Advanced Modern Macroeconomics

Employment

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- Dynamic labor market: supply, demand, comparative static changes.
- Replicate business cycle change in employment.
  - Goods productivity factor: wage moves cyclically
  - but no change in employment.
  - Substitution, income effects offset again in dynamic model
- Combine goods productivity and time endowment changes
  - wage, employment move cyclically.
- Add fixed wage to decrease in goods, time endowment
  - replicate a more severe recession,
  - excess supply of labor, or unemployed labor.
  - With less employment than when flexible wage.
- Tax on labor income shifts back supply, demand for output, labor
  - less output and employment; wage rate the same.
  - Tax policy important for employment level in dynamic model.

Building on the Last Chapters

- Last chapter presented baseline dynamic model.
- This chapter derives baseline dynamic labor market,
- presents business cycle employment theory, and tax effect
- Same examples as in Chapter 8:
  - Examples 8.1, 8.2, 8.3 extended to Examples 9.1, 9.2, 9.3.
  - Business cycle combines Examples 9.2 and 9.3.
  - dynamic extension of static business cycle in Chapter 3;
  - also goods and time endowment changes combined.
  - Capital stock change affects dynamic employment explanation.
- Fixed wage, surplus labor, similar to Chapter 3
  - shows dynamic model gives similar fixed wage analysis.
- Dynamic model labor tax compares to static model.
  - Decreases capital, shifts back labor demand, unlike Chapter 3.
  - Supply, demand both shift back, leave wage unchanged;
  - wage rose in Chapter 3.

Learning Objective

- Dynamic equilibrium change in employment
  - contrasted to unemployment caused when wage cannot adjust.
- Business cycle employment change dynamically generated
  - with same simple comparative static changes
  - in goods and time endowments,
  - but not with change in either one alone.
- Tax increases also cause employment to decrease
  - must be used cautiously in any recessionary policy.
- All viewed within dynamic context, with change in capital stock,
  - and additional shifts in supply, demand
  - not present in static analysis of Chapter 3.

Who Made It Happen

- Pigou 1933 Theory of Unemployment writes (p. 252)
  - "The implication is that such unemployment as exists at any time is due wholly to the fact that changes in demand conditions are continually taking place and that frictional resistances prevent the appropriate wage adjustments from being made instantaneously."
- Frictional unemployment of labor today viewed within equilibrium,
  - not with fixed wage, but main alternative is sluggish wage adjustment.
- Recessional wage inflexibility: Keynes's General Theory (1936)
  - 1930s Great Depression, excess labor supply, fixed wage.
- Modern business cycles: Kydland, Prescott (1982), Long, Plosser.
  - Goods productivity factor causes business cycles,
  - called "real business cycle theory".
  - Labor employment does not move as much as in data
- Hansen (1985), Rogerson (1988), rectify with another labor margin
  - enter the labor force or to leave it; "external margin".
  - relates here to exogenous change in time endowment.

Aggregate Labor Supply and Demand

$$c_t^d = \frac{1}{1+\alpha}\left(w_t T + \rho k_t\right);\; x_t = \frac{\alpha c_t^d}{w_t};\; l_t^s = T - x_t.$$

$$l_t^s = T - \frac{\alpha c_t^d}{w_t} = T - \frac{\frac{\alpha}{1+\alpha}\left(w_t T + \rho k_t\right)}{w_t} = T - \frac{\alpha}{1+\alpha}\left[T + \left(\frac{\rho}{w_t}\right) k_t\right].$$

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t.$$

$$w_t = \frac{\alpha \rho k_t}{T - (1+\alpha)\, l_t^s};\; w_t = \gamma A_G \left(\frac{k_t}{l_t^d}\right)^{1-\gamma}. \tag{1}$$

Example 9.1: Labor Market Equilibrium

Calibration: Assumed Parameter Values:
$\gamma = \frac{1}{3}$, $\alpha = 0.5$, $\rho = 0.03$, $T = 1$. $A_G = 0.15$;
Equilibrium: $k_t = 2.3148$.

$$Labor\ Supply \quad : \quad w_t = \frac{0.5\,(0.03)\,2.3148}{1 - (1.5)\, l_t^s}, \tag{2}$$

$$Labor\ Demand \quad : \quad w_t = \frac{1}{3}\,(0.15)\left(\frac{2.3148}{l_t^d}\right)^{\frac{2}{3}}. \tag{3}$$

Baseline Labor Market in Example 9.1

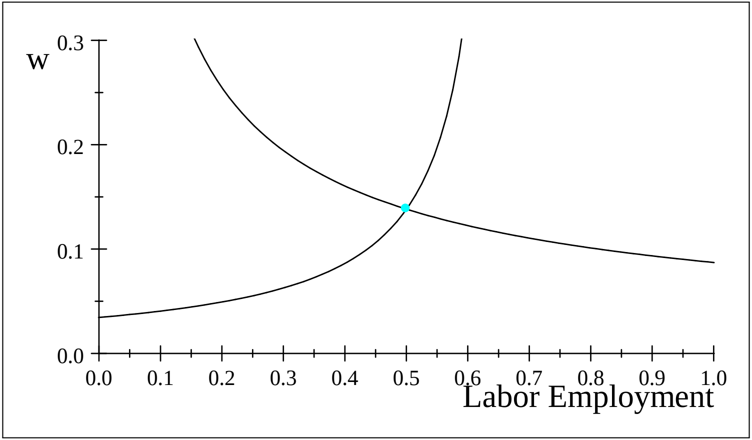


Figure 9.1. Labor Market in Baseline Dynamic Model of Example 9.1.

