# International Business Cycle and Financial Intermediation: Technical Appendix

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## Contents

# 1 The Model

In this section the model optimization problem and the equilibrium conditions are presented in detail. First, for the Home country and then for the Foreign country for each agent and/or sector.

## 1.1 The Home Country Household

For the home country consider the budget constraint in footnote *7*. In this case $i = H$ and also apply the calibration assumption that no adjustment cost is paid for changing domestic asset positions, i.e. $\chi_{DHH} = 0$, to get

$$\begin{aligned} P_{CH,t}C_{H,t} &= P_{CH,t}w_{H,t}(l_{H,t} + l_{QH,t}) + P_{CH,t}(1 + R_{DH,t})D_{HH,t} \\ &\quad + P_{CF,t}(1 + R_{DF,t})D_{HF,t} + P_{CH,t}\tau_{H,t} - P_{CH,t}D_{HH,t+1} \\ &\quad - P_{CF,t}D_{HF,t+1} + P_{CH,t}\frac{\chi_{DHF}}{2}\left(\frac{P_{CF,t}}{P_{CH,t}}D_{HF,t+1}\right)^2. \end{aligned} \tag{1}$$

Next normalize the budget constraint with the domestic consumption good price, $P_{CH,t}$, and use the definition of the real exchange rate, $RER_t \equiv P_{CF,t}/P_{CH,t}$, to get

$$\begin{aligned} C_{H,t} &= w_{H,t}t(l_{H,t} + l_{QH,t}) + (1 + R_{DH,t})D_{HH,t} + RER_t(1 + R_{DF,t})D_{HF,t} + \tau_{H,t} \\ &\quad - D_{HH,t+1} - RER_tD_{HF,t+1} + \frac{\chi_{DHF}}{2}\left(RER_{t+1}D_{HF,t+1}\right)^2. \end{aligned} \tag{2}$$

Then the household's problem in the Home country can be writen as

$$\begin{aligned} \mathcal{L} = E_0\sum_{t=0}^{\infty}\beta_H^t\{&\frac{\left[C_{H,t}\left(1 - l_{H,t} - l_{HQ,t}\right)^{a_H}\right]^{1-\theta_H}}{1-\theta_H} \\ + \lambda_{H,t}&\left[\begin{array}{c} w_{H,t}t(l_{H,t} + l_{QH,t}) + (1 + R_{DH,t})D_{HH,t} \\ +RER_t(1 + R_{DF,t})D_{HF,t} + \tau_{H,t} - D_{HH,t+1} \\ -D_{HH,t+1} - RER_tD_{HF,t+1} + \frac{\chi_{DHF}}{2}\left(RER_{t+1}D_{HF,t+1}\right)^2 - C_{H,t} \end{array}\right]\}. \end{aligned} \tag{3}$$

The resulting first order conditions are the following:

$$C_{H,t}: \quad \lambda_{H,t} = C_{H,t}^{-\theta_H}x_{H,t}^{a_H(1-\theta_H)}; \tag{4}$$

$$l_{H,t}: \quad \lambda_{H,t}w_{H,t} = A_HC_{H,t}^{1-\theta}x_{H,t}^{a_H(1-\theta_H)-1}; \tag{5}$$

$$l_{QH,t}: \quad \lambda_{H,t}w_{H,t} = A_HC_{H,t}^{1-\theta_H}x_{H,t}^{a_H(1-\theta_H)-1}; \tag{6}$$

$$D_{HH,t+1}: \quad \beta_H E_t \frac{\lambda_{H,t+1}}{\lambda_{H,t}}(1+R_{DH,t+1}) = 1; \tag{7}$$

$$D_{HF,t+1}: \quad \beta_H E_t \frac{\lambda_{H,t+1}}{\lambda_{H,t}} \frac{RER_{t+1}}{RER_t}(1+R_{DF,t+1}) = 1 + \chi_{DHF}\left(RER_t D_{HF,t+1}\right); \tag{8}$$

Then by combining the first order conditions one obtains the following equilibrium conditions:

$$a_H C_{H,t} = w_{H,t} x_{H,t}; \tag{9}$$

$$1 = \beta_H E_t \left\{ \left(\frac{C_{H,t}}{C_{H,t+1}}\right)^{\theta_H} \left(\frac{x_{H,t+1}}{x_{H,t}}\right)^{a_H(1-\theta_H)} (1+R_{DH,t+1}) \right\}; \tag{10}$$

$$E_t\left(\frac{RER_{t+1}}{RER_t}\right) = E_t\left\{\left[\frac{1+R_{DH,t+1}}{1+R_{DF,t+1}}\right](1+\chi_{DHF} RER_t D_{HF,t+1})\right\}; \tag{11}$$

where equation (9) is the marginal rate of substitution (MRS) between consumption and leisure; equation (10) is the Euler condition with respect to domestic deposits; and equation (11) is the uncovered interest parity equation for the home country household.

## 1.2 The Foreign Country Household

For the Foreign country household $i = F$ and apply the calibration assumption that no adjustment cost is paid for changing domestic asset positions, i.e. $\chi_{DFF} = 0$, to get

$$\begin{aligned} P_{CF,t} C_{F,t} &= P_{CF,t} w_{F,t}(l_{F,t} + l_{QF,t}) + P_{CF,t}(1+R_{DF,t}) D_{FF,t} \\ &+ P_{CH,t}(1+R_{DH,t}) D_{FH,t} + P_{CF,t}\tau_{F,t} - P_{CF,t} D_{FF,t+1} \\ &- P_{CH,t} D_{FH,t+1} + P_{CF,t}\frac{\chi_{DFH}}{2}\left(\frac{P_{CH,t}}{P_{Cf,t}} D_{FH,t+1}\right)^2. \end{aligned} \tag{12}$$

