Income Tax Evasion: Tax Elasticity, Welfare, and Revenue

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Abstract

This paper provides a general equilibrium model of income tax evasion. As functions of the share of income reported, the paper contributes an analytic derivation of the tax elasticity of taxable income, the welfare cost of the tax, and government revenue as a percent of output. It shows how an increase in the tax rate causes the tax elasticity and welfare cost to increase in magnitude by more than with zero evasion. Keeping constant the ratio of income tax revenue to output, as shown to be consistent with certain US evidence, a rising productivity of the goods sector induces less evasion and thereby allows tax rate reduction. The paper derives conditions for a stable share of income tax revenue in output with dependence upon the tax elasticity of reporting income. Examples are provided with less and more productive economies in terms of the tax elasticity of reported income, the welfare cost of taxation and the tax revenue as a percent of output, with sensitivity analysis with respect to leisure preference and goods productivity. Discussion focuses on how the tax evasion analysis may help explain such fiscal tax policy as the postwar US income tax rate reductions with discussion of tax acts and government fiscal multipliers. Fiscal policy with tax evasion included shows how tax rate reduction induces less tax evasion, a lower welfare cost of taxation, and makes for a stable income tax share of output.

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1 Introduction

The paper models tax evasion in a general equilibrium, representative agent, optimizing framework. Using closed form solutions, it derives the elasticity of labor income to a flat tax, the welfare cost of the labor income tax, and the effect of productivity and tax rate changes on tax revenue. The paper uses a certainty-equivalence model of tax evasion as an optimal choice of the share of earned income to report for tax purposes.[1] It contributes how the tax elasticity and welfare measures are based on structural utility, technology, and policy parameters rather than being behavioral, immutable, parameters. It is shown how rising goods sector productivity causes less tax elasticity, tax evasion, and welfare loss, and how falling income tax rates act similarly.[2]

The paper contributes analytic conditions for how income tax rate reductions can keep income tax revenue as a share of output constant over time. Given a trend up in goods sector productivity, the tax rate can trend down while keeping income tax revenue a constant share of output, as dependent upon one aspect: The tax elasticity of the tax revenue as a share of income. The higher is the elasticity in magnitude, the faster the tax rate can fall; as the tax elasticity magnitude falls over time due to the productivity rise, the rate of decrease in the income tax rate becomes smaller, while keeping the revenue share constant. This fleshes out how the nature of the tax elasticity provides a rationale for the continually lower tax rates experienced worldwide.

Given evidence of tax evasion through banks, the paper uses the microeconomics of the banking literature to present a model of tax evasion through the intermediation of the service using that banking production technology.[3] This technology is of standard form (Cobb-Douglas) and can be interpreted more broadly as supplying tax avoidance, such as tax expenditures and shelters, as well as evasion.[4] Here the probability of getting caught and fined is subsumed into a certainty equivalent parameterization of the productivity factor of the intermediation sector that supplies the tax evasion and/or

[1] See for example this decision of reporting income in Allingham and Sandmo (1972); see Hansen and Sargent (2005) on certainty equivalence; and see Ehrlich (1973, 1996), Becker 1968, and Klepper and Nagen (1989) on penalties and enforcement.

[2] This is the main point of Kopczuk (2005).

[3] For example, Alstadsæter et al. (2019a) present evidence on the amount of income hidden from taxation internationally, within banks, on the basis of wealth held in previously undisclosed bank accounts.

[4] See for example Weisbach (2003) on the distinction between tax avoidance and evasion, with for example tax expenditures being legal but evasive, so that a hard line between these actiivties is tenous.

avoidance service.[5]

The paper shows how tax evasion is optimally determined by setting the marginal cost of the evasion, which is the tax rate, equal to the marginal cost of avoidance through the intermediation sector. As enabled by the sectoral production approach, this marginal cost, in price-theoretic fashion, equals the marginal factor cost of intermediation labor divided by the marginal factor product of the labor. As the tax rate rises, the magnitude of the tax elasticity of taxable income rises. Tax evasion causes the magnitude of the tax rate elasticity of the reported income to be higher than in the economy without tax evasion for tax rates up to a moderately high threshold value of the tax rate. The role of tax evasion in raising the elasticity magnitude is supported by a literature finding it difficult to explain responses to tax reform through standard labor elasticities alone without considering evasion (Saez et al., 2012; Mertens and Ravn, 2014). Given the traditional relation between the magnitudes of the tax elasticity and of the welfare cost of the tax, the paper also shows how the welfare cost of the tax rises with the tax rate and how the welfare cost relates to the tax elasticity magnitude once evasion is allowed.

Certain evidence suggests developed countries tend to have less tax evasion than developing countries, that developed countries such as the US post-WWII have lowered their tax rates sequentially over time, and that developing countries rely less on income taxes as a source of government revenue while increasing such reliance on income taxes as they develop. This paper can explain these phenomena jointly through the effect of goods sector productivity on tax evasion. As this productivity trends upwards, tax evasion decreases and can drive such experience.

To illustrate the effect of productivity, an application is made with two example economies differing in productivity levels. It illustrates the effect of tax evasion on tax elasticities and welfare cost for low versus high productivity economies, typically characterized as less versus more developed countries (Lucas, 1990).[6] The less productive economy has more tax evasion, a higher tax elasticity of taxable income and a higher welfare cost of the tax up to a certain threshold tax rate. The less productive country

[5] On the financial intermediation approach to the production of such services using Cobb-Douglas technology, see for example Sealey and Lindley (1977), Clark (1984), Hancock (1985) and Degryse et al. (2009)

[6] See Lucas's (1990) explanation of the productivity differential between developed and less developed countries in terms of the wage rate.

also exhibits a lower Laffer curve with less tax revenue as a percent of income at each tax rate relative to the more productive economy.

Greater leisure preference causes more goods to leisure avoidance of the tax while less goods productivity causes more evasion and less goods relative to leisure. Sensitivity analysis illustrates the effect of these factors of leisure preference and goods sector productivity on the tax elasticity and welfare cost. Greater leisure preference and lower goods sector productivity each cause a higher tax elasticity in magnitude but have opposite effects on the welfare cost of the tax. When the differences in tax elasticity are due only to a lower goods productivity, it is shown that the higher tax elasticity corresponds also to a higher welfare cost, as in the Ramsey (1927) logic. A higher leisure preference causes a higher tax elasticity and a lower welfare cost, as an exception to Ramsey logic that may be important for public finance, for example to extensions to models of heterogeneous agents differing by both leisure preference and goods productivity.

The paper discusses developing country tax evidence, flat tax reform, government spending multipliers and US tax reform since the 1980s in terms of how tax evasion may factor into consideration of tax base broadening and tax rate reduction. Given rising productivity over time, the paper's analysis suggests a scenario by which future tax law would see falling income tax rates, increasingly less tax evasion, and possibly stable tax revenue as a share of output, such as in the US experience of a 7.7% average from 1947-2018 (Fig.1, Sect.3).

2 Related Literature

Kopczuk (2005, p. 2094) focuses on base broadening, the tax elasticity and its role in welfare loss. "This elasticity is the key parameter necessary to evaluate the deadweight loss of the income tax" and "that it is not a structural parameter depending only on underlying preferences and technology, but instead it depends on a non-rate aspect of the tax system (tax base) that can be manipulated by policy makers." Here the paper lays out the tax elasticity and welfare in a flat tax framework, with base broadening resulting from decreasing the tax rate itself and from goods sector productivity advance that both decrease evasion, the tax elasticity magnitude and the welfare cost of the tax.

Weisbach (2003, p. 9) stresses the importance of a welfare cost approach to analyzing tax avoidance and evasion: "The task is to determine how to classify and treat the various responses to taxation based directly on the welfare consequences."[7] He emphasizes that avoidance and evasion are related in economic consequences, with the former legal and the latter illegal. "There is also nothing sacred about a division of the world between evasion and avoidance." Weisbach (p. 12) analyses the effect of tax shelters on the elasticity of taxable income while noting that "It is easy to imagine how the avoidance/evasion type regime we have arises from this analysis." This paper provides both the avoidance through what Weisbach (p.14, footnote 1) calls the "labor/leisure distortion of an income tax" and evasion through the bank intermediary. The entanglement of legal evasion through tax expenditures and illegal evasion is left unwound in this paper in which the intermediation production of evasion implicitly allows for taking advantage of tax expenditures or shelters. See also Weisbach (2002, 2006) on tax expenditures and shelters.

Saez et al. (2012) discuss how taxable income changes with tax rates, in the sense that with lower tax rates the income base can rise if there is a rising price elasticity of taxable income relative to the tax rate as the price. This paper builds upon the insight that for the tax base to rise significantly, as some evidence after the 1981 and 1986 tax acts suggests (Saez et al.), then tax evasion could be decreasing as the tax rate falls. This paper models such optimal competitive equilibrium tax evasion, resulting in a greater elasticity of taxed income relative to the tax rate and a lower amount of evasion when the tax rate decreases.

Sandmo (2005) reviews the literature on modeling tax evasion, including the approach of deciding how much income one is to report at the time that the decision needs to be made; this paper follows this concept by also deciding how much income to report. Sandmo further allows for a probability of detection of evasion, and possible penalties, a common feature of this literature (for example, Becker, 1968, and Ehrlich, 1973, 1996). Here, this paper considers the probability of detection and being penalized all in terms of resulting in some average, economy-wide, productivity of avoiding/evading the taxes. For example, in utilizing "tax expenditures" in a way that constitutes a type of avoidance

[7] Weisbach (2003, p. 9) expands on the welfare approach to avoidance and evasion: "To put the problem in a welfarist framework, we cannot assume pre-existing definitions of tax avoidance and evasion. Instead, we must determine which responses to taxation will be treated in various fashions based directly on the welfare consequences of such treatment".

behavior, there may be little uncertainty involved in the process of legally decreasing taxable income yet there can be significant cost due to setting up various legal and accounting extensions. For tax avoidance/evasion, the paper here stipulates through an intermediation production process some average cost of avoiding and evading taxes, taking into account all of the probabilistic outcomes. This is a "certainty-equivalence" approach used in macroeconomics, an abstraction by which these factors are embedded into the productivity parameter of the intermediation sector (Gillman and Kejak 2014).

Gale and Brown (2013) emphasize the importance of "broadening the income tax base". They offer ways to eliminate special tax deductions and consider how a value added tax works to raise taxes internationally. The paper here uses only a flat income tax to show base broadening through less tax evasion. While a myriad of tax expenditures are reviewed by Gale and Brown, this paper abstracts also from the complex structuring of taxable income, through legal, accounting and investment banking fees, as being a part of the intermediation costs of evasion that are modeled; these costs can be thought of as including both the legal tax expenditures and illegal tax evasion that limit the income tax base.

A broad literature on reaction to specific tax law exists. For example, Alstadsæter et al. (2019b) show how tax avoidance is affected through shelters that are enhanced by social networks, using a study of a 2006 Norwegian tax reform. US tax law has stimulated a broad literature focused on tax expenditures and incentives. Kopczuk (2005) analyzes the 1986 US tax act in terms of base broadening; Poterba (2011) reviews tax expenditures; and Slemrod (2018) also considers the base broadening aspects of recent tax legislation. Cummins et al. (1994) discuss the incentive effect of tax reform with a focus on investment and Auerback (2002) considers the effect of tax reform on revenues in a dynamic context. Studies also consider how changing the labor tax effects government revenue while including human capital; for example, Holter et al. (2019) examine US Laffer curves with a flat income tax. Piketty et al. (2018) cover a rich literature in estimating the effects of income level on effective tax rates, although their "estimates of incomes at the top of the distribution are based on tax data, and hence disregard tax evasion" (p. 556). The paper here displays a Laffer curve as a result of evasion.