Excess Labor Demand is Zero at Equilibrium Wage

$$L(w_t) \equiv l_t^d - l_t^s;$$

$$L(w) = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t - \left(1 - \frac{\alpha}{1+\alpha}\left[1 + \left(\frac{\rho}{w_t}\right) k_t\right]\right) = 0;$$

$$L(w) = \left(\frac{0.15}{3w_t}\right)^{1.5} 2.3 - \left(1 - \frac{1}{3}\left[1 + \left(\frac{0.03}{w_t}\right) 2.3\right]\right) = 0.$$

$$w_t = 0.1389.$$

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t = \left(\frac{0.15}{3(0.1389)}\right)^{1.5} 2.3148 = 0.50,$$

$$l_t^s = 1 - \frac{\alpha}{1+\alpha}\left[1 + \left(\frac{\rho}{w_t}\right) k_t\right] = 1 - \frac{1}{3}\left(1 + \left(\frac{0.03}{0.1389}\right) 2.3\right) = 0.50.$$

- Excess demand $L(w) > 0$, $w_t < 0.1389$,
- Excess supply $L(w) < 0$, $w_t > 0.1389$.

L(w) Function for Example 9.1

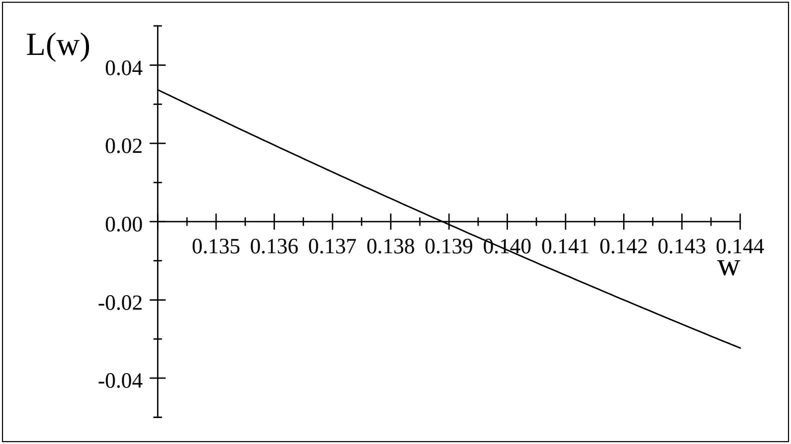


Figure 9.2. Implicit Solution of the Real Wage from $L(w) = 0$ in Example 9.1.

Productivity Increase: Example 9.2

- A_G rises 5%; $A_G = 0.15 \to A_G = (0.15)(1.05) = 0.1575$;
- $\gamma = \frac{1}{3}$, $\alpha = 0.5$, $\rho = 0.03$, $T = 1$; .k_t rises from 2.3148 to 2.6797.
- Labor demand affected directly and through capital increase;
- labor supply affected only through capital increase.

$$w_t = \frac{(0.03)\,2.6797}{1-(1.5)\,l_t^s}; \tag{4}$$

$$w_t = \frac{1}{3}(0.1575)\left(\frac{2.6797}{l_t^d}\right)^{\frac{2}{3}}. \tag{5}$$

Labor Market: Supply, Demand Shift Up, Wage Rises, Employment Same

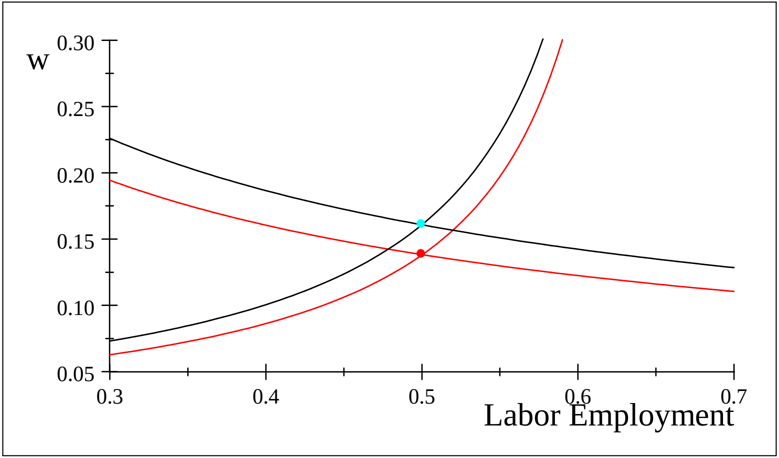


Figure 9.3. Increase in Productivity Raises *w* and Leaves Employment Unchanged in Example 9.2.

Productivity Increase: Excess Demand, Equilibrium Wage

$$
\begin{aligned}
L\left(w_t\right) &= \left(\frac{0.1575}{3w_t}\right)^{1.5} 2.6797 - \left(1 - \frac{0.5}{1.5}\left[1 + \left(\frac{0.03}{w_t}\right) 2.6797\right]\right) = 0; \\
w_t &= 0.16978; \\
l_t^d &= \left(\frac{0.1575}{3\left(0.161\right)}\right)^{1.5} 2.68 = 0.50 \\
&= \left(1 - \frac{0.5}{1.5}\left(1 + \left(\frac{0.03}{0.161}\right) 2.68\right)\right) = l_t^s.
\end{aligned}
$$

Graph of L(w): Excess Labor Demand with Productivity Increase

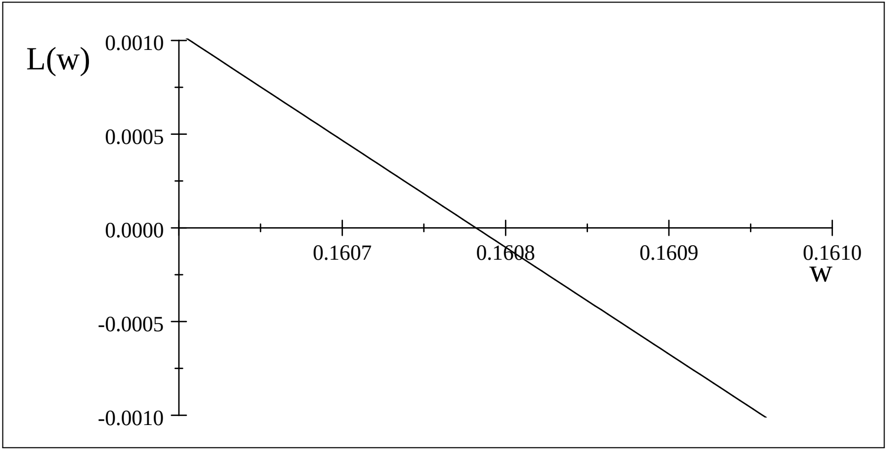


Figure 9.4. Excess Demand and the Real Wage in Example 9.2.