Normalizing the budget constraint with the domestic consumption good price, $P_{CF,t}$, and use the definition of the real exchange rate, $RER_t$, yields

$$\begin{aligned} C_{F,t} &= w_{F,t}(l_{F,t} + l_{QF,t}) + (1+R_{DF,t}) D_{FF,t} + \frac{(1+R_{DH,t}) D_{FH,t}}{RER_t} \qquad (13) \\ &+ \tau_{F,t} - D_{FF,t+1} - \frac{D_{FH,t+1}}{RER_t} + \frac{\chi_{DFH}}{2}\left(\frac{D_{FH,t+1}}{RER_t}\right)^2. \end{aligned}$$

Then given that the Lagrange multiplier of the Foreign household's budget constraint is denoted by $\lambda_{F,t}$ one obtain the following first order conditions:

$$C_{F,t}: \quad \lambda_{F,t} = C_{F,t}^{-\theta_F} x_{F,t}^{a_F(1-\theta_F)}; \tag{14}$$

$$l_{F,t}: \quad \lambda_{F,t} w_{F,t} = a_F C_{F,t}^{1-\theta_F} x_{F,t}^{a_F(1-\theta_F)-1}; \tag{15}$$

$$l_{FQ,t}: \quad \lambda_{F,t} w_{F,t} = a_F C_{F,t}^{1-\theta_F} x_{F,t}^{a_F(1-\theta_F)-1}; \tag{16}$$

$$D_{FF,t+1}: \quad \beta_F E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} (1 + R_{DF,t+1}) = 1; \tag{17}$$

$$D_{FH,t+1}: \quad \beta_F E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \frac{RER_t}{RER_{t+1}} (1 + R_{DH,t+1}) = 1 + \chi_{DFH} \left( \frac{D_{FH,t+1}}{RER_t} \right); \tag{18}$$

Then by combining the first order conditions one obtains the following equilibrium conditions:

$$a_F C_{F,t} = w_{F,t} x_{F,t}; \tag{19}$$

$$1 = \beta_F E_t \left\{ \left( \frac{C_{F,t}}{C_{F,t+1}} \right)^{\theta_F} \left( \frac{x_{F,t+1}}{x_{F,t}} \right)^{a_F(1-\theta_F)} (1 + R_{DF,t+1}) \right\}; \tag{20}$$

$$E_t \left\{ \frac{RER_{t+1}}{RER_t} \left[ 1 + \chi_{DFH} \left( \frac{D_{FH,t+1}}{RER_t} \right) \right] \right\} = E_t \left[ \frac{1 + R_{DH,t+1}}{1 + R_{DF,t+1}} \right]; \tag{21}$$

where equation (19) is the marginal rate of substitution between consumption and leisure; equation (20) is the Euler condition with respect to domestic deposits; and equation (21) is the uncovered interest parity equation for the foreign country consumer.

## 1.3 The Home Country Intermediate Goods Producer

For the Home country intermediate goods producer consider the profit function, $\Pi_{GH,t}$, in footnote *9* with $i = H$ and applying the calibration assumption that no adjustment cost is paid for changing domestic asset positions, i.e. $\chi_{QHH} = 0$, to get

$$\begin{aligned} \Pi_{GH,t} = {} & P_{XH,t} X_{H,t} - P_{CH,t} w_{H,t} l_{H,t} - P_{CH,t} i_{H,t} + P_{CH,t} Q_{HH,t+1} \\ & + P_{CF,t} Q_{FH,t+1} - P_{CH,t} \frac{\chi_{QFH}}{2} \left( \frac{P_{CF,t}}{P_{CH,t}} Q_{FH,t+1} \right)^2 \\ & - P_{CH,t}(1 + R_{QH,t}) Q_{HH,t} - P_{CF,t}(1 + R_{QF,t}) Q_{FH,t}. \end{aligned} \tag{22}$$

Normalizing the profit constraint with the domestic consumption good price, $P_{CH,t}$, using the definition of the real exchange rate, $RER_t$, and denoting the relative price $p_{XH,t} \equiv P_{XH,t}/P_{CH,t}$, yields

$$\begin{aligned} \pi_{GH,t} &= \frac{\Pi_{GH,t}}{P_{CH,t}} = p_{XH,t}X_{H,t} - w_{H,t}l_{H,t} - i_{H,t} + Q_{HH,t+1} \\ &+RER_tQ_{FH,t+1} - \frac{\chi_{QFH}}{2}\left(RER_tQ_{FH,t+1}\right)^2 - (1+R_{QH,t})Q_{HH,t} \\ &-(1+R_{QH,t})Q_{HH,t} - RER_t(1+R_{QF,t})Q_{FH,t}. \end{aligned} \tag{23}$$

Next, consider the loan requirement constraint in the Model section, which after applying $i = H$ becomes

$$P_{CH,t}Q_{HH,t} + P_{CF,t}Q_{FH,t} = P_{CH,t}k_{H,t}. \tag{24}$$

Normaling the the above with the domestic consumption good price results in

$$Q_{HH,t} + RER_tQ_{FH,t} = k_{H,t}. \tag{25}$$

Then substituting the capital accumulation constraint, the production technology and the loan requirement constraint into (23) the intermediate goods producer's problem in the home country becomes

$$\max_{\{Q_{HH,t},\ Q_{FH,t+1},\ l_{H,t}\}_{t=0}^{\infty}} \pi_{HG,t} = E_0\left(\beta_H^t\lambda_{H,t}\right)\left\{\begin{array}{c} p_{XH,t}A_He^{z_{H,t}}\left(Q_{HH,t}+RER_tQ_{FH,t}\right)^{\alpha_H}\left(l_{H,t}\right)^{1-\alpha_H} \\ -w_{H,t}l_{H,t} - Q_{HH,t}(\delta_H + R_{FQ,t}) \\ +(RER_t - RER_{t+1})Q_{FH,t+1} \\ -RER_tQ_{FH,t}(\delta_H + R_{QF,t}) \\ -\frac{\chi_{QFH}}{2}(RER_tQ_{FH,t+1})^2 \end{array}\right\}.$$