Section 3 examines some evidence on the US tax revenue to output ratio. Section 4 presents the model with a focus on the tax elasticity of income and the welfare cost of the tax. Section 5 provides examples with low and high productivity economies. Using the examples of Section 5, Sensitivity analysis is illustrated in Section 6 for the tax elasticity and welfare cost with respect to leisure preference and goods productivity. Discussion of the results is presented in Section 7, and the conclusion follows in Section 8.

3 Evidence on Income Taxes

For 1947 to 2019, Figure 1 graphs annual data of the ratio of US "personal taxes" to Gross Domestic Product (GDP).[8] It shows fluctuations around a stable trend with an average of 7.7%. During this time, top federal marginal income tax rate fell from over 90% in the 1950s to 37% in 2017, with compression of brackets and lower tax rates.

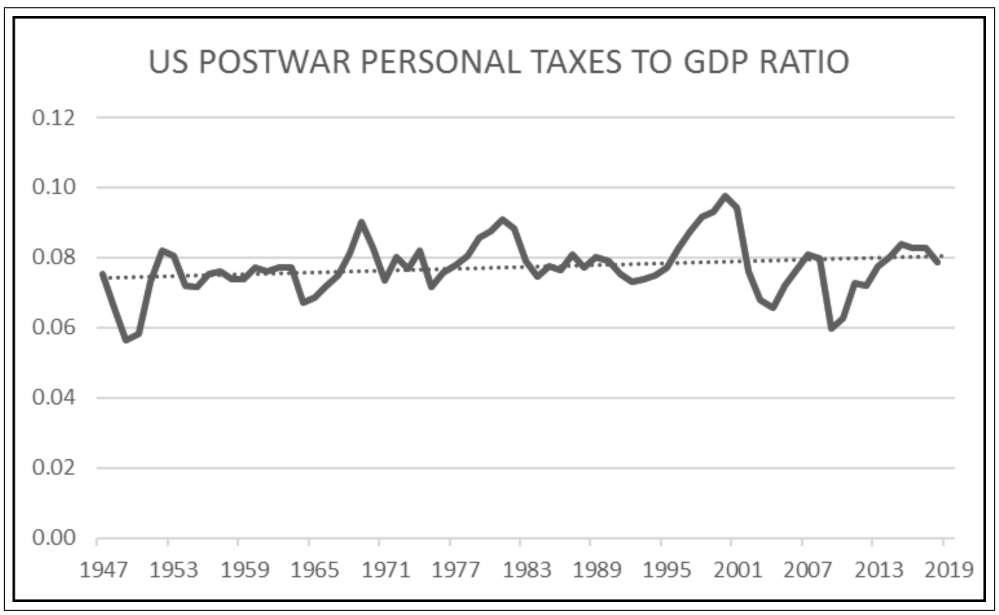

Figure 1. Stable around an average of 7.7%

International evidence, provided in World Bank tables, shows that from 1978 to 2017, the world tax revenue as a percent of GDP has fluctuated between 13% and

[8] U.S. Bureau of Economic Analysis (BEA), Federal government current tax receipts: Personal current taxes; from the Federal Reserve Bank of St. Louis; defined in BEA Table 3.4. as federal and state income taxes for 98% of the total reported in 2018; federal income taxes are 78% of that total.

15%.[9] The World Bank "taxes on income, profits and capital gains", from 1978 to 2017, has trended slightly upwards from 20% of government revenue to 23%.[10] Developing countries have seen a trend towards a rising share of income taxes. India, for example is one of the biggest, democratic, developing country examples; from 1974 to 2017, India's income taxes as a share of government revenue rose steadily from 21% to a peak of 50% in 2009, falling some after the 2008 crisis to 44%.[11] This is a characteristic of many developing countries, while developed countries have tended to follow the US with stable such shares.[12]

The model below provides an explanation of how tax evasion can explain a stable share of income taxes in GDP even as tax rates fall. It also explains how a rising share of tax revenue results with rising productivity, constant tax rates and less evasion.

4 A Model of Income Tax Evasion

Let the legal avoidance through "tax expenditures" be put aside formally with the focus strictly on tax evasion through the intermediation sector that supplies that service. Tax minimization efforts through accounting that take advantage of legal tax avoidance without effecting consumer behavior per se, except for the terms of paying for the service, might be thought of as being included in the following model of tax evasion.

With decentralized goods and bank production sectors, utility u at time t depends upon goods and leisure as denoted by c_t and x_t, respectively, in log utility fashion so that[13]

$$u\left(c_t, x_t\right) = \ln c_t + \alpha \ln x_t. \tag{1}$$

Let subscripts G and E denote goods and tax evasion production sectors. The representative consumer rents labor for an equilibrium price denoted by w_t to the goods and bank production sectors, with this labor denoted by l_{Gt} and l_{Et}, respectively, and the labor sum defined notationally as $l_t \equiv l_{Gt} + l_{Et}$. The labor and leisure sum to an endowment

[9] World Bank Development Indicators; World Bank ID: GC.TAX.TOTL.GD.ZS.
[10] World Bank Development Indicators: World Bank ID: GC.TAX.YPKG.RV.ZS.
[11] World Bank Development Indicators; World Bank ID: GC.TAX.YPKG.RV.ZS
[12] While stable since 1959, the US income tax share of GDP has decreased slightly since 1978.
[13] The model is a special, simplified, case of the Gillman and Kejak (2014) economy that includes both human and physical capital, with endogenous growth.

of 1 unit of time for each period t such that the allocation of time constraint is

$$1 = l_{Gt} + l_{Et} + x_t \equiv l_t + x_t. \tag{2}$$

The competitive output of goods denoted by y_t is produced as a linear function of labor l_{Gt}. With $A_G \in R_{++}$, output is

$$y_t = A_G l_{Gt}. \tag{3}$$

The goods producer maximizes profit such that the equilibrium price of labor is $w = A_G$.

The total labor income each period is deposited by the goods and bank producers directly into the consumer's bank deposit account, with these deposits denoted by d_{Et}. The consumer pays this intermediary for a service of hiding some fraction, denoted by $1 - a_{Et} \in [0, 1)$, of income so that this fraction of legally required income taxes is unpaid. The fraction of income $a_{Et} \in (0, 1]$ is reported. The fraction $1 - a_{Et}$ of income is successfully hidden without any probability of disclosure (such as in untraceable "offshore" or Swiss bank accounts).

Abstracting from exchange cost, the consumer purchases goods c_t (without exchange cost) using earned income, a government transfer Γ_t, a real endowment z, and profit from owning the bank that provides the income tax evasion. The government flat tax rate on income is τ; the total income earned is $w(l_{Gt} + l_{Et})$; and so the fraction of income reported to the government is $a_{Et} w (l_{Gt} + l_{Et})$. This makes the after-tax reported labor income equal to $a_{Et} w (l_{Gt} + l_{Et})(1 - \tau)$.

The consumer receives the income hidden by the bank, $(1 - a_{Et}) w (l_{Gt} + l_{Et})$, and pays the bank a competitively determined price, or fee, per unit of evaded income, denoted in real terms by p_{Et}. The total hidden income available to the consumer after paying the tax evasion service fee is $(1 - a_{Et}) w (l_{Gt} + l_{Et})(1 - p_{Et})$.

As owner of the intermediary bank by virtue of the deposited labor income d_{Et}, the consumer receives the residual dividends of the bank after the bank pays its labor cost. Denote the dividend yield per unit of deposited funds by r_{Et}, this being the producer surplus per unit of income processed. The total dividend income received by the consumer is $r_{Et} d_{Et}$ and assumed to be tax free.[14] While the total taxed income is $a_{Et} w (l_{Gt} + l_{Et})(1 - \tau)$, the total hidden reported income is the sum of the competitively

[14] The dividend income is assumed to be hidden by the bank; it can be made taxable which introduces a squared τ term that complicates the presentation of results while keeping them qualitatively the same.

priced hidden income after fees of $(1-a_{Et})\,w\,(l_{Gt}+l_{Et})\,(1-p_{Et})$ plus the producer surplus $r_{Et}d_{Et}$. In addition, the consumer receives the government transfer Γ_t and a fixed non-taxable endowment $z \geq 0$ of goods. The consumer's budget constraint is consumption c_t cannot exceed income:

$$a_{Et}w\,(l_{Gt}+l_{Et})\,(1-\tau)+(1-a_{Et})\,w\,(l_{Gt}+l_{Et})\,(1-p_{Et})+r_{Et}d_{Et}+\Gamma_t+z \geq c_t. \quad (4)$$

The consumer also faces the bank balance sheet constraint that the deposits in the bank d_E cannot exceed the earned wage income:

$$wl_t \geq d_{Et}. \quad (5)$$

The government returns all tax revenue through the lump sum transfer Γ_t so that only a substitution distortion of the tax τ is present in equilibrium. This makes the government budget constraint:

$$\Gamma_t = a_{Et}\tau w\,(l_{Gt}+l_{Et}) = a_{Et}\tau wl. \quad (6)$$

The consumer problem is to maximize the present discounted value of current utility in equation (1), subject to the time and income constraints, (2) and (4), plus the bank balance condition (5); the Appendix presents the problem and equilibrium conditions. Note that combining the resource constraints in equations (4) to (6), along with the bank production function of equation (8) below, and the bank zero profit condition in Appendix equation (38), gives back the resource constraint by which $c_t = wl_{Gt} + z$.

The quantity of real income evaded is denoted by q_{Et}. Tax evasion is produced using the inputs of labor time devoted to banking, l_{Et}, plus the bank labor income deposits, d_{Et}. The competitive intermediary problem is to maximize profit of revenue from the tax evasion service, $p_{Et}q_{Et}$, minus the cost of labor wl_{Et}, and minus payment of profit as dividend distributions to the consumer equal to $r_{Et}d_{Et}$, subject to the production technology. Given $\kappa \in [0,1)$, and $A_E \in R_+$ as a bank productivity parameter, the production function for intermediation is assumed to be Cobb-Douglas in form:

$$q_{Et} = A_E\,(l_{Et})^{\kappa}\,d_{Et}^{1-\kappa}. \quad (7)$$

Normalized by deposits, the share of income evading taxes, $1-a_E$, is defined by

$$1-a_{Et} \equiv \frac{q_{Et}}{d_{Et}} = A_E\left(\frac{l_{Et}}{d_{Et}}\right)^{\kappa} \leq 1. \quad (8)$$

This service for evasion of reported income, on a per unit of deposit basis, has a diminishing marginal product of labor per unit of deposits (l_{Et}/d_{Et}) which makes for an endogenously determined marginal cost of income evaded per unit of income ($\frac{q_{Et}}{d_{Et}}$) that is an upward-sloping convex function for $\kappa < 0.5$ as illustrated below in Figure 2b.[15]

Dropping time subscripts henceforth in describing the stationary equilibrium allocation, the consumer equilibrium condition in Appendix equation (34) implies that the tax rate equals the competitive price of the evasion service per unit of income that is evaded:

$$p_E = \tau. \tag{9}$$

From Appendix equations (32)-(37), the marginal rate of substitution between goods and leisure, denoted by $MRS_{c,x}$ is given by

$$MRS_{c,x} \equiv \frac{x}{\alpha c} = \frac{1}{w\left(1 - \tau + r_E\right)}. \tag{10}$$

The $MRS_{c,x}$ equation (10) shows how the deposit yield r_E (producer surplus per unit of income) decreases the effective tax rate on labor income.