Time Endowment Increase: Example 9.3

- T rises 5% from 1 to 1.05. $A_G = 0.15$, $\gamma = \frac{1}{3}$, $\alpha = 0.5$, $\delta_k = 0.03$, $\rho = 0.03$.
- $k_t = 2.4306$, a 5% increase.

$$c_t^d = \frac{1}{1+\alpha}\left(1.05 w_t + \rho k_t\right), \; x_t = \frac{\alpha c_t^d}{w_t}. \; l_t^s = 1.05 - x_t;$$

$$l_t^s = 1.05 - \frac{\alpha c_t^d}{w_t} = 1.05 - \frac{\alpha}{1+\alpha}\left(1.05 + \frac{\rho k_t}{w_t}\right)$$

$$w_t = \frac{\alpha \rho k_t}{1.05 - (1+\alpha)\, l_t^s} = \frac{0.5\,(0.052632)\,2.4306}{1.05 - (1.5)\, l_t^s}. \tag{6}$$

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t,$$

$$w_t = \gamma A_G \left(\frac{k_t}{l_t^d}\right)^{1-\gamma} = \frac{1}{3}(0.15)\left(\frac{2.4306}{l_t^d}\right)^{1-\frac{1}{3}}. \tag{7}$$

Labor Supply, Demand Shift Out; Wage Unchanged

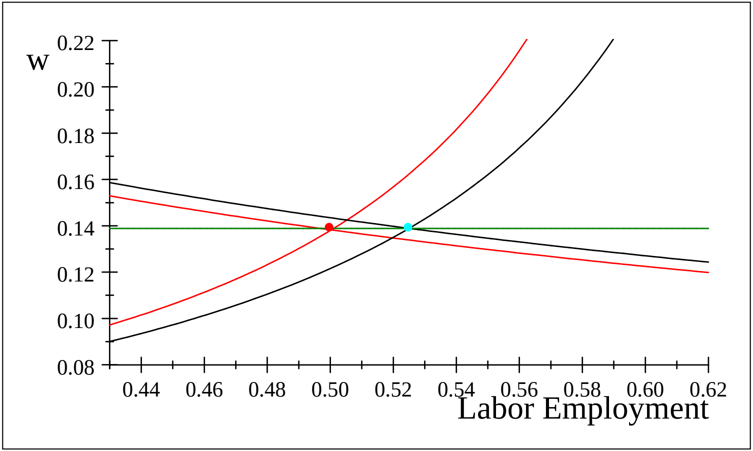


Figure 9.5. Increase in Time Endowment Increases Employment in Example 9.3.

Equilibrium Wage with Time Endowment Increase

$$L\left(w_t\right) \equiv \left(\frac{0.15}{3w_t}\right)^{1.5} 2.4306 - \left(1.05 - \frac{\left[1.05 + \left(\frac{0.03}{w_t}\right) 2.4306\right]}{3}\right) = 0,$$
$$\Longrightarrow \quad w_t = 0.13889.$$

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t = \left(\frac{0.15}{3\left(0.139\right)}\right)^{1.5} (2.43) = 0.525.$$
$$l_t^s = 1.05 - \frac{\alpha\left[1.05 + \left(\frac{\rho}{w_t}\right) k_t\right]}{1+\alpha}$$
$$= 1.05 - \frac{\left[1.05 + \left(\frac{0.03}{0.139}\right)(2.43)\right]}{3} = 0.525.$$

L(w): Excess Demand with Time Endowment Increase

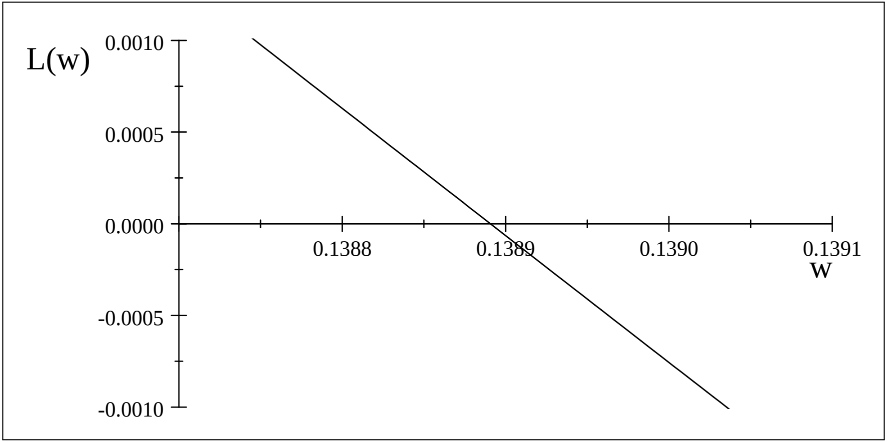


Figure 9.6. Implicit Solution of the Real Wage in Example 9.3.

Dynamic Model Employment Cycle Explanation: Example 9.4

- $A_G = 0.15 \longrightarrow A_G = (0.15)(1.05) = 0.1575$; $T = 1 \longrightarrow T = 1.05$.
- 5% increases; and same.$\gamma = \frac{1}{3}$, $\alpha = 0.5$, $\delta_k = 0.03$, $\rho = 0.03$.
- $k = 2.8137$; a 21.6% increase.

$$l_t^s = 1.05 - \frac{0.5}{1.5}\left[1.05 + \left(\frac{0.03}{w_t}\right)2.81\right], \quad (8)$$

$$l_t^d = \left(\frac{0.1575}{3w_t}\right)^{1.5} 2.81. \quad (9)$$

$$w_t = \frac{0.5\,(0.03)\,2.8137}{1.05 - (1.5)\,l_t^s}, \quad (10)$$

$$w_t = \frac{1}{3}(0.1575)\left(\frac{2.8137}{l_t^d}\right)^{1-\frac{1}{3}}. \quad (11)$$