The resulting first order conditions are:

$$l_{H,t}: \quad \lambda_{H,t}(1-\alpha_H)A_He^{z_{H,t}}\left[\frac{k_{H,t}}{l_{H,t}}\right]^{\alpha_H} = \lambda_{H,t}w_{H,t}; \tag{26}$$

$$Q_{HH,t}: \quad \lambda_{H,t}\left(R_{QH,t}+\delta_H\right) = \lambda_{H,t}\alpha_Hp_{XH,t}A_He^{z_{H,t}}\left(\frac{k_{H,t}}{l_{H,t}}\right)^{\alpha_H-1}; \tag{27}$$

$$\begin{aligned} Q_{FH,t+1} \quad : \quad & \beta_HE_tRER_{t+1}\lambda_{H,t+1}\left\{\alpha_Hp_{XH,t+1}A_He^{z_{H,t+1}}\left(\frac{k_{H,t+1}}{l_{H,t+1}}\right)^{\alpha-1} - (\delta_H + R_{FQ,t+1})\right\} = \\ & \lambda_{H,t}(RER_{t+1} - RER_t) + \lambda_{H,t}RER_t\chi_{QFH}(RER_tQ_{FH,t+1}). \end{aligned} \tag{28}$$

Next, by denoting the marginal product with respect to physical capital as $r_{H,t} = p_{XH,t}\alpha_HA_He^{z_{H,t}}\left[\frac{k_{H,t}}{l_{H,t}}\right]^{\alpha_H-1}$, the full set of equilibrium conditions and constraints of the intermediate goods producer in the Home country are:

$$r_{H,t} = R_{QH,t} + \delta_H; \tag{29}$$

$$X_{H,t} = A_H e^{z_{H,t}} (k_{H,t})^{\alpha_H} (l_{H,t})^{1-\alpha_H}; \tag{30}$$

$$X_{H,t} = X_{HH,t} + X_{HF,t}; \tag{31}$$

$$i_{H,t} = k_{H,t+1} - (1-\delta_H)k_{H,t}; \tag{32}$$

$$k_{H,t} = Q_{HH,t} + RER_t Q_{FH,t}; \tag{33}$$

$$w_{H,t} = p_{XH,t}(1-\alpha)A_H e^{z_{H,t}} \left[\frac{k_{H,t}}{l_{H,t}}\right]^{\alpha_H}; \tag{34}$$

$$r_{H,t} = p_{XH,t}\alpha A_H e^{z_{H,t}} \left[\frac{k_{H,t}}{l_{H,t}}\right]^{\alpha_H - 1}; \tag{35}$$

$$\begin{aligned} & 1 + \beta_H E_t \left\{ \left(\frac{C_{H,t}}{C_{H,t+1}}\right)^{\theta_H} \left(\frac{x_{H,t+1}}{x_{H,t}}\right)^{a_H(1-\theta_H)} \frac{RER_{t+1}}{RER_t} \left(R_{QH,t+1} - R_{QF,t+1}\right) \right\} \\ = \quad & E_t \left\{ \frac{RER_{t+1}}{RER_t} + \chi_{QFH}(RER_t Q_{FH,t+1}) \right\}; \end{aligned} \tag{36}$$

where equation (29) is obtained by using the first order conditions in (27) and using the definition of $r_{H,t}$. It equates the gross interest on domestic loans in the home country with the gross real return on physical capital net of depreciation. Equation (30) is the Cobb-Douglas production technology of the intermediate goods producer; equation (31) equates the sum of domestically and internationally sold intermediate goods to the total quantity produced; equation (32) is the physical capital law of motion; and equation (33) is the loan requirement constraint the firm faces. Equations (34) and (35) equate the marginal products to the real wage and the real return to physical capital. Lastly, by combining the first order conditions in (27) and (28) one obtains the home and foreign loan uncovered interest parity condition in equation (36).

## 1.4 The Foreign Country Intermediate Goods Producer

For the Foreign country intermediate goods producer consider the profit function, $\Pi_{GF,t}$, in footnote *9* with $i = F$ and applying the calibration assumption that no adjustment cost is paid for changing domestic asset positions, i.e. $\chi_{QFF} = 0$, to get

$$\Pi_{GF,t} = P_{XF,t}X_{F,t} - P_{CF,t}w_{F,t}l_{F,t} - P_{CF,t}i_{F,t} + P_{CF,t}Q_{FF,t+1} \tag{37}$$
$$+P_{CH,t}Q_{HF,t+1} - P_{CF,t}\frac{\chi_{QHF}}{2}\left(\frac{P_{CH,t}}{P_{CF,t}}Q_{HF,t+1}\right)^2$$
$$-P_{CF,t}(1+R_{QF,t})Q_{FF,t} - P_{CH,t}(1+R_{QH,t})Q_{HF,t}.$$

Normalizing the profit constraint with the domestic consumption good price, $P_{CF,t}$, using the definition of the real exchange rate, $RER_t$, and denoting the relative price $p_{XF,t} \equiv P_{XF,t}/P_{CH,t}$, yields

$$\pi_{GH,t} = \frac{\Pi_{GF,t}}{P_{CF,t}} = p_{XF,t}X_{F,t} - w_{F,t}l_{F,t} - i_{F,t} + Q_{FF,t+1} + \frac{Q_{HF,t+1}}{RER_t} \tag{38}$$
$$-\frac{\chi_{QHF}}{2}\left(\frac{Q_{HF,t+1}}{RER_t}\right)^2 - (1+R_{QF,t})Q_{FF,t} - \frac{(1+R_{QH,t})Q_{HF,t}}{RER_t}.$$