From the bank side, the Appendix provides the bank problem and equilibrium conditions (38-40) which imply that the marginal cost of tax evasion per unit of deposits, p_{Et}, equals the marginal factor cost w divided by the marginal factor product:[16]

$$p_E = \tau = \frac{w}{\kappa A_E \left(\frac{l_E}{d_E}\right)^{(\kappa-1)}}. \tag{11}$$

Equation (11) shows a "price-theoretic" derivation of the optimal tax evasion, and provides directly the solution for the factor input ratio:

$$\frac{l_E}{d_E} = \left(\frac{\tau \kappa A_E}{w}\right)^{\frac{1}{1-\kappa}}. \tag{12}$$

Second, the dividend return per unit of income r_E is a fraction of the tax rate τ as given in equilibrium by

$$r_E = \tau\left(1 - \kappa\right)\left(1 - a_E\right), \tag{13}$$

[15] Berk and Green (2004) assume a similar but exogenous upward sloping intermediation cost function for mutual funds supply.

[16] See Gillman and Kejak (2005) for a parallel condition to equation (11) in a monetary generalization of the Baumol (1952) condition; the marginal cost of avoiding the inflation tax through banking services equals the marginal benefit which is the inflation tax rate itself.

which equivalently writes as $r_E = \tau - [a_E \tau + \tau\kappa (1 - a_E)]$. This per unit producer surplus r_E equals the tax rate minus the shadow tax cost per unit of goods that is described and graphed below (Fig. 2a, 2b).

To solve for the equilibrium fraction of income which evades taxes, from equation (8) solve for the factor input ratio $\frac{l_E}{d_E}$ as $\frac{l_E}{d_E} = \left(\frac{q_E}{d_E} \frac{1}{A_E} \right)^{\frac{1}{\kappa}}$, substitute this input ratio into the bank equilibrium condition (11), and solve for $1 - a_E$:

$$1 - a_E \equiv \frac{q_E}{d_E} = A_E \left(\frac{p_E \kappa A_E}{w} \right)^{\frac{\kappa}{1-\kappa}} < 1. \tag{14}$$

Using equation (9) to substitute τ for p_E, and equation (5) by which $wl = d_E$, the demand for the fraction of income which is reported, a_{Et}, is stationary and equal to

$$a_E = 1 - A_E \left(\frac{\tau \kappa A_E}{w} \right)^{\frac{\kappa}{1-\kappa}} \leq 1. \tag{15}$$

The demand for reporting income falls as the tax rate τ rises, as the cost of labor w falls, and as the productivity in the bank sector A_E rises. Tax revenue per unit of labor income is $a_E \tau$. The evasion degree $(1 - a_E)$ also determines the evasion cost, per unit of goods, $\tau\kappa (1 - a_E)$. Equations (5), (12), and (14) imply that this cost is the value of labor per unit of labor income used in producing evasion. This is given by

$$\frac{wl_E}{wl} = w \left(\frac{\tau \kappa A_E}{w} \right)^{\frac{1}{1-\kappa}} = \tau \kappa A_E \left(\frac{\tau \kappa A_E}{w} \right)^{\frac{\kappa}{1-\kappa}} = \tau\kappa (1 - a_E) . \tag{16}$$

To illustrate the demand for the share of labor income reported, a_E, and the shadow cost of the reported income, Figure 2a graphs equation (15) as solved for τ; Figure 2b graphs the implied marginal cost of the share of tax evasion, $1 - a_E$.[17] These assume $\kappa = 0.36$ as in Gillman and Kejak (2014), that $A_E = 1.038$ and $w = 1.41$ for a more productive economy (Black) and that $A_E = 0.53$ and $w = 0.094$ for a less productive economy (Blue) as in Examples 1 and 2 in Section 5 below. For a given tax rate, they show how the demand for reported income is shifted up in a more productive economy, with a higher fraction of income reported and a higher marginal cost of production.

[17] The marginal cost for Figure 2B can be solved for τ from $1 - a_E = \frac{q_E}{c} = A_E \left(\frac{\tau\kappa A_E}{w} \right)^{\frac{\kappa}{1-\kappa}}$, such that $\tau = \frac{w(1-a_E)^{\frac{1-\kappa}{\kappa}}}{\kappa (A_E)^{\frac{1}{1-\kappa}}}$; here $\frac{1-\kappa}{\kappa}$ is the power coefficient on the quantity $1 - a_E$, so that marginal cost rises at an increasing rate if $\kappa < 0.5$.

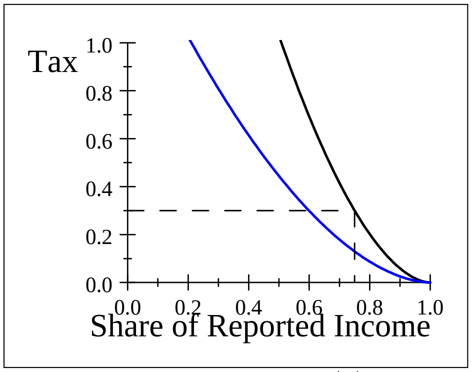

Fig. 2a: Demand, $a_E(\tau)$.

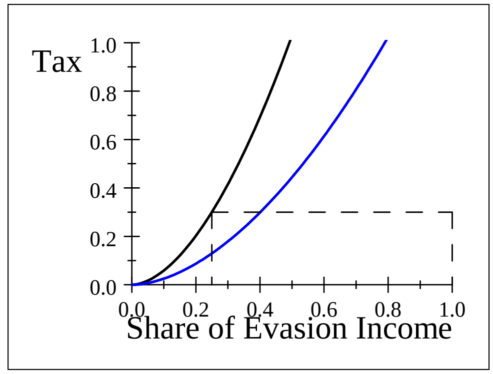

Fig. 2b: Marginal Cost, $1 - a_E$

Figures 2a and 2b show that the tax paid to the government per unit of income, $a_E\tau$, is given by the area of the dashed boxes. The second part of the cost paid by the consumer is the triangular area either under the demand for reported income in Figure 2a or equivalently under the marginal cost curve in Figure 2b. The former triangle is known as the lost consumer surplus for the reported income and the latter triangle as the cost of producing evasion per unit of income (integrating under the marginal cost curve). Since the marginal cost equals the tax rate τ, and the average cost is the fraction κ of the marginal cost, then $\tau\kappa$ is the average cost of evasion per unit of labor income and $1 - a_E$ is the amount of evasion per unit. This gives a total cost of $\tau\kappa(1 - a_E)$, per unit of income, which equals the integral sum of the marginal cost. This sum is also equal to the labor cost per unit of labor income evaded (as in equation 16); it also equals the lost consumer surplus under the demand function for reported income.

Taking the two areas together, the box and triangle under $\tau = 0.2$ in either Figure 2a or 2b, the total shadow tax cost equals the sum of the legal tax paid for reported income, $a_E\tau$, plus the average cost paid while evading the tax, $\tau\kappa(1 - a_E)$, per unit of labor income. The shadow cost of the tax per unit of income is therefore $a_E\tau + \tau\kappa(1 - a_E)$, which is a weighted average of the tax paid τ and the average cost with evasion, $\tau\kappa$, with the weights being the shares of taxes paid, a_E, and evaded, $1 - a_E$. Note that this cost of the tax is the effective tax rate $\tau - r_E$ and it can be rewritten using equation (13) to make clear that it is less than the actual tax as seen in the following:

$$\tau - r_E = a_E\tau + \kappa(1 - a_E)\tau = \tau[a_E(1 - \kappa) + \kappa] \leq \tau. \qquad (17)$$

The expression in equation (17) is useful for interpreting the $MRS_{c,x}$ of equation (10), which can be alternately written using equation (13) as

$$MRS_{c,x} = \frac{1}{w\left(1-\tau+r_E\right)} = \frac{1}{w\left(1-\tau\left[1-\left(1-\kappa\right)\left(1-a_E\right)\right]\right)} = \frac{1}{w\left(1-\tau\left[\kappa+a_E\left(1-\kappa\right)\right]\right)}. \tag{18}$$

The ratio of the marginal utilities of goods to leisure equals the ratio of the shadow prices of each. From equation (18), the shadow price of goods is 1 and the shadow price of leisure is the real wage minus the effective tax rate, or $w\left(1-\tau\left[\kappa+a_E\left(1-\kappa\right)\right]\right)$. Because the effective tax is less than the statutory tax, tax evasion decreases the goods to leisure avoidance channel. The effective tax rises with τ at a decreasing rate.

4.1 Tax Elasticity of Taxable Income

The elasticity of the government income tax revenue with respect to the tax rate is effected by the degree of reported income a_E. Define the taxable income (TI) by $a_E wl$, and denote the price elasticity of this reported income with respect to the tax rate as η_τ^{TI}. Since w is constant, this latter elasticity is the sum of the elasticity of a_E with respect to τ and the elasticity of labor l, with respect to τ.

Let $\eta_\tau^{a_E}$ denote the tax elasticity of the share of income reported, and η_τ^l denote the elasticity of labor supply to the tax rate such that $\eta_\tau^{TI} = \eta_\tau^{a_E} + \eta_\tau^l$. From equation (15), the share elasticity is

$$\eta_\tau^{a_E} = -\frac{\kappa}{1-\kappa}\frac{1-a_E}{a_E}. \tag{19}$$

Using the Appendix solution for labor (equation 45), it results that

$$\eta_\tau^l = -\frac{\alpha}{1+\alpha}\frac{a_E\tau\left[\left(1+\eta_\tau^{a_E}\right)\left(1-\tau\kappa\right)+\tau\kappa\right]}{\left\{1-\tau\left[\kappa+a_E\left(1-\kappa\right)\right]\right\}\left[1-\tau\kappa\left(1-a_E\right)-\frac{\tau a_E}{1+\alpha}\right]}, \tag{20}$$

and the income elasticity can be proved to be always negative for $\tau < 1$ and $a_E > \kappa$ which are weak restrictions.

$$\eta_\tau^{TI} = -\frac{\kappa}{1-\kappa}\frac{1-a_E}{a_E} - \frac{\alpha}{1+\alpha}\frac{a_E\tau\left[\left(1+\eta_\tau^{a_E}\right)\left(1-\tau\kappa\right)+\tau\kappa\right]}{\left\{1-\tau\left[\kappa+a_E\left(1-\kappa\right)\right]\right\}\left[1-\tau\kappa\left(1-a_E\right)-\frac{\tau a_E}{1+\alpha}\right]} < 1. \tag{21}$$

For the Example 1 application given below in Section (5.1), which is for a less productive economy with the share of reported income at $a_E = 0.60$ and an income tax rate of $\tau = 0.30$, the income elasticity to the tax rate is -0.465. For the second Example 2 below in Section (5.2), for a more productive economy with the share of reported income at $a_E = 0.75$ and an income tax rate of $\tau = 0.30$, the income elasticity to the tax rate is -0.333. For an economy with zero tax evasion, which results as the case when $A_E = 0$ and $a_E = 1$,

$$\eta_\tau^{TI}\big|_{A_E=0} = -\frac{\alpha}{(1+\alpha-\tau)}\frac{\tau}{(1-\tau)} < 1; \tag{22}$$

for both Examples 1 and 2 this income elasticity ($A_E = 0$) at $\tau = 0.30$ is -0.25.[18]

The income elasticity to the tax rate is lower than in both examples in which there is tax evasion. Consider that for the less productive economy of Example 1, the elasticity components are -0.376 from $\eta_\tau^{a_E}$ and -0.089 from η_τ^l. For the more productive economy of Example 2, the elasticity components are -0.187 from $\eta_\tau^{a_E}$ and -0.146 from η_τ^l. This illustrates how the tax evasion component $\eta_\tau^{a_E}$ can be dominant in the elasticity. It also shows a smaller labor elasticity magnitude when tax evasion exists since the burden is taken through reporting less income as well as substituting from goods to leisure. This is indicative as to why the -0.089 and -0.146 numbers for η_τ^l in the examples are less than when there is zero evasion and the labor elasticity is larger (in magnitude) at -0.25. In the latter case with $a_E = 1$, only substitution from goods to leisure enables tax avoidance and so the burden is greater on this aspect of optimization in response to taxation.