Expansionary Shift Out in Demand, Pivot in Supply

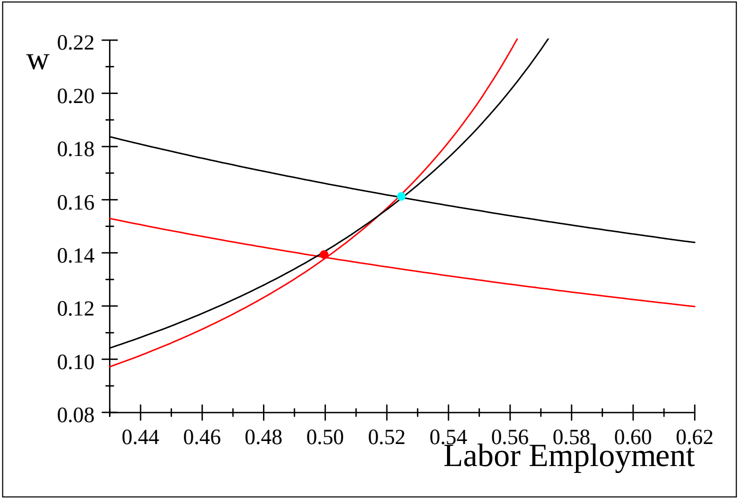


Figure 9.7. Expansionary Increase in Time Endowment and Goods Productivity in Example 9.4.

Equilibrium Wage and Employment

$$
\begin{aligned}
L(w_t) &\equiv \left(\frac{0.1575}{3w_t}\right)^{1.5} 2.81 - \left(1.05 - \frac{1}{3}\left[1.05 + \left(\frac{0.03}{w_t}\right) 2.81\right]\right) = 0, \\
l_t^d &= \left(\frac{(0.1575)}{3(0.161)}\right)^{1.5} (2.81) = 0.525 \\
&= 1.05 - \frac{\left(1.05 + \left(\frac{0.03}{0.161}\right)(2.81)\right)}{3} = l_t^s; \\
w_t &= 0.16078.
\end{aligned}
$$

Equilibrium Wage and Employment

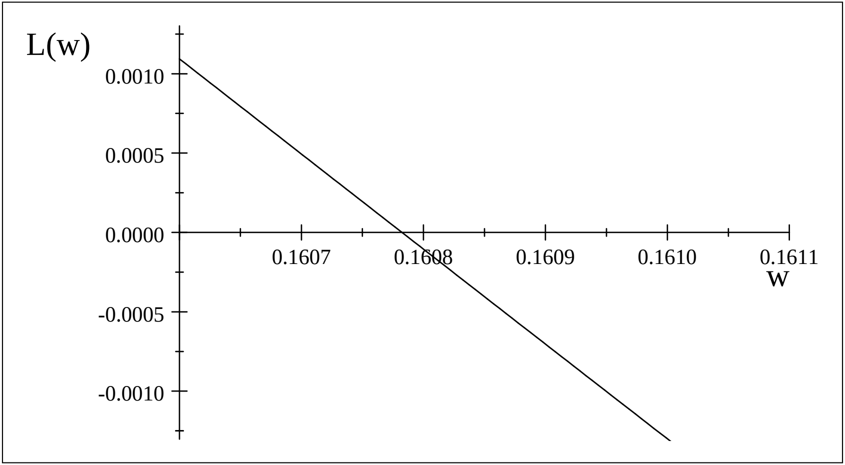


Figure 9.8. Implicit Solution of the Real Wage in Example 9.4.

Consumption to Output Ratio during Expansion

- $$c_t^d = \frac{1}{1+\alpha}\left(1.05w_t + \rho k_t\right) = \frac{1}{1.5}\left(\left(1.05\right)0.161 + 0.03\left(2.81\right)\right) = 0.169.$$
- 21.6%, same increase in capital.

$$\begin{aligned} y_t^d &= \frac{1}{1+\alpha}\left(1.05w_t + k_t\left[\rho + \left(1+\alpha\right)\delta_k\right]\right) \\ &= \frac{1}{1.5}\left(\left(1.05\right)0.161 + \left(0.03 + \left(1.5\right)0.03\right)\left(2.8\right)\right) \\ &= 0.25323, \end{aligned}$$

- 21.6%, the same as consumption.
- Consumption to output

$$\frac{c^d}{y^d} = \frac{0.16882}{0.25323} = \frac{2}{3}.$$

- Same as before expansion.

Example 9.5 Contraction

- $A_G = 0.15 \longrightarrow A_G = 0.1425$; $T = 1 \longrightarrow T = 0.95$.
- 5% decreases; and same.$\gamma = \frac{1}{3}$, $\alpha = 0.5$, $\delta_k = 0.03$, $\rho = 0.03$; $k = 1.8854$.

$$l_t^s = 0.95 - \frac{0.5}{1.5}\left[0.95 + \left(\frac{0.03}{w_t}\right)(1.89)\right],$$
$$l_t^d = \left(\frac{0.1425}{3w_t}\right)^{1.5}(1.89).$$

$$w_t = \frac{\alpha\rho k_t}{0.95 - (1+\alpha)\, l_t^s} = \frac{0.5\,(0.03)\,1.89}{0.95 - (1.5)\, l_t^s},$$
$$w_t = \gamma A_G \left(\frac{k_t}{l_t^d}\right)^{1-\gamma} = \frac{1}{3}(0.1425)\left(\frac{1.89}{l_t^d}\right)^{1-\frac{1}{3}}.$$

Decrease in Employment During Contraction

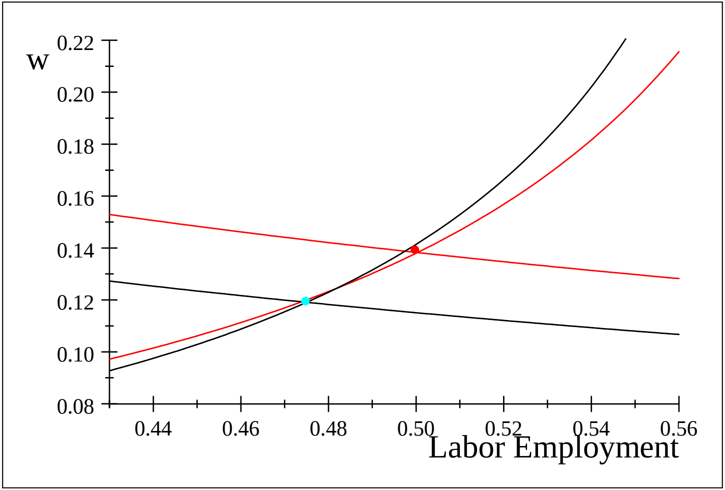


Figure 9.9. Contractionary Decrease in Time Endowment and Goods Productivity in Example 9.5.