Next, consider the loan requirement constraint, which after applying $i = F$ becomes

$$P_{CF,t}Q_{FF,t} + P_{CH,t}Q_{HF,t} = P_{CF,t}k_{F,t}. \tag{39}$$

Normaling the the above with the domestic consumption good price results in

$$Q_{FF,t} + \frac{Q_{HF,t}}{RER_t} = k_{H,t}. \tag{40}$$

Then substituting the capital accumulation constraint, the production technology and the loan requirement constraint into (23) the intermediate goods producer's problem in the home country becomes

$$\max_{\{Q_{FF,t},\ Q_{HF,t+1},\ l_{F,t}\}_{t=0}^{\infty}} \pi_{FG,t} = E_0\left(\beta_F^t \lambda_{F,t}\right) \left\{ \begin{array}{c} p_{XF,t}A_F e^{z_{F,t}}\left(Q_{FF,t} + \frac{Q_{HF,t}}{RER_t}\right)^{\alpha_F}(l_{F,t})^{1-\alpha_F} - w_{F,t}l_{F,t} \\ -Q_{FF,t}(\delta_F + R_{QF,t}) + \left(\frac{1}{RER_t} - \frac{1}{RER_{t+1}}\right)Q_{HF,t+1} \\ -\frac{Q_{HF,t}(\delta_H + R_{QH,t})}{RER_t} - \frac{\chi_{QHF}}{2}(\frac{Q_{HF,t+1}}{RER_t})^2 \end{array} \right\}.$$

The resulting first order conditions are:

$$l_{F,t}: \quad \lambda_{F,t}p_{XF,t}(1-\alpha_F)A_F e^{z_{F,t}}\left[\frac{k_{F,t}}{l_{F,t}}\right]^{\alpha_F} = \lambda_{F,t}w_{F,t}; \tag{41}$$

$$Q_{FF,t}: \quad R_{QF,t} + \delta_F = \lambda_{F,t}p_{XF,t}\alpha_F A_F e^{z_{F,t}}\left[\frac{k_{F,t}}{l_{F,t}}\right]^{\alpha_F - 1}; \tag{42}$$

$$
\begin{aligned}
Q_{HF,t+1} \quad : \quad & \beta_F E_t \frac{1}{RER_{t+1}} \lambda_{F,t+1} \left\{ \alpha_F p_{XF,t+1} A_F \left( \frac{k_{F,t+1}}{l_{F,t+1}} \right)^{\alpha_F - 1} - (\delta_F + R_{QH,t+1}) \right\} \\
= \quad & \lambda_{F,t} \left( \frac{1}{RER_{t+1}} - \frac{1}{RER_t} \right) + \lambda_{F,t} \frac{1}{RER_t} \chi_{QHF} \left( \frac{Q_{HF,t+1}}{RER_t} \right). \qquad (43)
\end{aligned}
$$

Next, by denoting the marginal product with respect to physical capital as $r_{F,t} = p_{XF,t} \alpha_F A_F e^{z_{F,t}} \left[ \frac{k_{F,t}}{l_{F,t}} \right]^{\alpha_F - 1}$, the full set of equilibrium conditions and constraints of the intermediate goods producer in the Home country are:

$$r_{F,t} = R_{QF,t} + \delta_F; \qquad (44)$$

$$X_{F,t} = A_F e^{z_{F,t}} (k_{F,t})^{\alpha_F} (l_{F,t})^{1-\alpha_F}; \qquad (45)$$

$$X_{F,t} = X_{FF,t} + X_{FH,t}; \qquad (46)$$

$$i_{F,t} = k_{F,t+1} - (1 - \delta_F) k_{F,t}; \qquad (47)$$

$$k_{F,t} = Q_{FF,t} + \frac{Q_{HF,t}}{RER_t}; \qquad (48)$$

$$w_{F,t} = p_{XF,t} (1 - \alpha_F) A_F e^{z_{F,t}} \left[ \frac{k_{F,t}}{l_{F,t}} \right]^{\alpha_F}; \qquad (49)$$

$$r_{F,t} = p_{XF,t} \alpha_F A_F e^{z_{F,t}} \left[ \frac{k_{F,t}}{l_{F,t}} \right]^{\alpha_F - 1}; \qquad (50)$$

$$
\begin{aligned}
& 1 + \beta_F E_t \left\{ \frac{RER_t}{RER_{t+1}} \left( \frac{C_{F,t}}{C_{F,t+1}} \right)^{\theta_F} \left( \frac{x_{F,t+1}}{x_{F,t}} \right)^{a_F(1-\theta_F)} [R_{QF,t+1} - R_{QH,t+1}] \right\} \\
= \quad & E_t \left\{ \frac{RER_t}{RER_{t+1}} + \chi_{QHF} \left( \frac{Q_{HF,t+1}}{RER_t} \right) \right\}; \qquad (51)
\end{aligned}
$$

where equation (44) is obtained by using the first order condition in (42) and using the definition of $r_{F,t}$. It equates the gross interest on domestic loans in the foreign country with the gross real return on physical capital net of depreciation. Furthermore, equation (45) is the Cobb-Douglas production technology of the intermediate goods producer; equation (46) equates the sum of domestically and internationally sold intermediate goods to the total produced; equation (47) is the physical capital law of motion; and equation (48) is the financing constraint that the firm faces. Equations (49) and (50) equate the marginal products to the real wage and the real return to physical capital. Lastly, by combining the first order conditions in (42) and (43) one obtains the home and foreign loan uncovered interest parity condition in equation (51).

## 1.5 The Home Country Final Good Producer

Nontradable final gooconsumption goods in the Home country are denoted by $Y_{H,t}$. Given parameteres $\gamma_H \in (0,1)$ and $\eta_H > 0$, these goods are competitively produced using domestic and foreign intermediate goods according to the following CES technology:

$$Y_{H,t} = \left[\gamma_H^{1/\eta_H} \left(X_{HH,t}\right)^{(\eta_H-1)/\eta_H} + (1-\gamma_H)^{1/\eta_H} \left(X_{FH,t}\right)^{(\eta_H-1)/\eta_H}\right]^{\gamma_H/(\eta_H-1)}.$$