The converse of only goods to leisure substitution can be seen by letting leisure preference α fall to zero. At $\alpha = 0$ zero leisure is consumed and it results that $\eta_\tau^l = 0$. The only way to avoid the tax is through evasion; $\eta_\tau^{TI} = \eta_\tau^{a_E} = -\frac{\kappa}{1-\kappa}\frac{1-a_E}{a_E}$. As in Figure 5 below, up to a certain threshold tax rate this would yield an underestimate of the tax elasticity, similar to how setting $A_E = 0$ and forcing $a_E = 1$ also yields an underestimate relative to the full model.

The bulk of the income elasticity magnitude with tax evasion comes from the tax evasion component itself ($\eta_\tau^{a_E}$) which can be multi-fold the magnitude of the labor supply elasticity component (η_τ^l). This key result is consistent with conclusions reached by Saez et al. (2012) that the labor supply elasticity alone is insufficient to explain responses to 1980s US tax rate cuts and that instead tax evasion or avoidance must be also factoring into the magnitude of response to tax rate changes.

[18] For Examples 1 and 2, $\kappa = 0.36$ and $\alpha = 1$; for Example 1 $w = 0.094$ and $A_E = 0.53$, while for Example 2 productivities are higher at $w = 1.41$ and $A_E = 1.03$.

4.2 Welfare Cost of Income Tax Evasion

With a higher tax elasticity of taxable income, by Ramsey (1927) logic the welfare cost of taxation is higher for a given tax rate τ if there exists tax evasion means. Because of greater sensitivity to the tax through tax evasion and the use of scarce resources in producing tax evasion, the welfare cost of taxation is higher, up to a certain tax rate threshold, than without evasion. Consider the following derivation of the consumer's closed form solution for the welfare cost of the income tax.[19]

The feasible, efficient, welfare cost of income taxation can be found by deriving the value of the transfer z that makes utility at the optimum equal to that utility level when $\tau > 0$. With the * superscript indicating the equilibrium solutions of goods and leisure, as given in the Appendix, let the standard indirect utility as evaluated at the equilibrium be denoted by $u(c^*, x^*) \equiv v(\tau, z)$, which is convenient since both c^* and x^* depend upon τ and z and all other parameters are kept constant. The welfare cost of τ is determined by solving for z from the following:

$$v(\tau, z) = v(0,0). \tag{23}$$

The solution for z is the compensating asset transfer necessary to make the consumer indifferent to facing the tax rate τ. It is a feasible equilibrium in which both goods and leisure are allowed to adjust to the asset transfer such that the minimum z is found and is thereby efficient.[20]

The solution for z/w is then found from

$$e^{[v(\tau,z)-v(0,0)]} = 1; \tag{24}$$

using the Appendix solutions for c and x, equations (43) and (44), and equations (1) and (24), and given that $a_E = 1 - A_E \left(\frac{\tau\kappa A_E}{w}\right)^{\frac{\kappa}{1-\kappa}}$ from equation (15), z as normalized by full income w is given by the following.

$$\frac{z}{w} = \{1 - \tau[\kappa + a_E(1-\kappa)]\}^{-\frac{1}{1+\alpha}} \left[1 - \tau\kappa(1-a_E) - \frac{\tau a_E}{1+\alpha}\right] - [1 - \tau\kappa(1-a_E)], \tag{25}$$

The welfare cost z/w is a function of the effective after-tax earnings rate, $1 - \tau + r_E = 1 - \tau[\kappa + a_E(1-\kappa)]$, the term $1 - \tau\kappa(1-a_E) - \frac{\tau a_E}{1+\alpha} = 1 - \tau[\kappa + a_E(1-\kappa)] +$

[19] See also Feldstein (1999) on the welfare cost of income taxation.

[20] In related work, it is clarified how compensating only goods consumption is not a feasible equilibrium; see Gillman (2020a).

$\alpha \left[1 - \tau\kappa \left(1 - a_E\right)\right]$ that depends upon the effective tax rate $\tau \left[\kappa + a_E \left(1 - \kappa\right)\right]$ (equation 17) plus the effectively decreased leisure preference due to the permanent income decrease from time in banking, $\alpha \left[1 - \tau\kappa \left(1 - a_E\right)\right]$. The last term subtracts the time endowment of one less the share of labor time used in banking $l_E/l = \tau\kappa \left(1 - a_E\right)$ (equation 16).

As the tax evasion share $1 - a_E$ approaches one, the welfare cost approaches a limiting value.[21] Without evasion, when $A_E = 0$ and $a_E = 1$, the welfare cost of the tax τ is

$$\frac{z}{w}\bigg|_{A_E=0} = (1 - \tau)^{-\frac{1}{1+\alpha}} \left(\frac{1 + \alpha - \tau}{1 + \alpha}\right) - 1. \tag{26}$$

The welfare cost results solely due to tax avoidance through substitution towards leisure. For zero leisure preference, $\alpha = 0$, the welfare cost z/w in equation (25) is given by

$$\frac{z}{w}\bigg|_{\alpha=0} = \tau \left(1 - a_E\right) - r_E = \tau\kappa \left(1 - a_E\right) = \frac{wl_E}{wl}. \tag{27}$$

Without leisure as a way to avoid the income tax τ, the welfare cost z/w exactly equals the value of time spent in the bank production of evasion per unit of labor income (equation 16).[22]

4.3 Stable Revenue with Tax Rate Reduction

The US has decreased tax rates over time while keeping the income tax revenue as a share of GDP stable, as shown in Figure 1. It can be established here how the tax rate can be reduced gradually over time if goods productivity trends up while keeping the revenue share constant. As both gradual tax rate reduction and rising goods productivity are seemingly stylized features of developed countries, the model can be used to derive the rate at which tax rates can be reduced during productivity growth and a stable tax revenue share of output, as a function solely of the elasticity of the demand for reporting income relative to the tax rate.

By equation (6), the total government tax revenue is $\tau a_E wl$, as denoted here by TR. The elasticity of this with respect to the tax rate τ is just 1 plus the elasticity of income η_τ^{TI}; $\eta_\tau^{TR} = 1 + \eta_\tau^{TI}$. This elasticity can of course be positive so that government revenue typically would rise as the tax rate rises.

[21] $\lim_{a_E \longrightarrow 0} \left(\frac{z}{w}\right) = (1 - \tau\kappa)^{-\frac{\alpha}{1+\alpha}} + \tau\kappa - 1.$

[22] This result is similar to a parallel monetary literature in Lucas (2000) and Gillman (2020b) in which the welfare cost of the inflation tax in this case is the cost of bank time used in avoiding the inflation tax through exchange credit.

As a percent of labor income, the tax revenue is simply τa_E. The next two propositions indicate conditions under which this tax revenue share changes as the tax rate changes.

Proposition 1 *As the tax rate τ decreases, the share of government revenue in output, $a_E\tau$, declines for $a_E > \kappa$, and rises if $a_E < \kappa$, forming a hump-shaped relation (Laffer curve).*

Proof. From equation (15), $\frac{\partial(a_E\tau)}{\partial\tau} = (a_E)\left(\frac{a_E-\kappa}{1-\kappa}\right) > 0$ for $a_E > \kappa$ and $\frac{\partial(a_E\tau)}{\partial\tau} < 0$ for $a_E < \kappa$. ■

This Proposition 1 shows the type of decrease in the tax revenue share of output as tax rates go down, for mature economies in which the share of reported income a_E would be expected to exceed the labor share in producing tax evasion, which generally is less than 0.5 (as in a typically convex marginal cost curve per unit of income as in Figure 2b).[23] Newly democratic countries, with low productivity such as the countries within the former Soviet Union block, might find that tax evasion is so pervasive ($a_E < \kappa$) that lowering the income tax rate would raise revenue as a share of output. For the high productive economies with a tradition of voluntary tax reporting then it would be expected that $a_E > \kappa$. In this case, Proposition 1 implies that tax reduction alone is unable to explain a stable share of income tax revenue in GDP except when $\tau = \kappa$.

For $a_E > \kappa$, by additionally allowing for the wage rate w to rise over time in line with the economy's productivity trend, the revenue share may be stable as a result of decreasing tax evasion as time becomes more valuable. During a trend down in tax rates, the rise up in productivity allows for stability of the tax revenue share in a way similar to the experience in Figure 1 above.

Proposition 2 *Given $|\eta_\tau^{a_E}| < 1$, the share of income tax revenue in output remains constant over time if the percentage reduction in the tax rate over time equals the percentage increase in goods productivity factored by a function of the tax elasticity of the share of reported income: Let $\dot{\Delta}$ denote the percentage change over time; then $\dot{\Delta}(a_E\tau) \equiv (100)\frac{\frac{\partial(a_E\tau)}{\partial t}}{a_E\tau} = 0$, if*

$$\dot{\Delta}\tau = -\dot{\Delta}w\frac{|\eta_\tau^{a_E}|}{1-|\eta_\tau^{a_E}|}. \tag{28}$$

[23] See Benk et al. (2005, 2008, 2010) for $\kappa < 0.5$ for inflation tax avoidance.

Proof. Using equations (15) and (19), and assuming A_E and κ are constant over time, then $\frac{\partial(a_E\tau)}{\partial t} = \frac{\partial(\tau)}{\partial t}a_E + \tau\frac{\partial(a_E)}{\partial t} = 0$, implies the equation (28) result that $\dot{\Delta}\tau = -\dot{\Delta}w\left[\frac{\left(\frac{\kappa}{1-\kappa}\right)\frac{1-a_E}{a_E}}{1-\left(\frac{\kappa}{1-\kappa}\right)\frac{1-a_E}{a_E}}\right] = -\dot{\Delta}w\left(\frac{\left|\eta_\tau^{a_E}\right|}{1-\left|\eta_\tau^{a_E}\right|}\right)$. ■

A constant tax share $a_E\tau$ would result for economies with $a_E > \kappa$ and $|\eta_\tau^{a_E}| < 1$ (such that the economy is on the left side of the Laffer curve) while experiencing a trend up in productivity w if the tax rate τ decreases over time (at a decreasing rate since $|\eta_\tau^{a_E}|$ falls if either w rises or τ falls). The reduction in tax rates, $\dot{\Delta}\tau$, would be smaller as w rises. Given a constant goods productivity increase such as by 2%, tax rates would decline over time initially by 2% if for example $\eta_\tau^{a_E} = -0.5$, and then by less that 2% as the elasticity magnitude falls while w increases. This assumes all other parameters are constant.