Equilibrium Wage, Excess Labor Demand, with Contraction

$$L(w_t) = \left(\frac{0.1425}{3w_t}\right)^{1.5} 1.89 - \left(0.95 - \frac{\left[0.95 + \left(\frac{0.03}{w_t}\right) 1.89\right]}{3}\right) = 0;$$

$$w_t = 0.1191.$$

$$0.475 = l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t = T - \frac{\alpha}{1+\alpha}\left[T + \left(\frac{\rho}{w_t}\right) k_t\right] = l_t^s;$$

$$0.475 = \left(\frac{(0.1425)}{3\,(0.12)}\right)^{1.5} (1.89) = 0.95 - \frac{1}{3}\left(0.95 + \left(\frac{0.03}{0.12}\right)(1.89)\right)$$

Equilibrium Wage, Excess Labor Demand, with Contraction

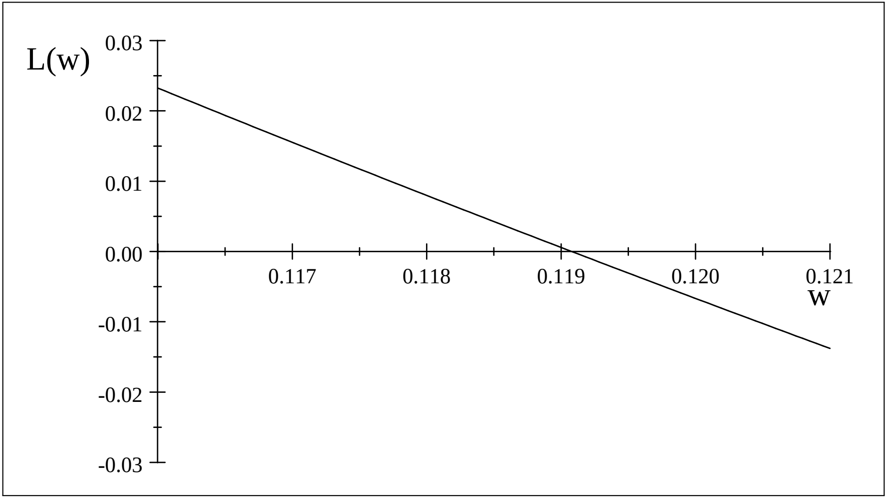


Figure 9.10. Excess Labor Demand during Contraction in Example 9.5.

Fixed Wage Unemployment and Depressions

- Big employment decrease: 1930s Depression, 2007-2010 recession.
  - Depression characterized as having "excess labor supply".
- Labor contracts often set a wage rate for some years.
  - can allow employers to reduce temporarily employment
  - automobile, airplane industries: large cyclic swings in output
- Model "normal" contraction in dynamic model, with fixed wage.
- Chapter 3: fixed wage causes surplus labor, 50% further employment decrease;
  - dynamic model: same 50% result when capital stock arbitrarily held fixed.
- Allowing capital to adjust, fixed wage during contraction:
  - much bigger drop in employment.

Example 9.6: Fixed Wage, Fixed Capital, During Contraction

- Marginal product of labor

$$MP_l = w_t = \gamma A_G \left(\frac{k_t}{l_t}\right)^{1-\gamma}.$$

$$\begin{aligned} l &= \left(\frac{\gamma A_G}{\bar{w}}\right)^{\frac{1}{1-\gamma}} \bar{k}.\ w = \bar{w} = 0.1389;\ k_t = \bar{k} = 2.3148. \\ l_t &= \left(\frac{(0.1425)}{3\,(0.1389)}\right)^{1.5} 2.3148 = 0.463. \end{aligned}$$

- Fractional employment change: $\frac{0.463-0.5}{0.5} = -0.074$, a 7.4% fall,
- similar to Chapter 3 fixed wage result;
    - 50% bigger than flexible wage.

Example 9.6: Fixed Wage, Variable Capital, During Contraction

- If capital stock readjusts, employment much lower

$$l_t^d = \left(\frac{(0.1425)}{3\,(0.1389)} \right)^{1.5} (1.8854) = 0.377,$$

- a fractional decrease of $\frac{0.337-0.5}{0.5} = -0.326$, or a 32% decrease.

$$l_t^s = \left(0.95 - \frac{0.5}{1.5} \left(0.95 + \left(\frac{0.03}{0.13889} \right) 1.8854 \right) \right) = 0.49$$

$$\textit{Excess Supply} : l_t^s - l_t^d = 0.498 - 0.377 = 0.121\,.$$

- Excess: $\frac{0.498-0.377}{0.377} = 0.32$, or 32% of employment at $l = 0.377$.
- Of total "labor force" 0.498, excess is $\frac{0.498-0.377}{0.498} = 0.24$, or 24%.
- Comparable to US unemployment during 1930s depression.
  - Unemployment reached 22%, compared to 24% in example here.
- Chapter 16 : use banking productivity decrease instead of fixed wage,
  - to generate big employment drop.