The final goods producer then minimizes its expenditures subject to the production technology

$$\min_{X_{HH,t},\ X_{FH,t}} P_{XH,t}X_{HH,t} + P_{XF,t}X_{FH,t}.$$

The optimal demand allocation across domestic and imported intermediate goods for a given level of $Y_{H,t}$ gives the demand equations:

$$X_{HH,t} = \gamma_H \left(\frac{P_{XH,t}}{P_{CH,t}}\right)^{-\eta_H} Y_{H,t} \tag{52}$$

$$X_{FH,t} = (1-\gamma_H) \left(\frac{P_{XF,t}}{P_{CH,t}}\right)^{-\eta_H} Y_{H,t} \tag{53}$$

From the demands functions and the budget constraint of the competitive final goods producer, $P_{CH,t}Y_{H,t} = P_{XH,t}X_{HH,t} + P_{XF,t}X_{FH,t}$, which follows from the zero profit condition, the price of the home consumption good, $P_{CH,t}$, is given as follows:

$$P_{CH,t} = \left[\gamma_H (P_{XH,t})^{1-\eta_H} + (1-\gamma_H)(P_{XF,t})^{1-\eta_H}\right]^{\frac{1}{1-\eta_H}}. \tag{54}$$

## 1.6 The Foreign Country Final Good Producer

Nontradable final gooconsumption goods in the Foreign country are denoted by $Y_{F,t}$. Given parameteres $\gamma_F \in (0,1)$ and $\eta_F > 0$, these goods are competitively produced using domestic and foreign intermediate goods according to the following CES technology:

$$Y_{F,t} = \left[\gamma_F^{1/\eta_F} \left(X_{FF,t}\right)^{(\eta_F-1)/\eta_F} + (1-\gamma_F)^{1/\eta_F} \left(X_{HF,t}\right)^{(\eta_F-1)/\eta_F}\right]^{\gamma_F/(\eta_F-1)}.$$

The final goods producer then minimizes its expenditures subject to the production technology

$$\min_{X_{FF,t},\ X_{HF,t}} P_{XF,t}X_{FF,t} + P_{XH,t}X_{HF,t}.$$

The optimal demand allocation across domestic and imported intermediate goods for a given level of $Y_{F,t}$ gives the demand equations:

$$X_{FF,t} = \gamma_F \left( \frac{P_{XF,t}}{P_{CF,t}} \right)^{-\eta_F} Y_{F,t} \tag{55}$$

$$X_{HF,t} = (1-\gamma_F) \left( \frac{P_{XH,t}}{P_{CF,t}} \right)^{-\eta_F} Y_{F,t} \tag{56}$$

From the demands functions and the budget constraint of the competitive final goods producer, $P_{CF,t}Y_{F,t} = P_{XF,t}X_{FF,t}+P_{XH,t}X_{HF,t}$, which follows from the zero profit condition, the price of the foreign consumption good, $P_{CF,t}$, is given as follows:

$$P_{CF,t} = \left[\gamma_F (P_{XF,t})^{1-\eta_F} + (1-\gamma_F)(P_{XH,t})^{1-\eta_F}\right]^{\frac{1}{1-\eta_F}} . \tag{57}$$

## 1.7 Trade Openness

The mass of world population is normalized to unity. The Home country household then lies on the interval $(0,\, n)$, where $0 < n < 1$; the Foreign household on the interval $(n,\, 1)$. Then following De Paoli (2009), the parameter determining the share of foreign goods in final consumption good production in the Home country, $(1-\gamma_H)$, is assumed to be a function of the relative size of the foreign economy, $1-n$, and of the degree of openness, $\upsilon$, more specifically, $1-\gamma_H = (1-n)\upsilon$. Similarly, for the foreign final goods producer, $\gamma_F = n\upsilon$.

## 1.8 The Home Country Financial Intermediary

For the Home country financial intermediary consider the profit function, $\Pi_{QH,t}$, in footnote *10* with $i = H$ and applying the calibration assumption that no adjustment cost is paid for changing domestic asset positions, i.e. $\chi_{BHH} = 0$, to get

$$\begin{aligned} \Pi_{QH,t} = \; & -P_{CH,t}Q_{HH,t+1} - P_{CF,t}Q_{HF,t+1} + (1+R_{QH,t})\left[P_{CH,t}Q_{HH,t} + P_{CF,t}Q_{HF,t}\right] \\ & +P_{CH,t}D_{HH,t+1} + P_{CF,t}D_{FH,t+1} - (1+R_{DH,t})\left[P_{CH,t}D_{HH,t} + P_{CF,t}D_{FH,t}\right] \\ & -P_{CH,t}w_{H,t}l_{QH,t} - P_{CH,t}B_{HH,t+1} - P_{CF,t}B_{FH,t+1} + (1+R_{H,t})P_{CH,t}B_{HH,t} \\ & +(1+R_{F,t})P_{CF,t}B_{FH,t} - P_{CH,t}\frac{\chi_{BFH}}{2}\left(\frac{P_{CF,t}}{P_{CH,t}}B_{FH,t+1}\right)^2 . \end{aligned} \tag{58}$$

Normalizing the profit constraint with the domestic consumption good price, $P_{CH,t}$, using the definition of the real exchange rate, $RER_t$, yields

$$
\begin{aligned}
\Pi_{QH,t} \quad = \quad & -Q_{HH,t+1} - RER_t Q_{HF,t+1} + (1+R_{QH,t})\left[Q_{HH,t} + RER_t Q_{HF,t}\right] \\
& + D_{HH,t+1} + RER_t D_{FH,t+1} - (1+R_{DH,t})\left[D_{HH,t} + RER_t D_{FH,t}\right] \\
& - w_{H,t} l_{QH,t} - B_{HH,t+1} - RER_t B_{FH,t+1} + (1+R_{H,t}) B_{HH,t} \\
& + (1+R_{F,t}) RER_t B_{FH,t} - \frac{\chi_{BFH}}{2}\left(RER_t B_{FH,t+1}\right)^2 . \qquad (59)
\end{aligned}
$$

The Home country's bank faces the following balance sheet constraint:

$$
Q_{HH,t} + RER_t Q_{HF,t} + B_{HH,t} + RER_t B_{FH,t} = D_{HH,t} + RER_t D_{FH,t}. \qquad (60)
$$

In this case unlike in Gillman (2011) deposits are not equal to loans, which allows for banks to have reserves held in the form of domestic or foreign bonds. Here, total deposits can be used for loan production, $D^{pr}_{H,t} = D_{HH,t} + RER_t D_{FH,t} - B_{HH,t} - RER_t B_{FH,t}$, or having reserves in the form of bonds. Then it follows that deposits used in loan production equal loans, i.e. $D^{pr}_{H,t} = Q_{HH,t} + RER_t Q_{HF,t}$.