Consider a simple, approximate, illustration of Proposition 2. Given $\dot{\Delta}w = 0.02$, let $\kappa = 0.36$ as in Gillman and Kejak (2014) and assume the other parameters of equation (15) such that $a_E = 0.75$, as in a high productive, low tax evasion, economy. Then the tax rate level would need to fall by approximately 21% over fifty-five years such as from 1965 to 2020.[24] This is about a two percent reduction in the level of τ every five years; such magnitudes are comparable to findings in Piketty et al. (2018).

The qualifications are numerous. For example, this assumes a flat rate tax and makes difficult comparison to economies with graduated marginal tax brackets. One way would be to consider the average legal tax rate on income after adjusting for income level as a comparison to this analysis (see Piketty et al., 2018). Other complications are how the legal tax expenditures and shelters are taken into account. As qualified, the Proposition 2 allows for a sense in which gradual tax rate reduction over time is a natural consequence of the steady productivity increase that exists in economies undergoing sustained growth.

Other assumptions can also be made. In particular one might consider that the ability to evade taxes through banking is rising (or falling) over time. Consider allowing A_E to change in this sense. If A_E is constant as assumed above, then the implicit assumption is that the technology of catching tax evasion is keeping up with the technology for evading taxes, with a standstill in A_E. In general, let A_E change. In this case equation

[24] Starting at 75% of income reported, assume the steady tax rate reduction would yield 80% of reported income at the end of 60 years. Then a simple average would be 77.5% and $\dot{\Delta}\tau \simeq -2\left(\frac{\frac{0.36}{1-0.36}\frac{1-0.775}{0.775}}{1-\frac{0.36}{1-0.36}\frac{1-0.775}{0.775}}\right) = -0.39$ per year, or about a 21% reduction over 55 years.

(28) becomes instead $\dot{\Delta}\tau = \left(\frac{1}{\kappa}\dot{\Delta}A_E - \dot{\Delta}w\right)\frac{|\eta_\tau^{a_E}|}{1-|\eta_\tau^{a_E}|}$. Given $|\eta_\tau^{a_E}| < 1$, this implies that $\dot{\Delta}\tau < 0$ only if $\frac{1}{\kappa}\dot{\Delta}A_E < \dot{\Delta}w$. A sufficiently large increase in A_E over time would reverse the result and imply a rising tax rate while the share of tax revenue is constant. Quantifying changes in A_E is difficult by nature of the tax evasion being largely a hidden industry. However, on the basis of this model, the observed downward trend in US average income tax rates in the US supports that $\frac{1}{\kappa}\dot{\Delta}A_E < \dot{\Delta}w$.

Further international evidence on the OECD in its "Taxing Wages" study for 2000 to 2019 shows a slightly declining average OECD "tax wedge" from 37.4 to 36.0. This wedge is defined as the ratio of all labor tax rates to both employees and employers relative to the total labor cost to employers. This could be viewed as additional evidence for developed countries towards a falling effective tax rate. It puts the 2019 US labor tax wedge at 30%; for India, the OECD "Taxing Wages in Selected Partner Economies" puts the labor tax wedge at 28%.[25]

5 Application

The following two examples apply the degree of tax evasion to economies with a lower and a higher goods sector productivity. This can be thought of in terms of less developed relative to more developed countries in that evidence finds less goods sectors productivity in less developed countries relative to more developed countries, while the bank sector productivity in tax evasion is less clear. The examples target the degree of tax evasion using evidence from Schneider and Enste (2000) and on the differences in goods sector productivity as used in Lucas (1990) for wage differentials.

5.1 Example 1: Less Productive Economy

Assume a tax rate of $\tau = 30\%$ for both economies with a 15-fold difference in goods sector productivities as in the Lucas (1990) wage differentials. And let the less productive economy have a 40% degree of labor income tax evasion at this tax rate while the more productive economy will have a 25% degree of tax evasion.[26] Letting $\kappa = 0.36$ (as in

[25] Egger et al. (2013) use the same "Taxing Wages" OECD methodology for their study on the probablity of corporate headquarter locations.

[26] For a literature review on estimates of shadow economy sizes internationally, see for example, Schneider and Enste (2000).

Gillman and Kejak 2014), set $A_E = 0.53$, and $w = 0.094$ such that the degree of evasion is [27]

$$1 - a_E = (0.53)^{\frac{1}{1-(0.36)}} \left(\frac{(0.30)\,(0.36)}{(0.094)} \right)^{\frac{(0.36)}{1-(0.36)}} = 0.40. \tag{29}$$

Given $\alpha = 1$, the only other parameter needed, this example implies an income elasticity to the tax rate of -0.465 and a welfare cost of the tax of z/w (equation 25) of 0.0267, or 2.67% of full income ($1 \cdot w$: The time endowment is one).[28] Consider that if zero tax evasion existed in the sense of setting $A_E = 0$, then the income elasticity would be -0.25 and the welfare cost $z/w = 0.016$ or 1.6% of full income, which are underestimates within the framework of the economy.

The economy with 60% of taxes being paid could reflect a less developed country in which the incentive to avoid taxes is relatively high because of lower goods and bank sector productivities than in developed economies. Such less productive economies are found for example in India, Russia and Eastern Europe.

5.2 Example 2: More Productive Economy

Now let w be 15-fold higher so that $w = 1.41$, as in Lucas (1990) for US versus India data. With the tax rate of $\tau = 0.30$, and a 25% degree of tax evasion as in developed economy estimates, the bank productivity is $A_E = 1.038$, which is about double the bank productivity in the less productive economy. This means the example has higher productivity in both goods and bank sectors while targeting the 25% evasion rate and the 15-fold difference in w :

$$1 - a_E = (1.038)^{\frac{1}{1-(0.36)}} \left(\frac{(0.30)\,(0.36)}{(1.41)} \right)^{\frac{(0.36)}{1-(0.36)}} = 0.25. \tag{30}$$

Again with $\alpha = 1$, the income elasticity and the welfare cost here are -0.333 and 0.022, respectively; the welfare cost is 2.2% of full income. These are comparatively lower in the more productive economy because the higher goods productivity induces less tax evasion. There is a lower waste of resources from a social planner point of view.

[27] For a lower tax rate of 20%, then increasing A_E by 15% to 0.612 would also yield a 40% evasion rate.

[28] Using the Appendix solution for output which equals consumption when $z = 0$ (see equation 48), $y = c = 0.040$, and that $w/y = \frac{(0.094)}{0.040}$, then $z/y = 0.063$.

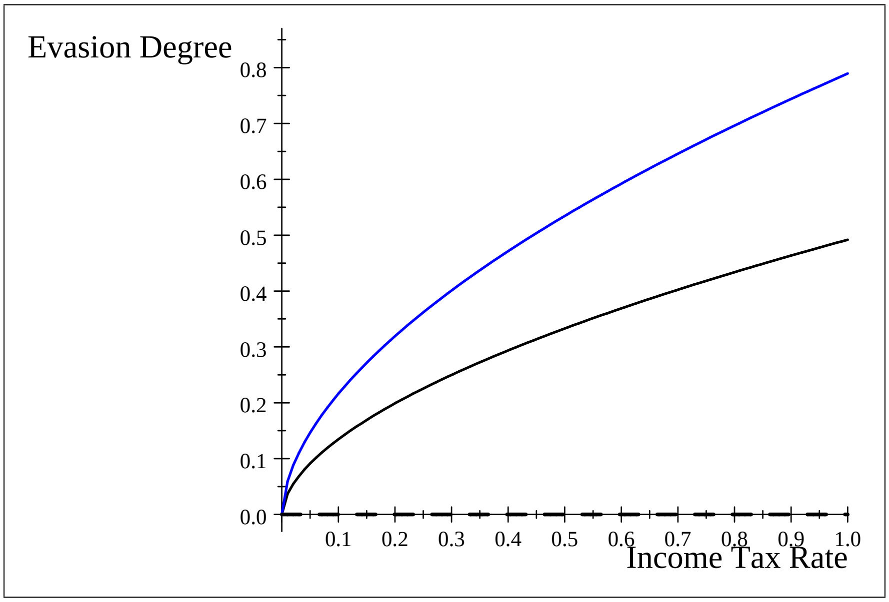

Figure 3: Degree of Tax Evasion, $1 - a_E$, with varying Tax rate τ.

Economies with 75% in reported income as in equation (30) are more in line with developed countries such as in Western Europe, the UK and the US. The examples illustrate how using the wage differentials from Lucas and estimates of tax evasion imply within this framework both more productive goods and banking sectors in the developed country. This could be viewed as a reasonable way to illustrate the more productive economy.[29]

Figure 3 graphs the degree of tax evasion, $1 - a_E$, for the less productive economy (blue) and the more productive economy (black) against the tax rate τ. The evasion rises at a decreasing rate as the marginal cost of evasion through the bank rises and as the substitution from goods towards leisure induces greater marginal utility loss from goods reduction and lesser marginal utility gains from leisure increases. The level of evasion is higher at every tax rate for the less productive economy. Without evasion, the dashed line at zero along the horizontal axis shows the alternative with $A_E = 0$.

Finally, for the Examples 1 and 2 above, Figure 4 graphs revenue as a share of output, $a_E \tau$. This revenue initially rises with the tax rate for both the more productive economy (solid curve) and the less productive economy (dashed curve). For the more productive

[29] I am grateful for this suggestion by an anonymous referee.

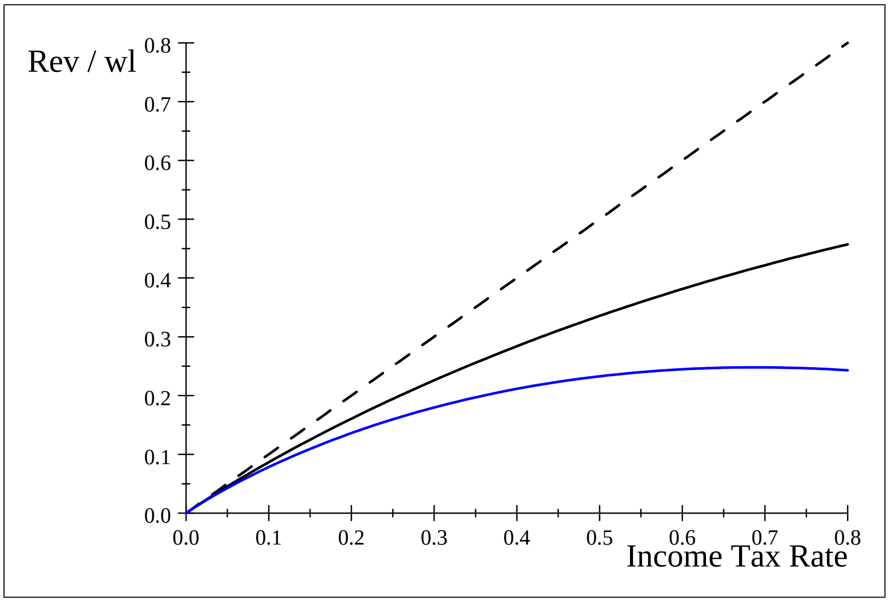

Figure 4: Revenue Share $\alpha_E \tau$ Increases with w, and with τ until Peak.

case with a reported income fraction of $a_E = 0.75$ and an average tax rate of $\tau = 0.3$, the tax revenue to labor income ratio is $(0.75)(0.3) = 0.225$, or 22.5% of labor income. For the less productive case with $a_E = 0.60$ at $\tau = 0.3$, the tax revenue percent of labor income is lower at 18%.

Figure 4 shows how the tax revenue share is higher for the more productive economy than the less productive one. It also illustrates a "Laffer curve" for the less productive economy, with the revenue ratio initially rising as the tax rate falls from very high levels and then declining the rest of the way. Before the peak in the Laffer curve, the graph shows that the less productive economy has a slower rate of increase in tax revenue as the tax rate rises.