Contractionary Excess Supply of Labor: 24% of Labor Force

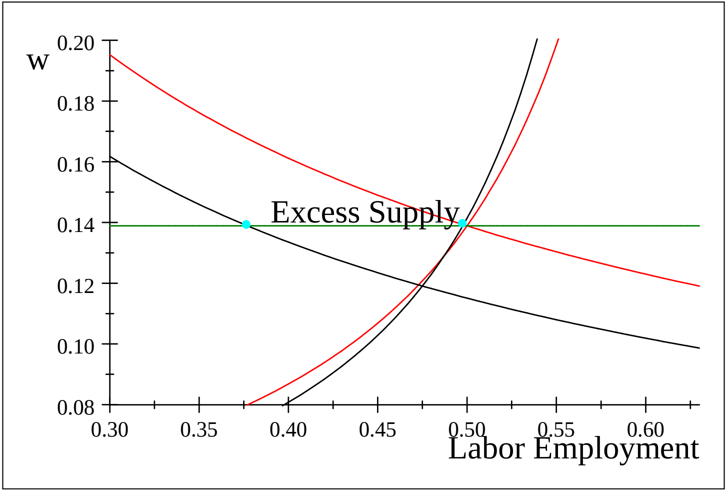


Figure 9.11. Excess Labor Supply with a Fixed Wage in Example 9.6.

Equilibrium with Labor Income Tax

Tax on labor income, at rate of τ_l; with a government budget constraint of

$$G_t = \tau_l w_t l_t^s.$$

$$c_t^d = w_t \left(1 - \tau_l\right) l_t^s + r_t k_t + G_t - k_{t+1} + k_t \left(1 - \delta_k\right).$$

$$x_t = \frac{\alpha c_t}{w_t \left(1 - \tau_l\right)} . l_t^s = T - x_t, r_t - \delta_k = \rho.$$

$$c_t^d = w_t \left(T - \frac{\alpha c_t}{w_t \left(1 - \tau_l\right)} \right) + \rho k_t = \frac{w_t T + \rho k_t}{1 + \frac{\alpha}{1 - \tau_l}}.$$

$$y_t^d = \frac{w_t T + \rho k_t}{1 + \frac{\alpha}{1 - \tau_l}} + \delta_k k_t = \frac{w_t T + k_t \left[\rho + \delta_k \left(1 + \frac{\alpha}{1 - \tau_l}\right)\right]}{1 + \frac{\alpha}{1 - \tau_l}}.$$

$$\frac{1}{w_t} = \frac{T}{y_t^d \left(1 + \frac{\alpha}{1 - \tau_l}\right) - k_t \left[\rho + \delta_k \left(1 + \frac{\alpha}{1 - \tau_l}\right)\right]}.$$

Example 9.7: 20% Labor Income Tax

- $A_G = 0.15,\ T = 1,\ \gamma = \frac{1}{3},\ \alpha = 0.5,\ \delta_k = 0.03,\ \rho = 0.03;$ $\tau_l = 0.20 : k_t = 2.0576.$
- Fractional decrease of $\frac{2.3148-2.0576}{2.3148} = 0.111$, or 11.1%.
- $AS - AD:$

$$\frac{1}{w_t} = \frac{1}{y_t^d\left(1+\frac{0.5}{1-0.2}\right) - 2.0576\left[0.03 + 0.03\left(1+\frac{0.5}{1-0.2}\right)\right]}, \tag{12}$$

$$\frac{1}{w_t} = \frac{\left(y_t^s\right)^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{\gamma}}\left(k_t\right)^{\frac{1-\gamma}{\gamma}}} = \frac{\left(y_t^s\right)^2}{\frac{1}{3}\left(0.15\right)^3\left(2.0576\right)^2}. \tag{13}$$

AS-AD Both Shift Back, Wage Unchanged

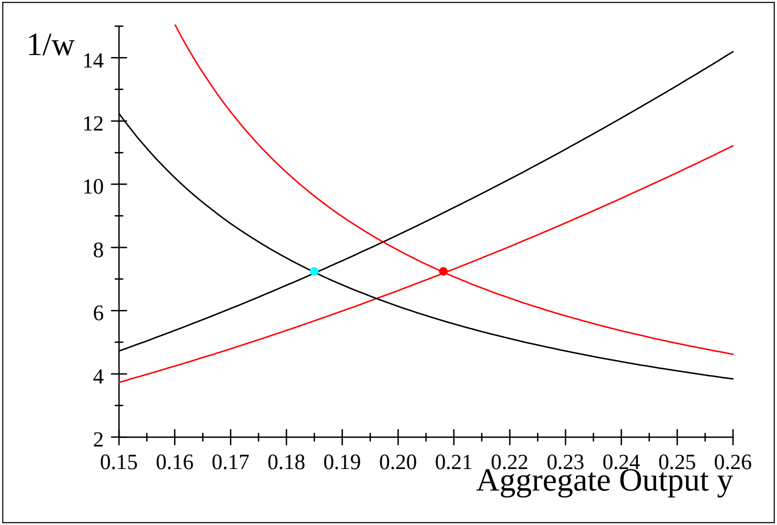


Figure 9.12. Equilibrium with a Labor Income Tax in Example 9.7.

Excess Labor Demand, Equilibrium Wage: 20% Labor Tax

$$0 = Y(w_t) \equiv \frac{w_t T + k_t \left[\rho + \delta_k \left(1 + \frac{\alpha}{1-\tau_l}\right)\right]}{1 + \frac{\alpha}{1-\tau_l}} - (A_G)^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}}$$

$$Y(w_t) = \frac{w_t + (2.0576)\left[0.03 + 0.03\left(1 + \frac{0.5}{1-0.2}\right)\right]}{1 + \frac{0.5}{1-0.2}} - (0.15)^{1.5}\left(\frac{\frac{1}{3}}{w_t}\right)^{\frac{1}{2}}(2.0576).$$

$$w_t = 0.13889.$$

Excess Labor Demand and Equilibrium Wage

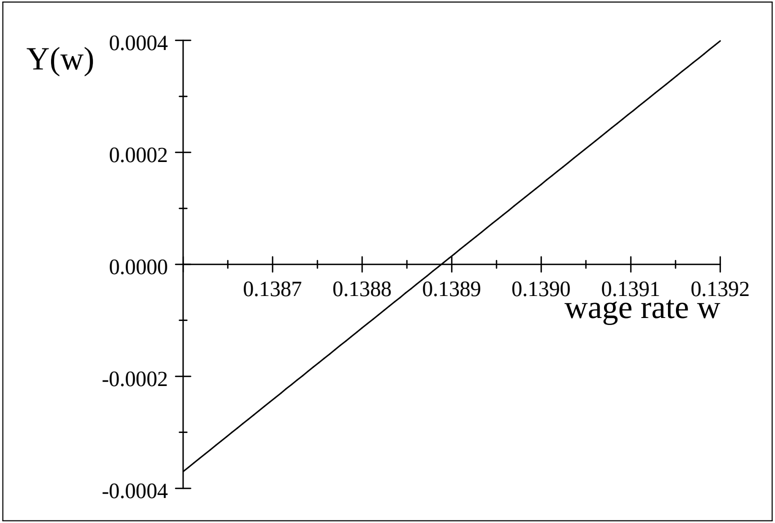


Figure 9.13. Excess Output Demand $Y\left(w_t\right)$ in Example 9.7.