The production of loans by the home country bank is subject to the financial intermediation CRS technology that requires labor from the representative consumer in the home country, $l_{QH,t}$, and deposits allocated to loan production, $D^{pr}_{H,t}$. With $A_{HQ} > 0$ representing the home banking sector's steady state productivity level and $\kappa_H \in (0,1)$, the total loans issued by the home country bank is

$$
Q_{H,t} = Q_{HH,t} + RER_t Q_{HF,t} = A_{QH} e^{z^Q_{H,t}} (l_{QH,t})^{\kappa_H} (D^{pr}_{H,t})^{1-\kappa_H}. \qquad (61)
$$

Then the home financial intermediary maximizes (59) subject to (60) and (61). Given that $\phi_{H,t}$ is the Lagrange multiplier associated with the loan production technology and $\theta_{H,t}$ is the multplier associated witht the balance sheet constraint, the resulting first order conditions are:

$$
l_{QH,t}: \quad w_{H,t} = \phi_{H,t} \kappa_H A_{QH} e^{z^Q_{H,t}} \left[\frac{l_{QH,t}}{D^{pr}_{H,t}}\right]^{\kappa_H - 1}; \qquad (62)
$$

$$
D_{HH,t+1}: \quad \beta_H E_t \frac{\lambda_{H,t+1}}{\lambda_{H,t}} \left\{ \begin{array}{c} (1+R_{DH,t+1}) \\ -\phi_{H,t+1}(1-\kappa_H) A_{QH} e^{z^Q_{H,t+1}} \left[\frac{l_{QH,t}}{D^{pr}_{H,t}}\right]^{\kappa_H} \\ -\theta_{H,t+1} \end{array} \right\} = 1; \qquad (63)
$$

$$
D_{FH,t+1}: \quad \beta_H E_t \frac{\lambda_{H,t+1} RER_{t+1}}{\lambda_{H,t} RER_t} \left\{ \begin{array}{c} (1+R_{DH,t+1}) \\ -\phi_{H,t+1}(1-\kappa_H) A_{QH} e^{z^Q_{H,t+1}} \left[\frac{l_{QH,t}}{D^{pr}_{H,t}}\right]^{\kappa_H} \\ -\theta_{H,t+1} \end{array} \right\} = 1; \qquad (64)
$$

$$Q_{HH,t+1}: \quad \beta_H E_t \frac{\lambda_{H,t+1}}{\lambda_{H,t}} \left\{(1+R_{QH,t+1}) - \phi_{H,t+1} - \theta_{H,t+1}\right\} = 1; \tag{65}$$

$$Q_{HF,t+1}: \quad \beta_H E_t \frac{\lambda_{H,t+1} RER_{t+1}}{\lambda_{H,t} RER_t} \left\{(1+R_{QH,t+1}) - \phi_{Ht+1} - \theta_{H,t+1}\right\} = 1; \tag{66}$$

$$B_{HH,t+1}: \quad \beta_H E_t \frac{\lambda_{H,t+1}}{\lambda_{H,t}} \left\{ \begin{array}{c} (1+R_{H,t+1}) \\ -\phi_{Ht+1}(1-\kappa_H) A_{QH} e^{z^Q_{H,t+1}} \left[\frac{l_{QH,t}}{D^{pr}_{H,t}}\right]^{\kappa_H} \\ -\theta_{H,t+1} \end{array} \right\} = 1. \tag{67}$$

$$\begin{aligned} B_{FH,t+1} \quad &: \quad \beta_H E_t \frac{\lambda_{H,t+1} RER_{t+1}}{\lambda_{H,t} RER_t} \left\{ \begin{array}{c} (1+R_{F,t+1}) \\ -\phi_{Ht+1}(1-\kappa_H) A_{QH} e^{z^Q_{H,t+1}} \left[\frac{l_{QH,t}}{D^{pr}_{H,t}}\right]^{\kappa_H} \\ -\theta_{H,t+1} \end{array} \right\} \\ &= \quad E_t \left\{1 + \chi_{BFH}(RER_t B_{FH,t+1})\right\}. \end{aligned} \tag{68}$$

Then the full set of equilibrium conditions and constraints of the financial intermediary in the Home country are:

$$Q_{HH,t} + RER_t Q_{HF,t} + B_{HH,t} + RER_t B_{FH,t} = D_{H,t} = D_{HH,t} + RER_t D_{FH,t}; \tag{69}$$

$$Q_{H,t} = A_{HQ} e^{z^Q_{H,t}} (l_{QH,t})^{\kappa_H} (D^{pr}_{H,t})^{1-\kappa_H}; \tag{70}$$

$$Q_{H,t} = Q_{HH,t} + RER_t Q_{HF,t}; \tag{71}$$

$$D^{pr}_{H,t} = D_{HH,t} + RER_t D_{FH,t} - B_{HH,t} - RER_t B_{FH,t}; \tag{72}$$

$$R_{QH,t} - R_{DH,t} = \frac{w_{H,t} l_{QH,t}}{Q_{H,t}}; \tag{73}$$

$$R_{DH,t} = R_{H,t}; \tag{74}$$

$$\begin{aligned} & E_t \left\{ \frac{RER_{t+1}}{RER_t} + \beta_F \left(\frac{C_{H,t}}{C_{H,t+1}}\right)^{\theta_H} \left(\frac{x_{H,t+1}}{x_{H,t}}\right)^{a_H(1-\theta_H)} \frac{RER_{t+1}}{RER_t} [R_{F,t+1} - R_{H,t+1}] \right\} \\ = \quad & E_t \left\{1 + \chi_{BFH}(RER_t B_{FH,t+1})\right\}. \end{aligned} \tag{75}$$

Equation (69) is the balance sheet constraint of the Home country bank; (70) is loan production technology; (71) is the definition of total Home country bank issued loans; and (72) is the definition of deposits used in credit production. Equation (73) is the equilibrium condition that defines the loan interest spread in the Home country, where the spread is driven by the labor cost of producing a unit of loan. Equation (74) is a no arbitrage condition equating the rate on bonds to the rate on deposits. Lastly, equation (75) is the expectational uncovered interest parity equation.