6 Sensitivity

There are three main aspects that determine the magnitude of the tax elasticity and the welfare cost of the income tax. First is the level of the tax itself. Second is the preference for leisure as given by the parameter α. Third, is the degree of tax evasion. For the latter, given constant structural parameters in the production of the intermediation service of evasion, in particular κ and A_E, the goods sector productivity level w along with the tax rate τ level determine the degree of evasion.

6.1 Leisure Preference

To see the sensitivity of the tax elasticity and the welfare cost first with respect to the leisure preference α, consider how this α affects the undistorted, first-best equilibrium leisure solution. Using Appendix equation (44), the first-best leisure is given simply by $x = \frac{\alpha}{1+\alpha}$. Consider three cases, each of which have been used in calibrations in the literature. With $\alpha = 0.5$, $x = \frac{1}{3}$ and undistorted labor is the other two-thirds of time; at $\alpha = 1$, $x = \frac{1}{2}$ and undistorted labor is the other one-half; and at $\alpha = 2$, $x = \frac{2}{3}$ and undistorted labor is the other one-third. Each calibration has justification in the macroeconomics literature; for example, Gomme and Rupert (2007) focus on a standard dynamic macroeconomic model calibration and target leisure near one-half.

As the leisure preference α rises, leisure is more attractive and the tax elasticity magnitude of taxable income rises both with and without evasion. With tax evasion ($A_E > 0$) at each α the tax elasticity is of greater magnitude (more negative) than when intermediation productivity is zero ($A_E = 0$) and $a_E = 1$, up to a moderately high level of τ.

In particular consider two sets of cases using the higher productivity Example 2 economy in which $w = 1.41$ and $A_E = 1.038$. i). With tax evasion: Given $\tau = 0.3$, with $\alpha = 0.5$ then $\eta_\tau^{TI} = -0.29$; with $\alpha = 1.0$ then $\eta_\tau^{TI} = -0.33$; and with $\alpha = 2.0$ then $\eta_\tau^{TI} = -0.37$. ii). With zero tax evasion ($A_E = 0$) : Given $\tau = 0.3$, with $\alpha = 0.5$ then $\eta_\tau^{TI}\big|_{A_E=0} = -0.18$; with $\alpha = 1.0$ then $\eta_\tau^{TI}\big|_{A_E=0} = -0.25$; and with $\alpha = 2.0$ then $\eta_\tau^{TI}\big|_{A_E=0} = -0.32$. Figure 5 graphs the magnitude of the tax elasticity η_τ^{TI} in equation (21) as the tax rate τ varies for the cases of $\alpha = 0.5$ (red) $\alpha = 1.0$ (black) and $\alpha = 2.0$ (green), both with tax evasion (solid lines) and without tax evasion (dashed lines with $A_E = 0$).

Figure 5 shows four basic concepts: 1) the tax elasticity rises with the tax rate. 2) With evasion, as the tax rate rises, this elasticity rises initially at an increasing rate and then a nearly linearly rate. 3) This elasticity is mildly sensitive to the leisure preference α, with increases in α associated with a higher elasticity magnitude for η_τ^{TI} at all τ; this

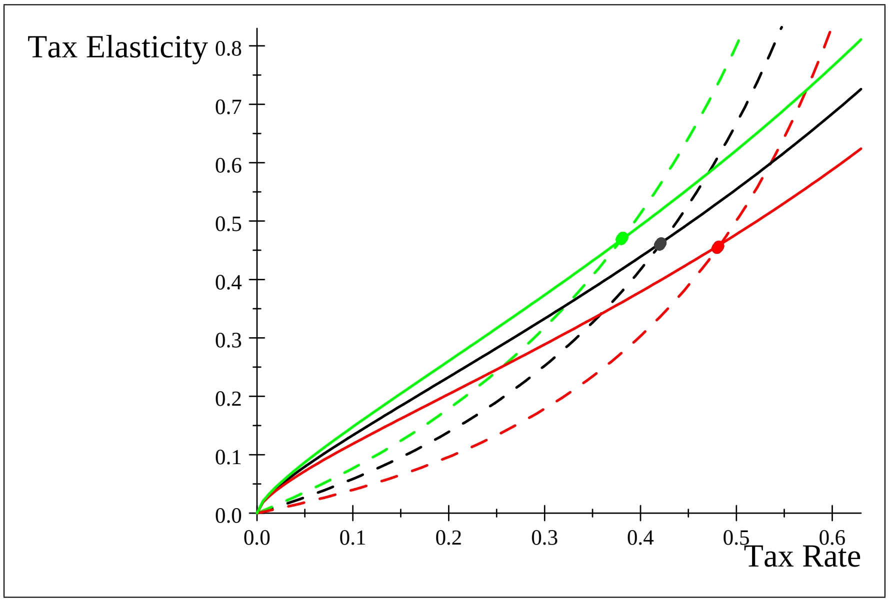

Figure 5: Elasticity: η_{τ}^{TI} : $A_E > 0$ (Solid) $A_E = 0$ (Dashed).

results since goods to leisure substitution is higher when leisure is more preferred. 4) The elasticity is higher when there exists tax evasion up to a certain moderately high threshold tax rate, with this threshold rising with decreases in α.

Figure 5 shows the tax threshold points referenced above. For any given α, these are the points at which the tax elasticity is equal for when $A_E(\alpha) = 0$ and $A_E(\alpha) > 0$. These green, black and red points occur at tax rates rising from about 0.38 to 0.42 to 0.48, respectively, as α falls from 2.0 (green) to 1.0 (black) to 0.5 (red). Below these thresholds, the tax elasticity is greater in magnitude with tax evasion than without it.

Consider also the sensitivity of the welfare cost estimate with respect to leisure preference. Figure 6 graphs the welfare cost of the income tax rate, z/w of equation (25), for similar sets of cases to those in Figure 5. The solid and dashed curves show z/w with evasion and without evasion ($A_E = 0$) and with α rising from 0.5 (red) to 1.0 (black) and to 2.0 (green). With evasion, a lower leisure preference causes a higher welfare cost even though it causes a lower magnitude of tax elasticity in Figure 5. With greater preference for leisure there is more goods to leisure substitution. Since a_E is unchanged by α, leaving the same degree of evasion, the only result as α increases is that leisure increases, goods production decreases, less income is produced, and so the total amount of evaded income and its cost is lower.

As α approaches zero, evasion occurs only through changes in a_E. The case with $\alpha = 0$ with evasion is graphed by the grey solid line. Here the welfare cost reduces to $\tau\kappa\,(1 - a_E)$ by equation (16). All evasion/avoidance activity is through real resource use. The result is that the welfare cost is higher than all other cases of α up to a threshold tax rate depending on the assumed $\alpha > 0$ for comparison. For the case of $\alpha = 0.5$ with evasion (red solid), the welfare cost with $\alpha = 0$ is higher up to a tax rate of about 0.42. This threshold is much higher for $\alpha = 1$. Optimal evasion/avoidance here ($\alpha = 0$) is highest when forced only through the degree of income reported since it cuts out the ability to balance that cost with avoidance through use of more leisure. Put differently, assuming zero leisure in the model implies an overstatement of the welfare cost if in fact leisure is taken by the agent.

In contrast, when there is zero evasion (dashed lines), Figure 6 shows that the ordering of welfare cost by α diverges from the positive relation to interest elasticity in Figure 5 and the negative relation to welfare cost with evasion. In this case without evasion, the welfare cost (which equals $(1-\tau)^{-\frac{1}{1+\alpha}}\left(\frac{1+\alpha-\tau}{1+\alpha}\right) - 1$ from equation 26) sees a rise as α rises from zero and then a decrease as α rises above a level close to 1. These results leave the $\alpha = 0.5$ case (red dashed) with a higher welfare cost than the $\alpha = 2.0$ case (green dashed line). Together with the results with evasion, Figure 5 shows how the Ramsey positive connection between the magnitude of the price elasticity (where here the price of reported income is the tax rate τ) and the welfare cost is fragile without evasion with respect to leisure preference α and is broken with evasion with respect to α.

Finally, consider that for the evasion case, the welfare cost rises with the tax rate at an increasing rate initially and then more linearly as in the tax elasticity graph. The welfare cost is higher with tax evasion and avoidance than with only tax avoidance through leisure up to the thresholds marked by the green, black and red points, which are 0.38, 0.43, and 0.52 respectively. This shows a rising threshold tax rate as leisure preference is lower, as is consistent with the tax elasticity Figure 5.

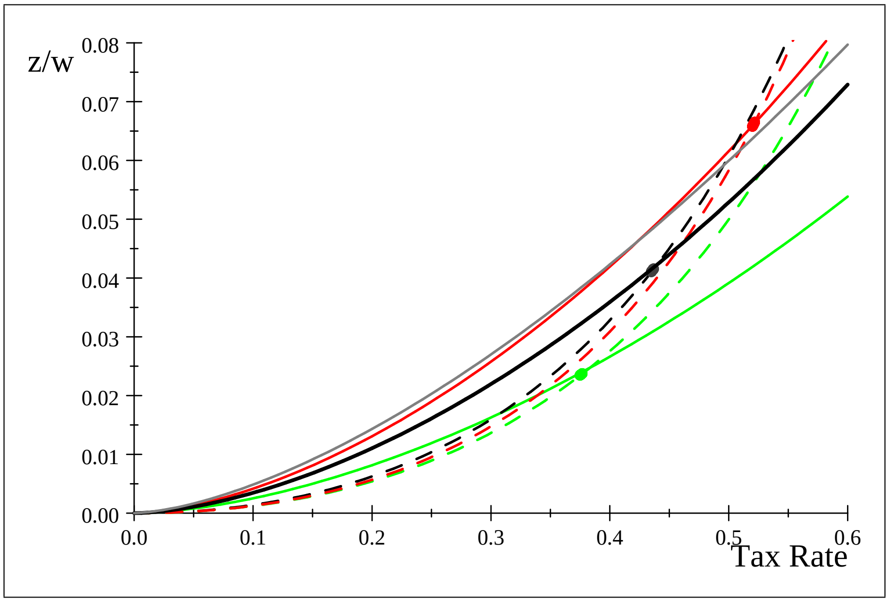

Figure 6: Welfare Cost z/w : $A_E > 0$ (Solid) $A_E = 0$ (Dashed).

6.2 Goods Sector Productivity

Leisure preference has zero effect on the degree of tax evasion in terms of the share of income reported, a_E, or on the tax revenue as a share of output $a_E\tau$. In contrast, the productivity level of the economy w affects a_E as well as the tax elasticity, welfare cost and the share of tax revenue in total output.

For the examples in Section 5, let all of the parameters be the same except for goods productivity, which will be 0.094 and 1.41 for the less and more productive economies, respectively. Here set the bank productivity to the 0.53 level of Example 1 for both economies. This means that the Example 2 economy experiences only this difference in w with a lower degree of evasion resulting at 0.07 instead of 0.25 for $\tau = 0.3$, since $A_E = 0.053$ instead of 1.038.

Figure 7a and Figure 7b illustrate the tax elasticity and welfare cost of the tax rate for each the less productive (blue) and more productive (black) economies both with tax evasion (solid curves) and without tax evasion ($A_E = 0$, dashed line). The case without evasion is the same for both economies in both figures for each the elasticity and welfare cost since these both depend only upon the tax rate and leisure preference ($\alpha = 1$ here), as seen in equations (22) and (26) respectively. In Figure 7a, the less productive economy has a higher tax elasticity of income, in magnitude, than the more productive economy; both are higher in magnitude than the zero evasion case (dashed) up to a threshold tax level as given by the intersecting points, which are at 0.35 and 0.54 for the more and less productive economies, respectively.