Consumption and Output

- $$c_t^d = \frac{w_t T + \rho k_t}{1 + \frac{\alpha}{1-\tau_l}} = \frac{0.13889 + 0.03\,(2.0576)}{1 + \frac{0.5}{1-0.2}} = 0.123\,46.$$
- Fractional decrease of $\frac{0.13889 - 0.123\,46}{0.13889} = 0.111$, or 11.1%,
  - same as percent decrease in capital stock.
- $$\begin{aligned} y_t^d &= \frac{w_t T + k_t \left[\rho + \delta_k \left(1 + \frac{\alpha}{1-\tau_l}\right)\right]}{1 + \frac{\alpha}{1-\tau_l}} \\ &= \frac{0.13889 + (2.0576)\left(0.03 + 0.03\left(1 + \frac{0.5}{1-0.2}\right)\right)}{1 + \frac{0.5}{1-0.2}} \\ &= 0.18519. \end{aligned}$$
- Fractional decrease: $\frac{0.208\,32 - 0.18519}{0.20832} = 0.111$, or 11.1%, same as consumption.
- Consumption to output ratio stays same:

Labor Market with 20% Labor Income Tax

- $$l_t^s = T - \frac{\alpha \left(\frac{w_t T + \rho k_t}{1 + \frac{\alpha}{1-\tau_l}} \right)}{w_t \left(1 - \tau_l\right)}. \tag{14}$$

$$w_t = \left(\frac{\alpha \rho k_t}{\left[1 + \frac{\alpha}{1-\tau_l}\right] \left(1 - \tau_l\right) \left(T - l_t^s\right) - \alpha T} \right), \quad w_t = \gamma A_G \left(\frac{k_t}{l_t^d} \right)^{\frac{2}{3}},$$

$$w_t = \left(\frac{0.5 \left(0.03\right) \left(2.058\right)}{\left(1 + \frac{0.5}{1-0.2}\right) \left(1 - 0.2\right) \left(1 - l_t^s\right) - 0.5} \right), \quad w_t = \frac{\left(0.15\right)}{3} \left(\frac{\left(2.058\right)}{l_t^d} \right)$$

$$w_t = 0.1389; \; l_t^d = \left(\frac{\left(0.15\right)}{3 \left(0.1389\right)} \right)^{1.5} \left(2.0576\right) = 0.444. \tag{15}$$

- Ffractional decrease in employment of $\frac{0.5 - 0.44439}{0.5} = 0.111$, or 11.1%,
  - same as decrease in capital stock, consumption, output.
  - Government spending $G = w_t \tau_l l_t = \left(0.13889\right) \left(0.20\right) \left(0.444\right) = 0.012$.

Labor Market with Tax: Supply, Demand Shift Back, Same Wage

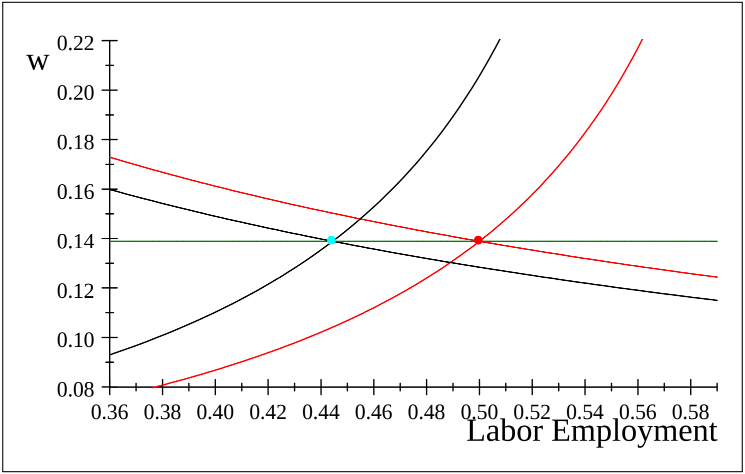


Figure 9.14. Increase in Time Endowment Shifts out Labor Supply in Example 9.7.

Isocost, Isoquant, Input Ratio with Labor Tax

Budget Line

$$
\begin{aligned}
0.18519 &= y_t = w_t l_t + r_t k_t \, (0.13889)\, l_t + (0.06)\, k_t, \\
k_t &= \frac{0.18519}{0.06} - \frac{(0.13889)\, l_t}{0.06}.
\end{aligned} \tag{16}
$$

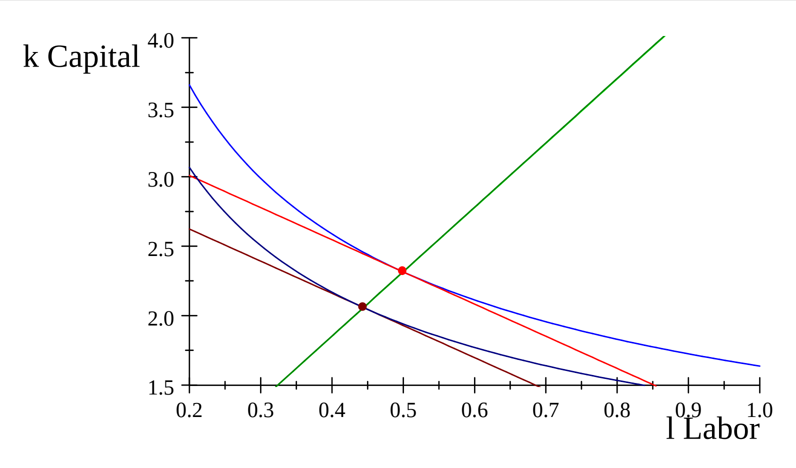
Isoquant curve

$$
\begin{aligned}
0.18519 &= y_t^s = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} = 0.15 \left(l_t^d\right)^{\frac{1}{3}} (k_t)^{\frac{2}{3}}; \\
k_t &= \left(\frac{(0.18519)}{0.15 \left(l_t^d\right)^{\frac{1}{3}}} \right)^{\frac{3}{2}} = \frac{\left(\frac{0.18519}{0.15}\right)^{\frac{3}{2}}}{\left(l_t^d\right)^{\frac{1}{2}}}.
\end{aligned} \tag{17}
$$