## 1.9 The Foreign Country Financial Intermediary

For the Foreign country financial intermediary consider the profit function, $\Pi_{QF,t}$, in footnote *10* with $i = F$ and applying the calibration assumption that no adjustment cost is paid for changing domestic asset positions, i.e. $\chi_{BHH} = 0$, to get

$$\begin{aligned}\Pi_{QF,t} = {} & -P_{CF,t}Q_{FF,t+1} - P_{CH,t}Q_{FH,t+1} + (1+R_{QF,t})\left[P_{CF,t}Q_{FF,t} + P_{CH,t}Q_{FH,t}\right] \\ & + P_{CF,t}D_{FF,t+1} + P_{CH,t}D_{HF,t+1} - (1+R_{DF,t})\left[P_{CF,t}D_{FF,t} + P_{CH,t}D_{HF,t}\right] \\ & - P_{CF,t}w_{F,t}l_{QF,t} - P_{CF,t}B_{FF,t+1} - P_{CH,t}B_{HF,t+1} + (1+R_{F,t})P_{CF,t}B_{FF,t} \\ & + (1+R_{H,t})P_{CH,t}B_{HF,t} - P_{CF,t}\frac{\chi_{BHF}}{2}\left(\frac{P_{CH,t}}{P_{CF,t}}B_{HF,t+1}\right)^2. \end{aligned} \tag{76}$$

Normalizing the profit constraint with the domestic consumption good price, $P_{CF,t}$, using the definition of the real exchange rate, $RER_t$, yields

$$\begin{aligned}\Pi_{QF,t} = {} & -Q_{FF,t+1} - \frac{Q_{FH,t+1}}{RER_t} + (1+R_{QF,t})\left[Q_{FF,t} + \frac{Q_{FH,t}}{RER_t}\right] \\ & + D_{FF,t+1} + \frac{D_{HF,t+1}}{RER_t} - (1+R_{DF,t})\left[D_{FF,t} + \frac{D_{HF,t}}{RER_t}\right] \\ & - w_{F,t}l_{QF,t} - B_{FF,t+1} - \frac{B_{HF,t+1}}{RER_t} + (1+R_{F,t})B_{FF,t} \\ & + \frac{(1+R_{H,t})B_{HF,t}}{RER_t} - \frac{\chi_{BHF}}{2}\left(\frac{B_{HF,t+1}}{RER_t}\right)^2. \end{aligned} \tag{77}$$

The Foreign country's bank faces the following balance sheet and productive deposits constraints:

$$Q_{FF,t} + \frac{Q_{FH,t}}{RER_t} + B_{FF,t} + \frac{B_{HF,t}}{RER_t} = D_{FF,t} + \frac{D_{HF,t}}{RER_t} \tag{78}$$

$$D_{F,t}^{pr} = D_{FF,t} + \frac{D_{HF,t}}{RER_t} - B_{FF,t} - \frac{B_{HF,t}}{RER_t} \tag{79}$$

The production of loans by the foreign country bank is subject to the financial intermediation CRS technology that requires labor from the representative consumer in the home country, $l_{QF,t}$, and deposits allocated to loan production, $D^{pr}_{F,t}$. With $A_{QF,t}$ representing the home banking sector's exogenous productivity process and $\kappa_F \in (0,1)$, the total loans issued by the home country bank is

$$Q_{F,t} = Q_{FF,t} + \frac{Q_{FH,t}}{RER_t} = A_{QF} e^{z^Q_{F,t+1}} (l_{QF,t})^{\kappa_F} (D^{pr}_{F,t})^{1-\kappa_F}. \tag{80}$$

Then the foreign financial intermediary maximizes (77) subject to (78) and (80). Given that $\phi_{F,t}$ is the Lagrange multiplier associated with the loan production technology and $\theta_{F,t}$ is the multplier associated witht the balance sheet constraint, the resulting first order conditions are:

$$l_{QF,t}: \quad w_{F,t} = \phi_{F,t} \kappa_F A_{QF} e^{z^Q_{F,t}} \left[ \frac{l_{QF,t}}{D^{pr}_{F,t}} \right]^{\kappa_F - 1}; \tag{81}$$

$$D_{FF,t+1}: \quad \beta_F E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \left\{ \begin{array}{c} (1 + R_{DF,t+1}) \\ -\phi_{F,t+1}(1-\kappa_F) A_{QF} e^{z^Q_{F,t+1}} \left[ \frac{l_{QF,t}}{D^{pr}_{F,t}} \right]^{\kappa_F} \\ -\theta_{F,t+1} \end{array} \right\} = 1; \tag{82}$$

$$D_{HF,t+1}: \quad \beta_F E_t \frac{\lambda_{F,t+1} RER_t}{\lambda_{F,t} RER_{t+1}} \left\{ \begin{array}{c} (1 + R_{DF,t+1}) \\ -\phi_{F,t+1}(1-\kappa_F) A_{QF} e^{z^Q_{F,t+1}} \left[ \frac{l_{QF,t}}{D^{pr}_{F,t}} \right]^{\kappa_F} \\ -\theta_{F,t+1} \end{array} \right\} = 1; \tag{83}$$