Figure 7b shows that the welfare cost for the less productive economy (blue) is higher than for the more productive economy (black solid) while rising at a slower rate as τ increases. With evasion the welfare cost is higher up to a threshold rate than for zero evasion (dashed line) for both economies. This threshold is 0.39 for the less productive economy and 0.48 for the more productive economy. These thresholds are where the welfare cost with evasion intersects that with zero evasion.

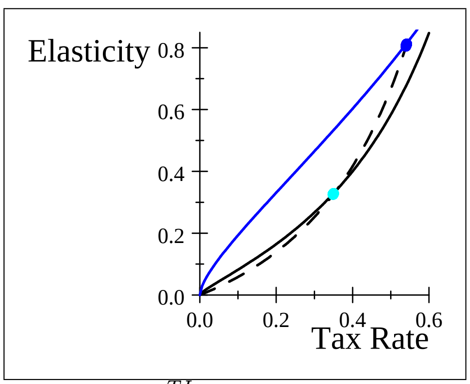

Fig. 7a. η_{τ}^{TI} : High vs. Low w

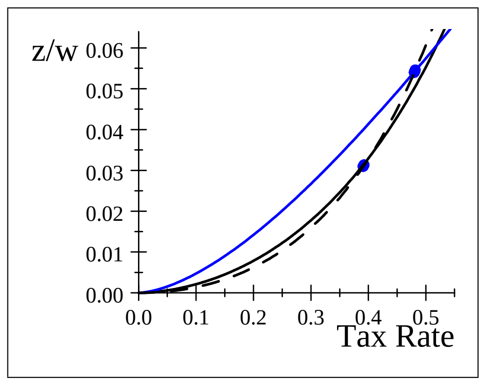

Fig. 7b. $\frac{z}{w}$: High vs. Low w

7 Discussion

Countries such as India have seen a trend up in their labor income revenue as a share of taxes, with a coinciding rising in goods sector productivity. The World Bank database shows that India has seen its income tax revenue as a share of total revenue trend upwards with a doubling from 21% in 1974 to 44% in 2017,[30] as noted above in Section 3, while India's personal income tax rate for the highest bracket has seen a modest rise from 30% in 2004 to 36% in 2020 (Krishna et al., 2018). KLEMS data shows India's total factor productivity rising by nearly 50% from 2000-2015, with labor productivity rising

[30] Note that this includes labor income, profit and capital gains which is broader than the concept of the paper's model which can be extended to include capital and the capital income tax with similar results.

by 150% in the same period.[31] Given the evasion analysis of the paper, the productivity increase would allow the government to rely more on this form of taxation for revenue. Less tax evasion would occur and more labor income revenue as a share of total labor income would result.

The form of the tax is a separate but related issue. Evidence shows that less developed countries rely significantly more on sales taxes, which can be regressive but relatively easy to collect. As Pirttilä and Tarp (2019, p. 968) emphasize: "This means that public finance solutions that work well in developed countries are not necessarily suitable for developing economies"... "tax systems in developing countries need to be designed with the small formal sector in mind." As countries develop and productivity rises and evasion declines, the paper suggests how governments might be able to rely more on income taxes for revenue, and less on sales taxes, as has been seen in India.

Using the implications of this paper, the current global trend towards flatter rates on income taxes may result in part from rising output productivity such that the formal sector increases, less tax evasion occurs, and the welfare cost of taxation is lower. These consequences would make attractive such policy.[32] There is a significant literature examining how flat tax reform has progressed.[33]

In the deterministic framework presented above without uncertainty the tax distortion affects the intratemporal marginal rate of substitution between goods and leisure, and the degree of tax avoidance, while leaving intertemporal margins undistorted. Adding uncertainty through stochastic shocks such as to the goods and bank sector productivities, and extending the model to include physical capital, would extend the results by cre-

[31] "It is compiled from National Accounts Statistics (NAS), published annually by the Central Statistical Organization, Government of India and supplemented by Input-Output tables, Annual Survey of Industries & National Sample Survey Organizations (NSSO) surveys on employment & unemployment." according to Krisha et al. (2018)." See: https://scholar.harvard.edu/files/jorgenson/files/pl09a_india_worldklems2018_dkd.pdf?m=1528208436

[32] "Over 20 countries in the world, including five central and eastern European Member States and seven EU neighbouring countries, have introduced a so-called "flat tax" (initially the three Baltic countries in 1994-1995, followed since 2001 by a second wave of countries including Russia, Serbia, Ukraine, Slovakia, Georgia, Romania, the former Yugoslav Republic of Macedonia, Montenegro and Albania – see table). The "flat tax" concept is usually associated with the academic works of Hall and Rabushka (1983, 1985) who consider a single tax rate applied to both personal and corporate income beyond a given threshold or "basic allowance" (ECB Monthly Bulletin, September 2007, p. 81).

[33] Gorodnichenko et al. (2009), for example, examine flat tax reform, tax evasion and welfare in Russia; Azacis and Gillman (2010) show welfare effects of flat tax reform in the Baltics, including transition dynamics; and Holter et al. (2019), and Alstadsæter et al. (2019a) address Laffer curves, flat and progressive taxes, and inequality.

ating an intertemporal wedge due to expectations of the degree of tax evasion/avoidance activity that were to take place in the next period. This wedge could help explain certain dynamics through the expected adjustment to the future shadow cost of consumption created by the need to finance government expenditure through taxation, which becomes important once uncertainty over government spending and finance becomes a factor as it does especially during crisis periods. An example of such a tax avoidance wedge explaining asset prices through departures from fundamentals is found through stochastic expectations of inflation tax avoidance for example in Csabafi et al. (2020).

Intertemporal wedges from distortions that cause departures from fundamentals have been shown to be essential in understanding the effects of government spending and fiscal as well as monetary policy. For example, Gali et al. (2007) show how departures from fundamentals affect intertemporal consumption through the "Euler" equation as a result of rule-of-thumb credit-constrained consumers. Gali et al. show that credit-constrained consumers can benefit from government spending because it reduces the wedge between the marginal rate of substitution in consumption and the marginal product of labor which in turn also affects the intertemporal Euler equation. Through this intertemporal wedge they show how aggregate consumption can rise with increased government expenditure, depending upon the tax structure used to finance the spending. The consumption rises by more to the degree that the tax increases are more associated with an increase in debt rather than with an increase in current spending.[34] Since the tax rate is part of the wedge between consumption and labor in Gali et al., and since evasion in this paper reduces the effective tax rate as in equation (10) above, then including evasion in a Gali et al. framework could amplify the decrease in their wedge that causes a higher spending multiplier; this effect may be further enhanced if credit-constrained consumers also have lower labor productivity and hence more tax evasion.

The effect on revenue from tax reduction also brings interest to estimating Laffer curves. Without introducing tax evasion, this literature typically makes use of the total taxable income and can produce Laffer curves in a variety of ways (see for example Trabandt and Uhlig 2011). In this paper, the alternative of using the tax revenue share of output without evasion is trivial since then $a_E = 1$ and $a_E\tau = \tau$ as in Figure 4. With

[34] Gali et al. (2007) qualify their results by finding that the financing by tax increases needs to be more associated with an increase in debt than with an increase in current spending.

tax evasion, a focus on income tax revenue as a share of output can be examined and extended to flat tax reform also within endogenous growth models so as to pick up how the growth rate can rise from such reform.[35]

For the US, tax rate reform has occurred through the 1981, 1984, and 1986 Tax Acts, along with the more recent 2001 and 2017 Tax Acts. These have been viewed as having the effect of expanding the tax base, flattening the income tax rates, and lowering tax rates.[36] While the headline tax acts of 1981, 1986 and 2017 are well-known, consider for example perhaps the least known of these, the Tax Reform Act of 1984.[37] This act ended a decade of Congressional moratoriums on new IRS regulations on how to define the taxability of non-statutory fringe benefits by codifying through law how the taxable income should be delineated for a broad set of categories. It reversed the direction of defining the tax base in continually narrower terms through newly devised fringe benefits; this reversal helped to allow further reform as a result of better establishing a broader income tax base.

The Tax Reform Act of 1986 limited tax deductions, such as repealing the 10% investment tax credit in order to use the revenue so gained to lower the corporate tax rate, which fell from 46% to 33% at the time.[38] Kopczuk (2005, p. 2112) concludes that "Focusing on broad income, I demonstrated that its elasticity is a non-trivial function of tax policy on estimates for the 1986 Act." Further (p. 2115): "This analysis suggests that taxable income is indeed sensitive to the tax base changes." These types of results are modeled in the paper above as the tax base expands when evasion decreases, and with the tax elasticity falling in magnitude with tax rate reductions. Recently, the US 2017 Tax Cuts and Jobs Act limited other "tax expenditures" that narrow the tax base while lowering effective personal income tax rates and bringing the corporate tax rate down to a more internationally competitive 21%.

[35] See Stokey and Rebelo (1995), Turnovsky (2000) and Azacis and Gillman (2010) on flat tax change with endogenous growth; Auerback (2002), for example, discusses the "dynamic scoring" of the 2001 US Tax Act that considers changes in the economy's growth rate.

[36] Slemrod (2018) discusses features of the 2017 Act that broaden the tax base on the basis of some long-standing, well-known, issues leftover from previous tax reforms.

[37] Piketty et al. (2018, p. 600) analyze many US tax acts and note in contrast, for example, that "The 2013 tax reform has partly reverted the long-run decline in top tax rates."

[38] Eliminating the 10% investment tax credit was estimated by the US Congressional Joint Tax Committee to be able to raise revenue sufficient to pay for a seven percent decrease in the corporate tax rate from 46% to 39%; the elimination of this credit became part of the Regan Treasury proposal and the final Act; see McClure and Zodrow (1987) for other details.

The policy of tax reform has lessened tax evasion by giving less incentive for evasion. Devising tax law can take into account how tax reform will affect the economy, as Kopczuk (2005) emphasizes. Certainly, tax revenue may fall initially after a tax rate reduction.[39] Evidence on the US postwar share of income tax in GDP being stable, combined with the Proposition 2 that entertains an upwards productivity trend, however provides evidence and theory on how tax rate reduction combined with productivity growth may leave the share of revenue in GDP constant. If the government spending, which rises with economic growth and which tax revenue finances, can deliver a social insurance system (broadly taken here) that becomes increasingly efficient in providing different income classes with better incentives and welfare, then the advantage of rising productivity allows both for rising welfare from less tax distortion as well as rising welfare from better expenditure.

Tax avoidance/evasion will occur as long as people act in terms of optimally setting the marginal cost of paying the tax equal to the expected marginal benefit of avoiding/evading the tax. It may be that the rising aggregate productivity trend allows tax rate reduction that gives less incentive for distortionary behavior. This paper has abstracted from uncertainty and presented a perfect foresight deterministic model of tax evasion where, in the end, there is an amount, on average, of productivity in the sector enabling the tax avoidance/evasion. This can be through accounting, lawyers, banks, or a variety of institutions designed to enable minimum tax payments, either as allowed under statutory law or as is prohibited. Qualified as being a flat tax approach without income distribution considerations, the model presented offers a representative agent macroeconomic approach to present optimal tax evasion through the sector that we think of broadly as providing it: Financial and/or other intermediation that induces the optimal use of productive resources to avoid/evade taxes in a competitive setting. It remains to extend the analysis to include comparison amongst other taxes, to allow for dynamic physical capital accumulation, and to include uncertainty.