Input ratio

$$
\frac{k_t}{l_t} = \frac{2.0576}{0.444} = 4.6342. \tag{18}
$$

Isocost, Isoquant Shift Up, Same Wage and Input Ratio



Example 9.15. Factor Market Equilibrium with Labor Income Tax τ_l, in Example 9.7 (lower) and Baseline (upper).

Labor Tax: Production Function, Utility Level, Budget Line

$$c_t^d = y_t^s - i_t = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma} - \delta_k k_t,$$
$$c_t^d = (0.15) \left(l_t^d\right)^{\frac{1}{3}} (2.0576)^{\frac{2}{3}} - (0.03)(2.0576); \quad (19)$$

$$u = \ln c_t + \alpha \ln x_t = \ln c_t + \alpha \ln (1 - l_t),$$
$$-2.3853 = \ln 0.12346 + 0.5 \ln (1 - 0.444),$$
$$c_t = \frac{e^{-2.3853}}{(1 - l_t)^{0.5}}. \quad (20)$$

$$c_t^d = w_t l_t^s (1 - \tau_l) + \rho k_t^s + G$$
$$c_t^d = (0.13889)(1 - 0.20) l_t^s + (0.03)(2.0576) + 0.012. \quad (21)$$

Tax: Intersection, Not Tangency, Between Utility, Production

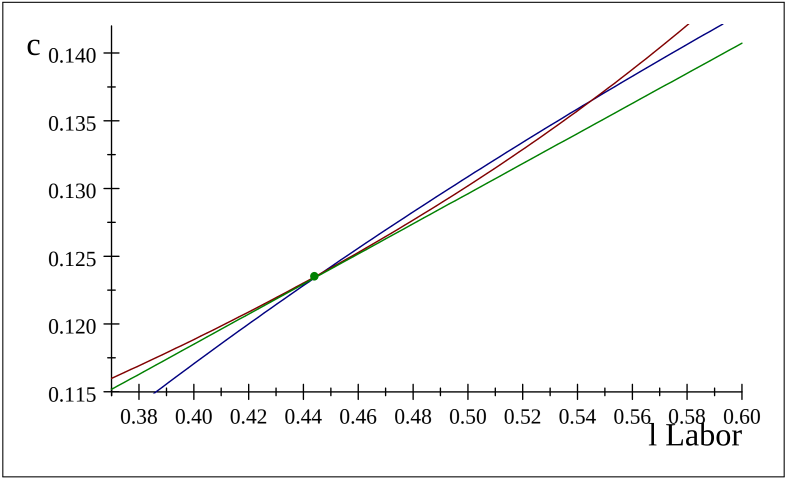


Figure 9.16. General Equilibrium Goods and Labor with Labor Tax in Example 9.7.

Application: Hours per Week, Taxes and Regulations

- Marginal productivity rises, but hours per week stay the same.
  - Implies with rising productivity, work week remains same.
  - International data not consistent constant hours of work per week.
- Post WWII hours declined: UK, France, Germany, Australia, Canada.
  - But rose in USA: yet all countries have upward trend in productivity.
- Standard dynamic model cannot explain.
  - Many alternative explanations: open question of active research.
- Case of France: restricted workweek to 35 hours in Feb 2000
  - further restriction from 39 hour week by President Mitterrand.
  - Idea (Meade, 1937): allow more employed; each employed works less, Many consider it is regulation on labor market like an implicit tax.
  - Restriction relaxed May 2002 to May 2005, and since May 2007.
  - France's hours dropped sharply from 2000 to 2002; up in 2003.
- Implicit taxes in US down as union membership has trended down.
- Ohanian, Raffo and Rogerson (2008) well explain European-US hours
  - by including taxes, implicit taxes on labor market, in standard model.

Hours Worked Per Week: US, Canada, Europe, Australia

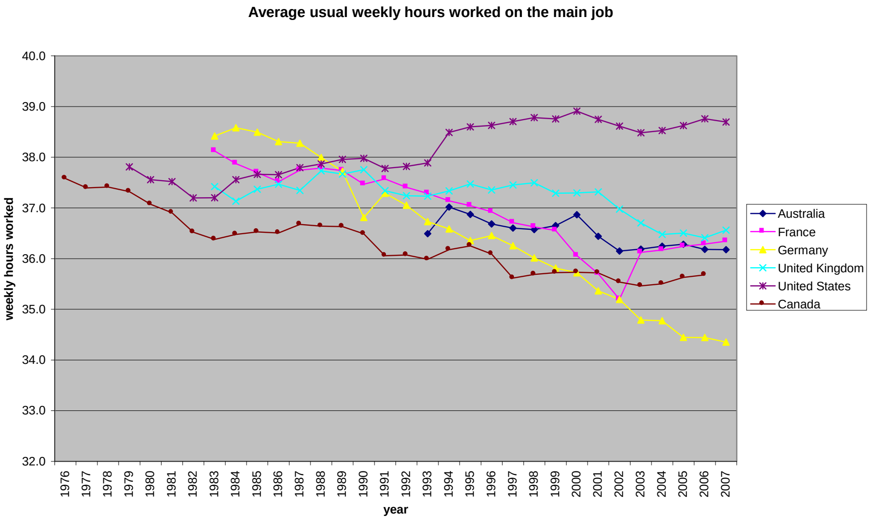


Figure 9.17.