figure 8.14. Excess Output Demand $Y\left(w_t\right)$ with a Time Endowment Increase

Consumption and Output

$$c_t^d = \frac{1}{1.5}\left(\left(1.05\right)0.1389 + 0.03\left(2.43\right)\right) = 0.1458,$$

$$y_t^d = \frac{1}{1.5}\left[1.05\left(0.139\right) + 2.43\left(0.03 + \left(1.5\right)0.03\right)\right] = 0.219.$$

$$\frac{c^d}{y^d} = \frac{0.14584}{0.21876} = \frac{2}{3}.$$

Again, the same consumption and savings rate.

Input Market with Time Endowment Increase

Budget Line:

$$
\begin{aligned}
0.218\,76 &= y_t = w_t l_t + r_t k_t = (0.13889)\, l_t + (0.06)\, k_t, \\
k_t &= 3.646 - (2.314\,8)\, l_t. \qquad (33)
\end{aligned}
$$

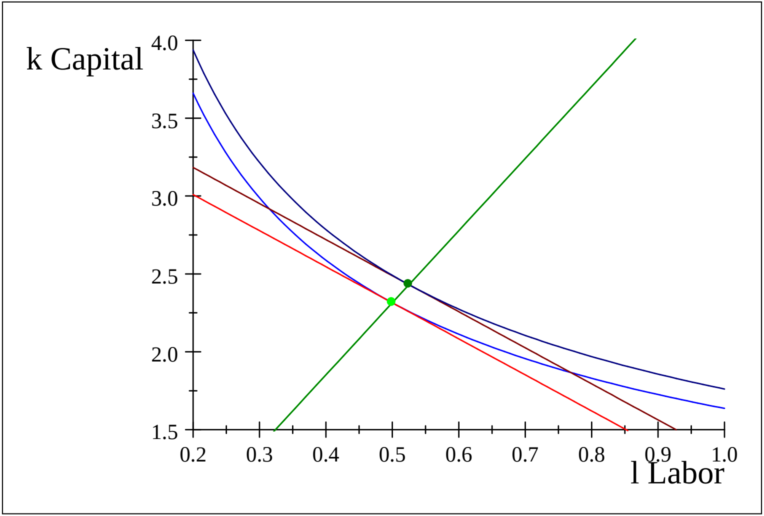
Isoquant curve:

$$
\begin{aligned}
0.218\,76 &= y_t^s = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} = 0.15 \left(l_t^d\right)^{\frac{1}{3}\gamma} (k_t)^{\frac{2}{3}}, \\
k_t &= \left(\frac{(0.218\,76)}{0.15\left(l_t^d\right)^{\frac{1}{3}}}\right)^{\frac{3}{2}} = \frac{1.7612}{\left(l_t^d\right)^{\frac{1}{2}}}. \qquad (34)
\end{aligned}
$$

Input ratio:

$$
\begin{aligned}
l_t^d &= \left(\frac{0.218\,76}{0.15\,(2.4306)^{\frac{2}{3}}}\right)^{3} = 0.52505, \\
\frac{k_t}{l_t} &= \frac{2.4306}{0.52505} = 4.6293.
\end{aligned}
$$

Employment Rises, Factor Ratio Unchanged



Example 8.15. Factor Market Equilibrium with Time Endowment Increase of Example 8.3.

New Production, Utility, Budget Line

$$
\begin{aligned}
c_t^d &= y_t^s - i_t = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} - \delta_k k_t, \\
c_t^d &= (0.15) \left(l_t^d\right)^{\frac{1}{3}} (2.4306)^{\frac{2}{3}} - (0.03)(2.4306), \\
c_t^d &= 0.271\,16 \left(l_t^d\right)^{\frac{1}{3}} - 0.072918; \qquad (35)
\end{aligned}
$$

$$
\begin{aligned}
-2.2475 &= u = \ln c_t + \alpha \ln (1.05 - l_t), \\
&= \ln 0.14584 + 0.5 \ln (1.05 - 0.52505), \\
c_t &= \frac{e^{-2.2475}}{(1.05 - l_t)^{0.5}}. \qquad (36)
\end{aligned}
$$

$$
c_t^d = w_t l_t^s + \rho k_t^s = (0.139)\, l_t^s + (0.03)(2.43). \qquad (37)
$$

Production Curve Pivots Up, Labor Rises

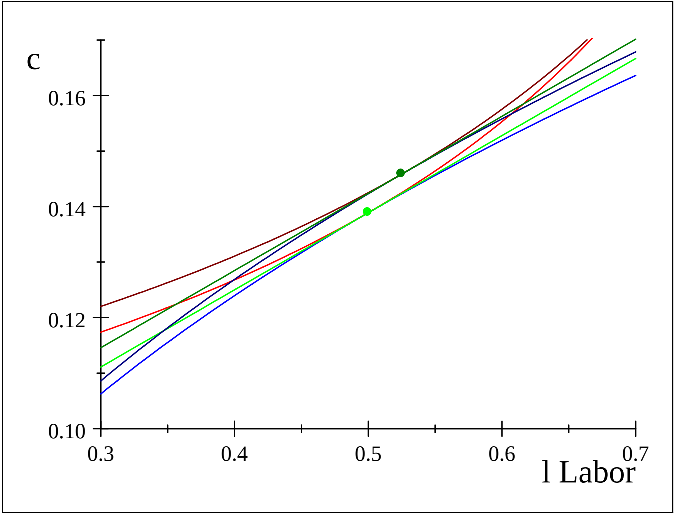


Figure 8.16. General Equilibrium with Time Endowment Increase of Example 8.3.

Application: AS, AD Shifts

- Comparative static changes: both AS and AD often shift.
  - A result of the role of capital stock, which changes.
  - Empirical identification as supply or demand shift hazardous.
- A_G change causes shifts both AS, AD.
  - Characterized as Supply shift
  - Yet consumer's permanent income rises.
  - AS shifts out by more than AD in example.
- Time endowment causes equal shift out in AS, AD
  - Supply side shift? or Demand side shift?
- Both changes at once: "net" AS shift,
  - but two very different factors causing result.
- Characterizing business cycle as supply shift
  - obscures AD changes,
  - obscures other factors, like time change.

Application: A View of Supply Side Economics

- Arnold Harberger 1998: "real cost reduction" of A_G growth.
- Economy reduces marginal cost, enables economic growth.
  - A_G increase causes marginal cost to shift down.
  - And AS curve shifts out.
  - Harberger: research, development, reduces cost.
- "Supply Side Economics" : focus on A_G as main force.
  - Can view as goods endowment increase, given technology.
- Say's Law: AS shifts out, "supply creates own demand"
  - marginal cost decrease causes relative price of output to fall
  - consumer moves along, down, AD.
  - But, with dynamic $AS - AD$, capital stock also shifts AD.

Application: Supply Side and Growth

- Harberger: prescription for long term growth
  - engender reductions in marginal cost.
  - Improving national institutions for market economy,
  - easing trade restrictions,
  - and minimizing taxes.
- Also traditional growth view:
  - time endowment focus instead of goods endowment.
  - Through infrastructure of education.
  - Increasing education also "supply-side economics".
  - Endogenous growth in Part 5, with human capital.