$$Q_{FF,t+1}: \quad \beta_F E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \{ (1 + R_{QF,t+1}) - \phi_{F,t+1} - \theta_{F,t+1} \} = 1; \tag{84}$$

$$Q_{FH,t+1}: \quad \beta_F E_t \frac{\lambda_{F,t+1} RER_t}{\lambda_{F,t} RER_{t+1}} \{ (1 + R_{QF,t+1}) - \phi_{Ft+1} - \theta_{F,t+1} \} = 1; \tag{85}$$

$$B_{FF,t+1}: \quad \beta_F E_t \frac{\lambda_{F,t+1}}{\lambda_{F,t}} \left\{ \begin{array}{c} (1 + R_{F,t+1}) \\ -\phi_{Ft+1}(1-\kappa_F) A_{QF} e^{z^Q_{F,t+1}} \left[ \frac{l_{QF,t}}{D^{pr}_{F,t}} \right]^{\kappa_F} \\ -\theta_{F,t+1} \end{array} \right\} = 1. \tag{86}$$

$$\begin{aligned} B_{HF,t+1} \quad &: \quad \beta_F E_t \frac{\lambda_{F,t+1} RER_t}{\lambda_{F,t} RER_{t+1}} \left\{ (1 + R_{H,t+1}) - \phi_{Ft+1}(1-\kappa_F) A_{QF} e^{z^Q_{F,t+1}} \left[ \frac{l_{QF,t}}{D^{pr}_{F,t}} \right]^{\kappa_F} - \theta_{F,t+1} \right\} \\ &= \quad 1 + \chi_{BHF} \frac{B_{HF,t+1}}{RER_t}. \end{aligned} \tag{87}$$

Then the full set of equilibrium conditions and constraints of the financial intermediary in the Foreign country are:

$$Q_{FF,t} + \frac{Q_{FH,t}}{RER_t} + B_{FF,t} + \frac{B_{HF,t}}{RER_t} = D_{FF,t} + \frac{D_{HF,t}}{RER_t}; \tag{88}$$

$$Q_{F,t} = A_{QF} e^{z^Q_{F,t}} (l_{QF,t})^{\kappa_F} (D^{pr}_{F,t})^{1-\kappa_F}; \tag{89}$$

$$Q_{F,t} = Q_{FF,t} + \frac{Q_{FH,t}}{RER_t}; \tag{90}$$

$$D^{pr}_{F,t} = D_{FF,t} + \frac{D_{HF,t}}{RER_t} - B_{FF,t} - \frac{B_{HF,t}}{RER_t}; \tag{91}$$

$$R_{QF,t} - R_{DF,t} = \frac{w_{F,t} l_{QF,t}}{Q_{F,t}}; \tag{92}$$

$$R_{DF,t} = R_{F,t}; \tag{93}$$

$$\begin{aligned} & E_t \left\{ \frac{RER_t}{RER_{t+1}} + \beta_F \left( \frac{C_{F,t}}{C_{F,t+1}} \right)^{\theta_F} \left( \frac{x_{F,t+1}}{x_{F,t}} \right)^{a_F(1-\theta_F)} \frac{RER_t}{RER_{t+1}} [R_{H,t+1} - R_{F,t+1}] \right\} \\ = \; & E_t \left\{ 1 + \chi_{BFH} \frac{B_{HF,t}}{RER_t} \right\}. \end{aligned} \tag{94}$$

Equation (88) is the balance sheet constraint of the Foreign country bank; (89) is loan production technology; (90) is the definition of total Foreign country bank issued loans; and (91) is the definition of deposits used in credit production. Equation (92) is the equilibrium condition that defines the loan interest spread in the Foreign country, where the spread is driven by the labor cost of producing a unit of loan. Equation (93) is a no arbitrage condition equating the rate on bonds to the rate on deposits. Lastly, equation (94) is the expectational uncovered interest parity equation.

### 1.10 The Home Country Government

The government in the Home country makes purchases, $P_{CH,t}G_{H,t}$, which are a constant share of output, $G_{H,t} = \left(\gamma_{H,gov}\right) e^{z^{gov}_{H,t}} Y_{H,t},$, and which are financed solely by issuing real bonds that can be traded domestically and internationally. Then the government budget constraint is as follows:

$$\begin{aligned} P_{CH,t}G_{H,t} \;=\; & P_{CH,t}B_{HH,t+1} + P_{CF,t}B_{HF,t+1} \\ & -(1+R_{H,t}) \left[ P_{CH,t}B_{HH,t} + P_{CF,t}B_{HF,t} \right]. \end{aligned} \tag{95}$$

Normalizing the government budget constraint with the domestic consumption good price, $P_{CH,t}$, and using the definition of the real exchange rate, $RER_t$, yields:

$$G_{H,t} = B_{HH,t+1} + RER_t B_{HF,t+1} - (1 + R_{H,t}) \left[ B_{HH,t} + RER_t B_{HF,t} \right]. \quad (96)$$

## 1.11 The Foreign Country Government

The government in the Foreign country makes purchases, $P_{CF,t} G_{F,t}$, which are a constant share of output, $G_{F,t} = \left( \gamma_{F,gov} \right) e^{z_{F,t}^{gov}} Y_{F,t,}$, and which are financed solely by issuing real bonds that can be traded domestically and internationally. Then the government budget constraint is as follows:

$$\begin{aligned} P_{CF,t} G_{F,t} &= P_{CF,t} B_{FF,t+1} + P_{CH,t} B_{FH,t+1} \\ &\quad - (1 + R_{F,t}) \left[ P_{CF,t} B_{FF,t} + P_{CH,t} B_{FH,t} \right]. \end{aligned} \quad (97)$$

Normalizing the government budget constraint with the domestic consumption good price, $P_{CF,t}$, and using the definition of the real exchange rate, $RER_t$, yields:

$$G_{F,t} = B_{FF,t+1} + \frac{B_{FH,t+1}}{RER_t} - (1 + R_{F,t}) \left[ B_{FF,t} + \frac{B_{FH,t}}{RER_t} \right].$$