To step back in perspective, consider that Lucas and Stokey (1983) make mainstream that inflation and fiscal taxes are the similar qualitatively. They show how tax smoothing

[39] Holter et al. (2019), for example, find that income tax rate reduction lowers tax revenue depending in degree on the nature of progressivity of the code; Trabandt and Uhlig (2011) estimate US and European Laffer curves and calculate the percent of labor and capital income tax cuts that are self-financing.

is optimal for both fiscal and inflation taxes. Debt borrowing in times of crisis is seen as optimal with the tax collection then smoothed over time. Debt repayment has even better prospects if the tax rates can decline through less tax evasion as civil society allows greater human capital accumulation, technological advance and rising productivity. In addition, fiscal spending multipliers can be affected by the debt/tax mix and by evasion, a topic for further research.

Money demand behavior and its elasticity to the interest rate represent optimal inflation tax avoidance and the inflation tax elasticity of the demand for using government supplied money. Just as monetary policy takes into account money demand as based on inflation tax avoidance, optimal fiscal policy would seem to need to take into account the demand for reporting income as based on income tax avoidance/evasion and the elasticity of reported income with respect to the tax rate. This can be applied across the international spectrum of countries with differences in aggregate productivity and in the ability to evade taxes. Design of fiscal tax law can incorporate more fully how households respond to tax law change through the both the formal industries and less formal channels set up to supply the evasion means.

8 Conclusion

The paper presents a model of tax evasion based on a microeconomic, financial intermediation, technology that is based in the banking literature and that produces the evasion service. With a tax on labor income, the analysis studies the elasticity of reported income with respect to the tax as well as the welfare cost of the tax. It is shown how tax evasion increases the tax elasticity of reported income and increases the welfare cost of such taxation. Examples of tax elasticities and welfare cost estimates are provided for less and more productive economies. A stable share of income tax revenue in output while goods productivity trends higher is shown to imply a certain trend downwards in the tax rate. This gives a perspective on the direction in which tax law may evolve and how this may affect its welfare cost internationally.

Appendix: Equilibrium Conditions and Solution

The consumer problem is

$$\begin{aligned} &\underset{c_t,x_t,a_{Et},d_{Et}}{Max} \ln c_t + \alpha \ln x_t \\ &+\lambda_t \left[a_{Et} w (1-x_t)(1-\tau) + (1-a_{Et}) w (1-x_t)(1-p_{Et}) + r_{Et} d_{Et} + \Gamma_t + z - c_t\right] \\ &+\varphi_t \left[w(1-x_t) - d_{Et}\right], \end{aligned} \tag{31}$$

with equilibrium conditions when the Lagrangian multipliers are binding as follows:

$$c_t : \frac{1}{c_t} - \lambda_t = 0; \tag{32}$$

$$x_t : \frac{\alpha}{x_t} - \lambda_t \left[a_E w (1-\tau) + (1-a_{Et}) w (1-p_{Et})\right] - \varphi_t w = 0; \tag{33}$$

$$a_E : \lambda_t \left[(1-\tau) - (1-p_{Et})\right] = 0; \tag{34}$$

$$d_{Et} : \lambda_t r_{Et} - \varphi_t = 0; \tag{35}$$

$$\lambda_t : a_{Et} w (1-x_t)(1-\tau) + (1-a_{Et}) w (1-x_t)(1-p_{Et}) + r_{Et} d_{Et} + \Gamma_t + z - c_t = 0; \tag{36}$$

$$\varphi_t : w(1-x_t) - d_{Et} = 0. \tag{37}$$

Only for the case when $\kappa = 1$ would there lack a unique and well-defined equilibrium with tax evasion. Given $\kappa \in [0,1)$, and $A_E \in R_+$, then $a_{Et} \in (0,1]$.

The bank problem with the production function for q_{Et} substituted in from equation (7) and equilibrium conditions are given by

$$\max_{l_{Et},d_{Et}} \Pi_{Et} = p_{Et} A_E (l_{Et})^{\kappa} (d_{Et})^{1-\kappa} - w_t l_{Et} - r_{Et} d_{Et}; \tag{38}$$

$$l_{Et:} : w = \kappa p_{Et} A_E \left(\frac{l_{Et}}{d_{Et}}\right)^{(\kappa-1)}; \tag{39}$$

$$d_{Et} : r_{Et} = (1-\kappa) p_{Et} A_E \left(\frac{l_{Et}}{d_{Et}}\right)^{\kappa}. \tag{40}$$

In equilibrium the time subscripts can be dropped. Equations (34) and (40) imply that $\tau = p_E$ and $r_E = (1-\kappa)\tau A_E \left(\frac{l_E}{d_E}\right)^{\kappa}$, such that equation (13) results above with $r_E = \tau(1-\kappa)(1-a_E)$. Substituting in that $\Gamma_t = a_E \tau w l$, that $d_E = wl$ and that $\tau = p_E$, the budget constraint of equation (36) writes as

$$c = wl(1-\tau) + r_E wl + a_E \tau wl + z = wl\left[1 - \tau(1-a_E) + r_E\right] + z. \tag{41}$$

The second main equation is the marginal rate of substitution between goods and leisure, which from equation (10), or the first-order conditions above, allows solving for leisure in terms of consumption as

$$x=\frac{\alpha c}{w\left(1-\tau+r_E\right)}. \tag{42}$$

We have the solutions for $r_E=\tau\left(1-\kappa\right)\left(1-a_E\right)$ and $a_E=1-A_E\left(\frac{p_E\kappa A_E}{w}\right)^{\frac{\kappa}{1-\kappa}}$ from the bank equilibrium equations (13) and (15) above, as well as from the Appendix conditions 38, 39 and 40. Given the r_E and a_E solutions, equations 41 and 42 are in two variables with two unknowns such that both c and x may be solved:

$$\begin{aligned}
c &= wl\left[1-\tau\left(1-a_{Et}\right)+r_E\right]+z=w\left(1-x\right)\left[1-\tau\left(1-a_E\right)+r_E\right]+z;\\
c &= w\left[1-\frac{\alpha c}{w\left(1-\tau+r_E\right)}\right]\left[1-\tau\left(1-a_E\right)+r_E\right]+z;\\
c &= \frac{w\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{1+\alpha\frac{\left[1-\tau\left(1-a_E\right)+r_E\right]}{\left(1-\tau+r_E\right)}}=\frac{w\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{1+\alpha\left(1+\frac{a_E\tau}{\left(1-\tau+r_E\right)}\right)}.
\end{aligned} \tag{43}$$

For leisure in turn the solution is

$$\begin{aligned}
x &= \frac{\alpha c}{w\left(1-\tau+r_E\right)}=\frac{\alpha w\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{w\left(1-\tau+r_E\right)\left[1+\alpha\left(1+\frac{a_E\tau}{\left(1-\tau+r_E\right)}\right)\right]};\\
&= \left(\frac{\alpha}{1+\alpha}\right)\frac{\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{1-\tau+r_E+\left(\frac{\alpha}{1+\alpha}\right)\tau a_E}.
\end{aligned} \tag{44}$$

The leisure solution gives the solution for labor using the time constraint of $l=1-x:$

$$l=1-\frac{\alpha\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{\left(1-\tau+r_E\right)\left\{1+\alpha\frac{\left[1-\tau\left(1-a_E\right)+r_E\right]}{\left(1-\tau+r_E\right)}\right\}}=1-\left(\frac{\alpha}{1+\alpha}\right)\frac{\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{1-\tau+r_E+\left(\frac{\alpha}{1+\alpha}\right)\tau a_E}. \tag{45}$$

The solution for time in banking l_E in turn comes from the solution for l_E/d_E in equation (12), or from the Appendix bank conditions. Given that $d_E=wl$ and that l is solved, l_E is found as

$$l_E=\tau\kappa\left(1-a_E\right)l=\tau\kappa\left(1-a_E\right)\left(1-\left(\frac{\alpha}{1+\alpha}\right)\frac{\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{1-\tau+r_E+\frac{\alpha\tau a_E}{1+\alpha}}\right). \tag{46}$$

Goods output is given by $y=wl_G$. Since $l_G=1-x-l_E$, and from equation (16) that

$w\left(\frac{\tau\kappa A_E}{w}\right)^{\frac{1}{1-\kappa}} = \tau\kappa\left(1-a_E\right)$, goods output follows as

$$\begin{aligned} y &= w\left[1-\left(1-\frac{\alpha}{1+\alpha}\frac{\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{1-\tau+r_E+\frac{\alpha\tau a_E}{1+\alpha}}\right)\right] \\ &\quad -w\left[\tau\kappa\left(1-a_E\right)\left(1-\frac{\alpha}{1+\alpha}\frac{\left[1-\tau\left(1-a_E\right)+r_E+\frac{z}{w}\right]}{1-\tau+r_E+\frac{\alpha\tau a_E}{1+\alpha}}\right)\right] \\ &= \frac{w\left(1-\tau+r_E-\alpha\frac{z}{w}\right)}{1+\alpha-\frac{a_E\tau}{\left[1-\tau(1-a_E)+r_E\right]}}. \end{aligned} \tag{47}$$

The final part of the equilibrium is to confirm that the social resource constraint is respected by which $y+z=c$. Substituting in the above solution for y and adding z, and rewriting, it results that

$$y+z = c; \tag{48}$$

$$\frac{w\left(1-\tau+r_E-\alpha\frac{z}{w}\right)}{1+\alpha-\frac{a_E\tau}{\left[1-\tau(1-a_E)+r_E\right]}}+z = \frac{w\left[1-\tau\left(1-a_E\right)+r_E\right]+z}{1+\frac{\alpha\left[1-\tau(1-a_E)+r_E\right]}{\left[1-\tau+r_E\right]}} = \frac{w\left[1-\tau\left(1-a_E\right)+r_E\right]+z}{1+\alpha\left(1+\frac{\tau a_E}{1-\tau+r_E}\right)}.$$

Consider output and consumption with $z=0$, and note that with $r_E=\tau\left(1-\kappa\right)\left(1-a_E\right)$

$$1-\tau\left(1-a_E\right)+r_E = 1-\tau\kappa\left(1-a_E\right); \tag{49}$$

$$1-\tau+r_E = 1-\tau\left[a_E+\kappa\left(1-a_E\right)\right]. \tag{50}$$

Output and consumption equal the permanent income of the (Beckerian, 1965) full value of time $w\cdot 1$ minus the value of time used up in evasion through bank labor per unit of income wl, which is $w\tau\kappa\left(1-a_E\right)$, as factored by $\frac{1}{1+\alpha\left(1+\frac{\tau a_E}{1-\tau+r_E}\right)}$, which is the fraction of permanent income consumed. The latter fraction rises as τ rises from zero. The tax τ causes less permanent income $w\left[1-\tau\kappa\left(1-a_E\right)\right]$ and effectively more leisure preference such that a higher fraction of a smaller permanent income is consumed. This is the effect of the tax distortion with tax evasion. Without tax evasion ($a_E=1$), as τ increases, permanent income falls by more and the fraction of permanent income consumed rises by more such that output and consumption fall by more. The lesser distortion with tax evasion results despite that fact that tax evasion wastes resources as a fraction of income equal to the value of bank time per unit of income, or $\frac{wl_E}{wl}=\tau\kappa\left(1-a_E\right)$.

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